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# Mixed Convection over an Isothermal vertical plate Embedded in a Porous medium with Variable Viscosity and Heat Generation with Nanofluid

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**Abstract:** *The effects of nanofluid Prandtl number, variable viscosity and heat generation of mixed convection adjacent to an isothermal vertical plate which is embedded in a porous medium is presented in this paper. The partial differential equations governing the problem transformed into ordinary differential equations with similarity transformation is used to reduced to nondimensional form are solved numerically subject to appropriate boundary conditions by the use of MATLAB code. The flow and heat transfer quantities of similarity equations are found to be the functions of different parameters such as Prandtl number, viscosity and heat generation parameter mixed convection parameter. The nondimensional boundary layer velocity and temperature, Skin friction, heat transfer coefficient, velocity and temperature fields are studied and discussed with the help of a table and graphs with reasonable conclusions.*

**Keywords:** *Mixed Convection, Porous medium, Darcy model, Heat generation, Variable viscosity, oxide nanofluid, Prandtl number*

## I. INTRODUCTION

A lot of engineering process is concerned in the transfer of thermal energy. In the many industries heat must be added, removed, or stimulated from one method stream to another. These processes acquire a source for energy heating and process fluid heating or cooling. The enhancement of heating or cooling in industrial applications may build a reduction in energy, slash process time, progress thermal rating and lengthen the working life of equipment. A few processes are even affected qualitatively by the action of enhanced heat transfer. The effective of high performance thermal systems for heat transfer augmentation has become accepted nowadays of work has been performed to gain an accepting of the heat transfer performance for their useful significance to heat transfer enhancement. Thus the initiation of high heat flow processes has formed significant demand for innovative technologies to improve heat transfer. There are a number of methods to improve the heat transfer effectiveness. The different methods to improve the heat transfer are the extended surfaces, application of vibration to the surfaces and adopting the micro channels. The heat transfer coefficient can be also enhanced by increasing the thermal conductivity of the working fluid. Water, ethylene glycol, and engine oil are the generally used heat transfer fluid, but these fluids have relatively lower thermal conductivity when they are compared with solids. High thermal conductivity of solids can be used to enhance the thermal conductivity of a fluid by adding small solid particles to that fluid. The possibility of the usage of such suspensions of solid particles with sizes on the order of 2 millimetres or micrometers was previously investigated by a number of researchers. It is known that a flow condition where both free and forced convection property is of comparable order is called mixed convection. The analysis of combined convection flow finds In several industrial application such as nuclear reactors exposed to winds, electronic devices cooled by exhaust fans and heat exchangers placed in a low velocity currents. The two dimensional laminar mixed convection flow along a vertical flat plate and extensive studies have been analysed on this type of flow [1–4]. The application of two dimensional can be found in the areas of reactor safety, combustion flames and solar collectors, as well as building energy conservation [5]. The two dimensional model has been used many researchers with mixed convection analysis boundary layer flow, for micro polar fluids or for the flow through packed medium [6-12]. Thermal diffusion effects are also analysed by Jha and Singh [13] and Kafoussias [14]. The thermal diffusion effects with MHD flow with mass transfer can be analysed by Alam and Sattar [15]. Alam et al. [16] studied the above mention effect on unsteady MHD free convection and mass transfer flow past an impulsively started vertical porous plate. Hence, the objective of the present paper is to study the above mentioned thermal-diffusion effects as well as heat generation effects on steady combined free-forced convection and heat transfer flow past a semi-infinite vertical porous flat plate embedded in a porous medium with different nanofluids say  $Al_2O_3, ZnO, CuO, SiO_2$ . The volumetric heat generation term may exert a strong influence on the heat transfer and as a result, also on the fluid flow and the results are presented graphically.

## II. MATHEMATICAL FORMULATIONS

Let an isothermal flat plate be embedded vertically in a porous medium saturated with a viscous incompressible fluid with internal generation heat generation/absorption is present. The plate is maintained at a constant temperature,  $x$ -axis is taken vertically along the plate and  $y$ -axis perpendicular to it so that the plate can be described by  $y = 0$ . Orientation of the plate can be seen in Fig-1. Using the boundary layer and Boussinesq approximations the equations governing the problem, i.e., the Continuity equation, the Darcy's law and the energy equations are presented as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial(p-p_\infty)}{\partial x} + (\rho - \rho_\infty)g + \frac{\mu}{K}(u - U_\infty) \tag{2}$$

$$\frac{\partial P}{\partial y} + \frac{\mu}{K}v = 0 \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_m}{\rho c_p} \left( \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q_0}{\rho c_p} (T - T_\infty) \tag{4}$$

Here  $u, v$  are fluid velocity components,  $T$  is fluid temperature,  $K$  is permeability,  $k_m$  is effective thermal conductivity of the fluid saturated porous medium, and  $Q_0$  heat generation/absorption constant.

The appropriate boundary conditions are:

$$T_0 = T_\infty + A, v = 0 \text{ at } y = 0, x \geq 0 \tag{5}$$

$$T \rightarrow T_\infty, u \rightarrow U_\infty \text{ at } y \rightarrow \infty, x \geq 0 \tag{6}$$

The following nondimensional parameters are introduced to transform the Eq.(1) to Eq.(4)

$$f(\eta) = \frac{\psi}{\alpha_m Pe_x^{0.5}}, \theta(\eta) = \frac{T - T_\infty}{T_0 - T_\infty}, \tag{7}$$

$$\eta = \frac{y}{x} Pe_x^{0.5}, Pe_x^{0.5} = \frac{U_\infty x}{\alpha_m}, Q_0 = \frac{Q_0 \rho c_p U_\infty}{x} \tag{7}$$

The governing equations are transformed

$$f'' \left[ 1 + \gamma_\mu \left( \theta - \frac{1}{2} \right) \right] + \gamma_\mu (f' - 1) \theta' = \theta' \xi \tag{8}$$

$$\theta'' = -2Q\theta - f\theta' \tag{9}$$

Where  $\xi = \frac{Ra_x}{Pe_x}$  is the mixed convection parameter

$$\mu = \mu_f \left( 1 + \gamma_\mu (\theta - 0.5) \right) \tag{10}$$

where  $\gamma_\mu$  is viscosity variation coefficient.

The boundary condition in Eq. (5) and (6) in terms of  $f$  and  $\theta$  are

$$\eta = 0, f = 0, \theta = 1 \tag{11}$$

$$\eta \rightarrow \infty, f' \rightarrow 0, \theta \rightarrow 0 \tag{12}$$

## III. METHOD OF SOLUTIONS

The nonlinear ordinary differential equations of Eq.(8),and Eq.(9)converted into first order differential equations and solved with the transformed boundary conditions in Eq.(11) and Eq.(12) with MATLAB code. In the present study the Al<sub>2</sub>O<sub>3</sub> SiO<sub>2</sub>,ZnO, CuO, oxide nanofluid with the Pr numbers[6.474,6.76,7.52,8.48]numbers[6.474,6.76,752,8.48] with different Grashof numbers are considered in the present study.

## IV. THERMOPHYSICAL PROPERTIES OF NANOFLUIDS

Metal and metallic oxide nanoparticles when dispersed in small quantities in a base liquid such as water, ethylene glycol, etc are observed to possess higher values of thermal conductivity compared to base liquid. Experiments conducted by Choi [18] with Carbon Nano Tubes (CNT) in engine oil at 1.0% volume concentration, obtained thermal conductivity enhancement of 160% of the base liquid value. The determination of thermo-physical properties of metal nanoparticles like Cu and metal oxides such as Al<sub>2</sub>O<sub>3</sub>, TiO<sub>2</sub>, CuO, SiO<sub>2</sub>, etc in different base fluids and the parameters influencing them is undertaken by various investigators The well known theoretical models of Maxwell [19] and Hamilton and Crosser [20] predicted lower values of thermal conductivity compared to experimental observations at higher temperatures. Hence experimental determination of thermal conductivity of various nanofluids is undertaken as the theoretical models are still pursued. It is fairly established that the thermal conductivity and viscosity of nanofluids is influenced by concentration and temperature. The influence of particle size on the properties has not been

considered in the development of regression equations by these investigators. The thermal conductivity and viscosity of various nanofluids are determined experimentally at different particle sizes, temperatures and concentration by many [21- 27] . Experiments for the estimation of nanofluid convective heat transfer coefficients are undertaken with particle size in the range of 20 to 170nm, temperature range of 20-700C, volume concentration of less than 4.0% with Al<sub>2</sub>O<sub>3</sub>, Cu, CuO, SiC, TiO<sub>2</sub>, ZrO<sub>2</sub>, etc nanoparticles dispersed in water. The viscosity and thermal conductivity data of metal and their oxide nanoparticles dispersed in water and available in literature is used in the development of regression equations. The nanofluid density and specific heat are determined using the mixture relations given by

**A. Density,  $\rho_{nf}$**

Applying the principle of mass conservation of the two species in a finite control volume of the nano fluid, the nanofluid density can be obtained from the relation

$$\rho_{nf} = \phi\rho_p + (1 - \phi)\rho_w \tag{13}$$

where  $\phi$  is the volumetric fraction of nano particles in the base fluid.

**B. Specific Heat,  $C_{pnf}$**

The thermal conservation of energy of the two species in a finite control volume will yield the overall

$$C_{pnf} = (1 - \phi)C_{pw} + \phi(\rho C_p)_w \tag{14}$$

The bulk material properties listed in are used in the development of regression equations for density and specific heat. Vajjha and Das [28] considered specific heat ratio to be dependent on concentration and bulk temperature in the development of regression equation. Hence equations applicable for metal and their oxide nanoparticles dispersed in water are

$$\rho_{nf} = \rho_w(0.9973 + 0.03479\phi + 0.0000619T_b) \tag{15}$$

$$C_{pnf} = (1.036 - 0.0298\phi - 0.001037T_b) \tag{16}$$

with an average deviation of less than 3.7% where  $\phi$  is in percent and  $T_b$  in o C. Nguyen et al. [29] conducted experiments for the determination of viscosity of Al<sub>2</sub>O<sub>3</sub> and CuO nanofluids in water at different concentrations and particle sizes in the ambient temperatures of 22 and 25oC. Experiments revealed the viscosity of Al<sub>2</sub>O<sub>3</sub> with particle sizes of 36 and 47nm and that of CuO with 29nm size predicted close values for volume concentration less than 4%. The dependence of viscosity ratio on concentration and particle diameter by Sarit et al. [30] and specific heat ratio on concentration and bulk temperature by Vajjha and Das [28] has been considered in the development of regression equations.

**C. Thermal Conductivity And Viscosity**

Based on these observations, it is can be stated that viscosity and thermal conductivity of nanofluids are influenced by volume concentration, temperature and particle size. Hence, the available experimental data in literature for a maximum temperature of  $T_{max} = 70^0$  C and particle size of  $d_{max} = 50$  nm is used to develop regression equation valid for various metals and their oxide nanoparticles dispersed in water

$$k_{nf} = (0.9808 + 0.0142\phi + 0.003883T_b - 0.00068dp) \tag{13}$$

$$\mu_{nf} = (0.9042 + 0.1245\phi + 0.0043dp - 0.0001206T_b) \tag{14}$$

for  $0 \leq \phi \leq 3.7$  ,  $20 \leq T_b \leq 70$  ,  $20 \leq dp \leq 150$  with a maximum deviation of less than 10% where  $\phi$  is in percent,  $T_b$  in<sup>o</sup>C and  $dp$  in nm. Yurong He et al. [31] observed the shear viscosity to increase and thermal conductivity to decrease with particle size.

**V. RESULTS AND DISCUSSIONS**

Fig 1.0 & Fig 2.0 represents the variation of velocity and temperature profiles with variation of mixed convection parameter for different nanofluid at heat absorption parameter  $Q = -1$  and viscosity variation parameter  $\gamma_\mu = 0.5$ . The velocity and temperature profiles are increases with increasing the mixed convection parameters.

Fig3.0& Fig4.0 represents with variation of velocity profiles and gradients of velocity profiles with nondimensional distance at of  $\gamma_\mu = 0.5, 0.0, -0.5$  ,  $Q = -1$  for different nanofluid with different mixed convection parameter at  $Q = -1$ . The profoiles are decreses with mixed convection parameter and increses with viscosity parameter. Fig5.0 represents the variation of temperature

profiles with nondimensional distance with variation of of velocity profiles with nondimensional distance at of  $\gamma_\mu = 0.5, 0.0, -0.5, Q = -1$  for different nanofluid with different mixed convection parameters. The fig6.0 represents the variation of temperature gradient with non dimensional distance for different values of mixed convection parameter. The temperature gradients are increases with increasing the viscosity parameter.

Fig7.0 variation of nondimensional temperature with nondimensional distance for different values of  $Q$  for different nanofluids with mixed convection parameters . The Temperature profiles are decreases with increasing the values of  $Q$  and mixed convection parameter decreases with increasing the viscosity parameter.

Fig8.0 & fig9.0 Represents the variation of velocity and velocity gradients with nondimensional distance. The profiles are decreases with increasing the mixed convection parameter and heat absorption coefficient

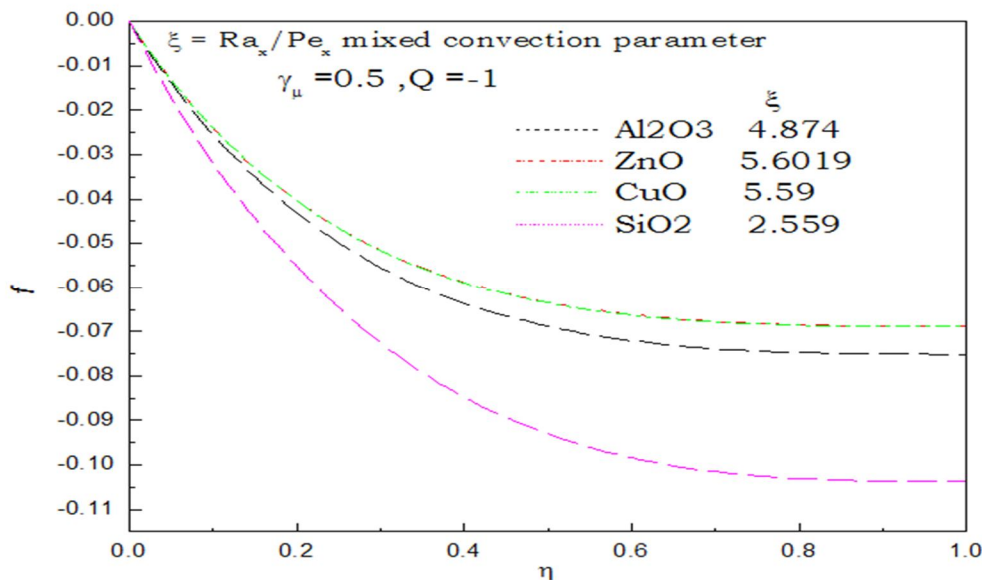


Fig 1.0 Effect of mixed convection parameter on velocity profiles  $\gamma_\mu = 0.5, Q = -1$  for different nanofluid

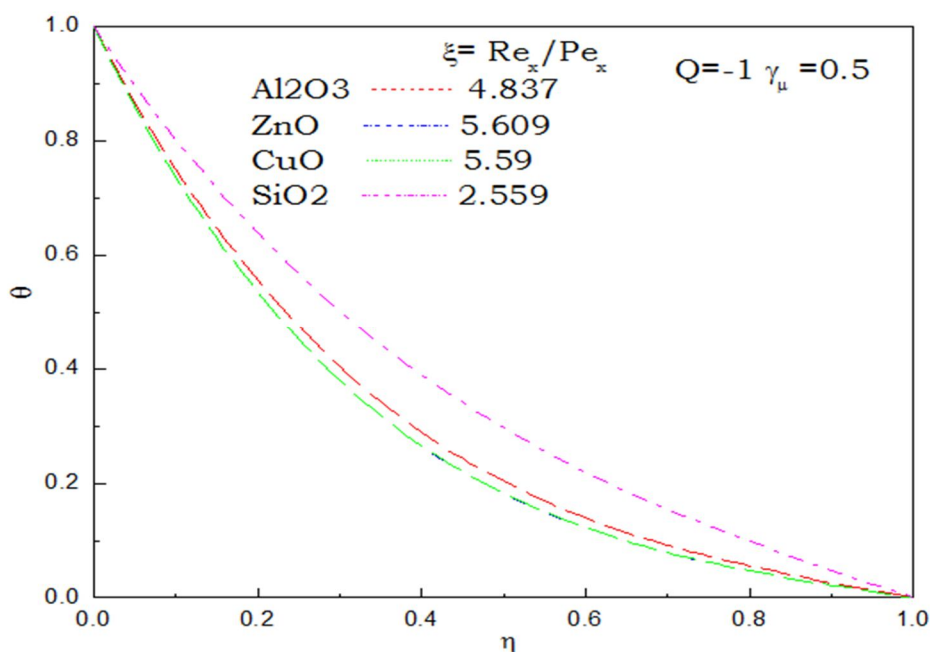


Fig2.0Effect of mixed convection parameter on temperature profiles  $\gamma_\mu = 0.5, Q = -1$  for different nanofluid

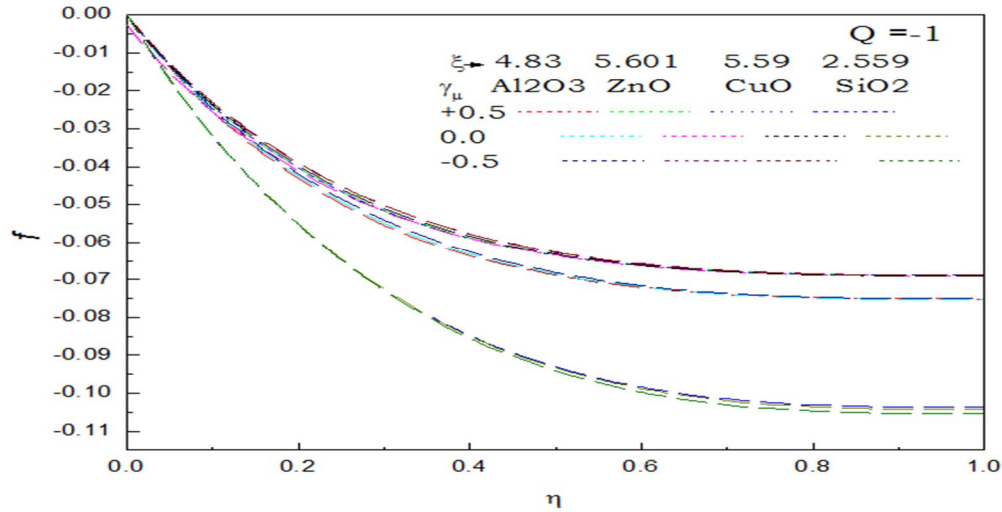


Fig 3.0 Effect of velocity profiles with different values of  $\gamma_\mu = 0.5, 0.0, -0.5, Q = -1$  for different nanofluid various mixed convection parameters .

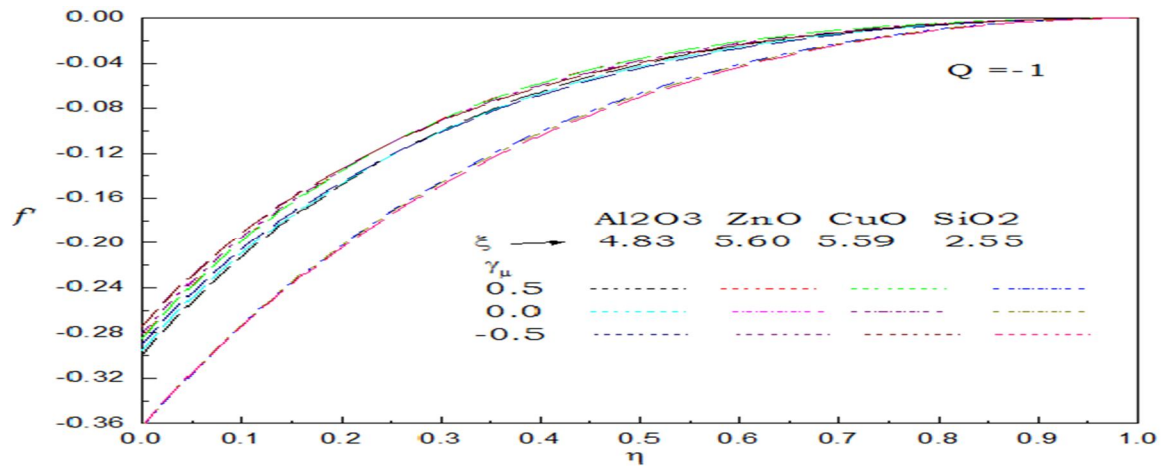


Fig 4.0 Effect of velocity gradient with profiles for different nanofluid  $\gamma_\mu = 0.5, 0.0, -0.5, Q = -1$  for different nanofluid with different mixed convection parameters

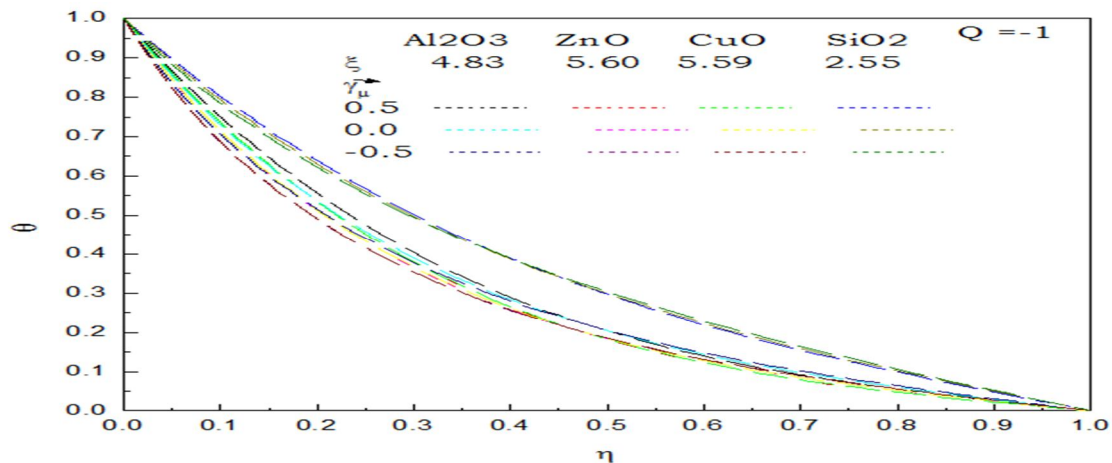


Fig5.0 variation of temperature profiles with  $\gamma_\mu = 0.5, 0.0, -0.5, Q = -1$  for different nanofluids with different mixed convection parameters

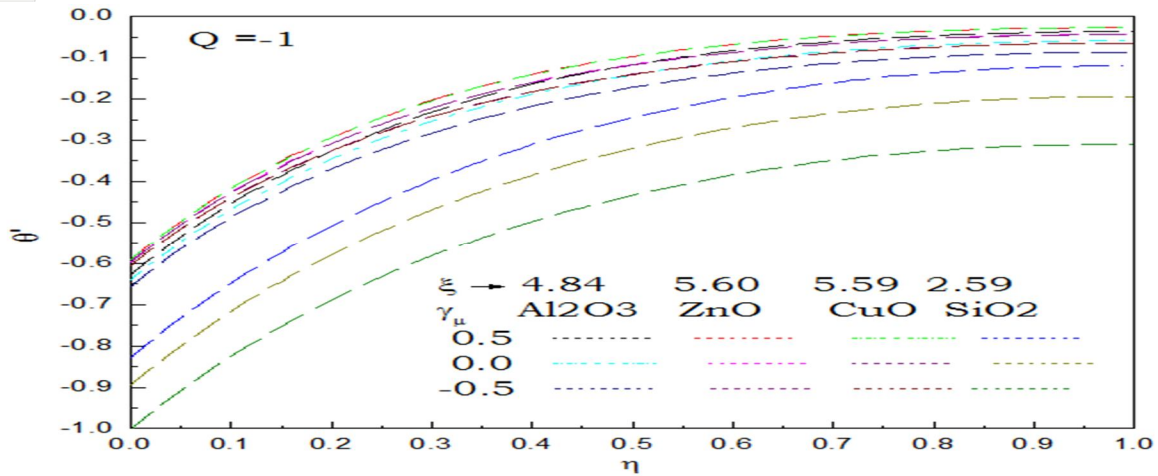


Fig 6.0 variation of temperature gradients with  $\gamma_\mu = 0.5, 0.0, -0.5, Q = -1$  for different nanofluids with various mixed convection parameters

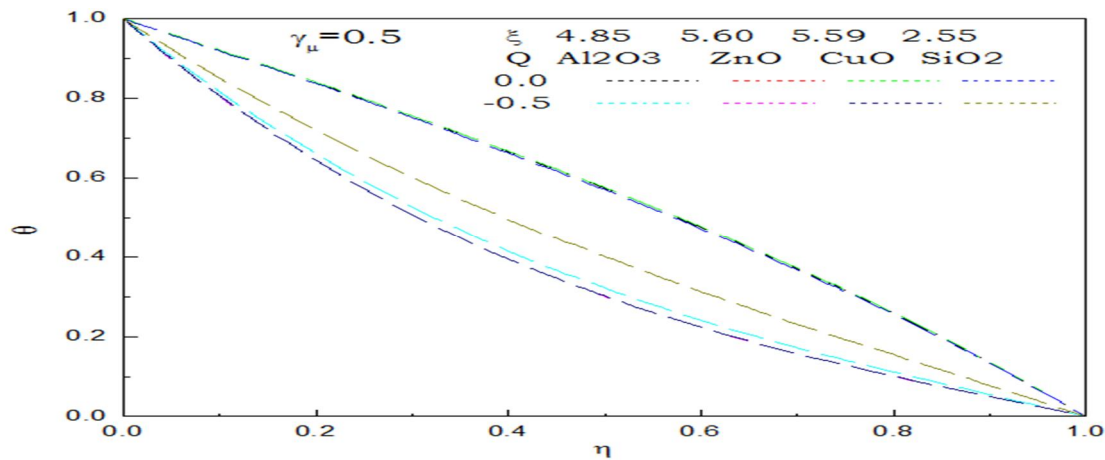


Fig7.0 variation temperature with nondimensional distance with different values of  $Q$  with different nanofluids with different mixed convection parameters at  $\gamma_\mu = 0.5$

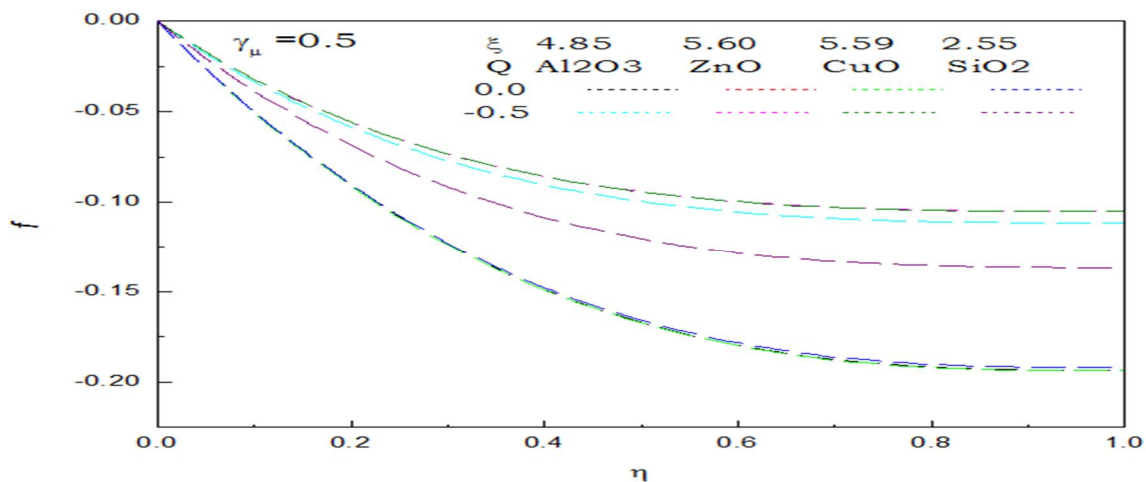


Fig 8.0 variation of velocity profiles with nondimensional distance at  $Q = 0.0, -0.5$  for different nanofluids with different mixed convection parameters

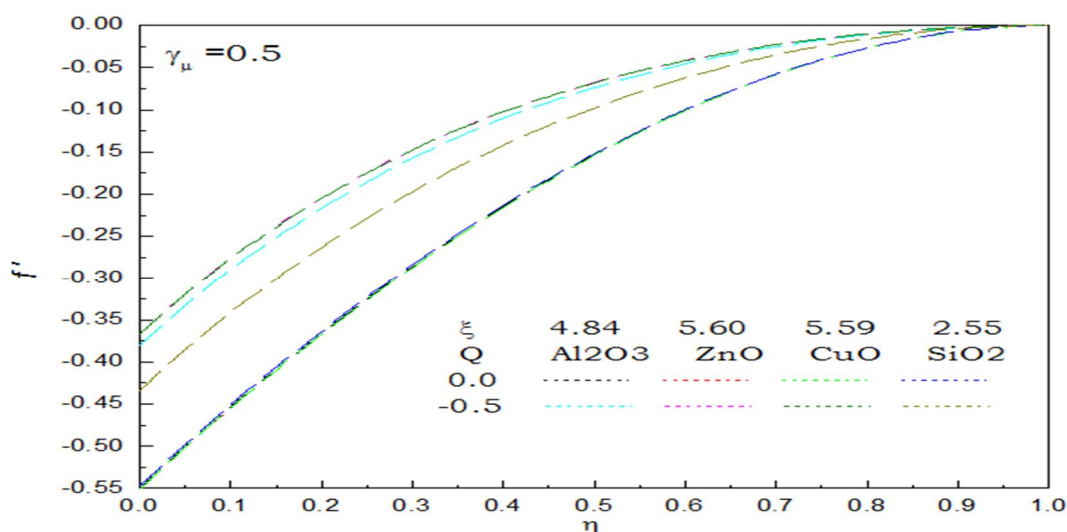


Fig9.0 Variation of velocity profiles with nondimensional distance at  $Q = 0.0, -0.5$  for different nanofluids with different mixed convection parameters.

## VI. CONCLUSIONS

This paper analyses fluid flow and heat transfer as affected by mixed convection parameters and properties of oxide nanofluid variable viscosity and heat generation in similarity solutions of mixed convection adjacent to an isothermal vertical plate embedded in a porous medium. Numerical solutions for momentum and heat transfer are obtained by employing MATLAB bvp4c code. The following conclusions are reported from the numerical and graphical results.

- 1) In case of assessing flow for heat absorption  $Q$  (negative = -1) keeping the constant for different values of viscosity parameter and mixed convection parameter the velocity and temperature profiles are decreases with increasing  $\xi$ , mixed convection parameter and  $\gamma_\mu$  viscosity parameter.
- 2) For the values of heat absorption  $Q = 0$  to  $-0.5$  the velocity and temperature profiles are decreases with increasing  $Q$  fro, 0 to  $-0.5$ .

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