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Radial Radio Mean Labeling of Mongolian Tent and Diamond Graphs

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Abstract: ***Let*** $G = (V, E)$ ***be a graph. In this paper, we are introducing a new graph labeling technique called radial radio mean labeling of graphs. A radial radio mean labeling of*** G ***is a function*** $f: V(G) \to \{1, 2, \dots, n\}$ ***satisfying the condition*** $d(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + r(G)$ ***where*** $d(u, v)$ ***represents the distance between any two vertices*** $u, v \in G$ ***and*** $r(G)$ ***represents the radius of*** G***. The span of a radial radio mean labeling*** f ***is the largest integer in the range of*** f ***and it is denoted by*** $span(f)$***. The radial radio mean number of*** G***, denoted by*** $rrmn(G)$ ***is the minimum span taken over all radial radio mean labeling of*** G***. In this paper, we have obtained the radial radio mean number of Mongolian tent and diamond graphs.***

Keywords: Radio labeling, Radial radio labeling, Radial radio mean labeling, Radial radio mean graceful labeling, Mongolian tent graph

I. INTRODUCTION

In this paper we have considered only simple, finite and connected graphs. In telecommunication systems, the most challenging problem is to assign frequencies to different radio channels such that there is no interference between any two transistors and also we have to minimize the usage of frequencies [5]. This problem can be modified as an graph theoretic problem where the radio transmitters represents the vertices and adjacent transmitters are connected by an edge [2].

The problem of assigning frequencies to the radio transmitters is called as frequency assignment problem which was introduced by William Hale [8]. In graph theory, the assignment of integers to vertices, edges or to both based on some condition is known as graph labeling [4]. Gary Chartrand et al. [3] were motivated by this frequency assignment problem and introduced a new graph labeling called Radio labeling. A radio labeling of a graph G is a function $f: V(G) \to \{1, 2, \dots, n\}$ such that $d(u, v) + |f(u) - f(v)| \geq 1 + diam(G)$. Motivated by this, KM. Kathiresan and S. Vimalajenifer [6] introduced the concept of radial radio labeling. A radial radio labeling f of G is a function $f: V(G) \to \{1,2, \dots, n\}$ satisfying the condition, $d(u, v) + |f(u) - f(v)| \geq 1 + r(G)$ where $r(G)$ represents the radius of the graph G and $u, v \in V(G)$. The above condition is known as radial radio condition. The span of a radial radio labeling f is the largest integer in the range of f. The radial radio number is the minimum span taken over all radial radio labelings of G and is denoted by $rr(G)$. That is, $rr(G) = \min_{f} \max_{v \in V(G)} f(v)$, where the minimum runs over all radial radio labelings of G.

Motivated by this work, in this paper we propose new graph labeling technique called radial radio mean labeling. A radial radio mean labeling of G is a function $f: V(G) \to \{1, 2, \dots, n\}$ satisfying the condition $d(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + r(G)$ where $d(u, v)$ represents the distance between the vertices u, v and $r(G)$ represents the radius of the graph G. The span of a radial radio mean labeling f is the largest integer in the range of f and is denoted by $span(f)$. The radial radio mean number of G, denoted by $rrmn(G)$ is the minimum span taken over all radial radio mean labeling of G. Also we say radial radio mean labeling as radial radio mean graceful labeling, if $|V(G)| = rrmn(G)$.

Before we proceed to the next section, we give the bounds of Complete graph, Complete bipartite graph and star graphs.

1) *Theorem 1.1.* The radial radio mean number of complete graphs K_n, $rrmn(K_n) = n$.
2) *Theorem 1.2.* The radial radio mean number of complete bipartite graphs $K_{m,n}$, $rrmn(K_{m,n}) = m + n$.
3) *Theorem 1.3.* The radial radio mean number of star graph S_n, $rrmn(S_n) = n + 1$.

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In radio labeling, if $V(G) = rn(G)$ then the labeling is said to be radio graceful labeling. Similarly we say that radial radio mean labeling to be radial radio mean graceful labeling if $V(G) = rrmn(G)$. It can be seen that Complete graph, Complete bipartite graph and star graph satisfies the condition $V(G) = rn(G)$ and hence they are said to be radial radio mean graceful labeling. Further in this paper, we have obtained the radial radio mean number of Mongolian tent and diamond graphs. We have also proved them to be radial radio mean graceful labeling.

II. RADIAL RADIO MEAN NUMBER OF MONGOLIAN TENT GRAPH

In this section, the radial radio mean number of Mongolian tent graph has been investigated.

1) *Definition 2.1.* [7]. The Mongolian tent graph is a graph obtained from Ladder graph L_n by adding a new vertex z above the ladder and joining each vertex $v_{1,j}, 1 \le j \le n$ with z. See figure 1.

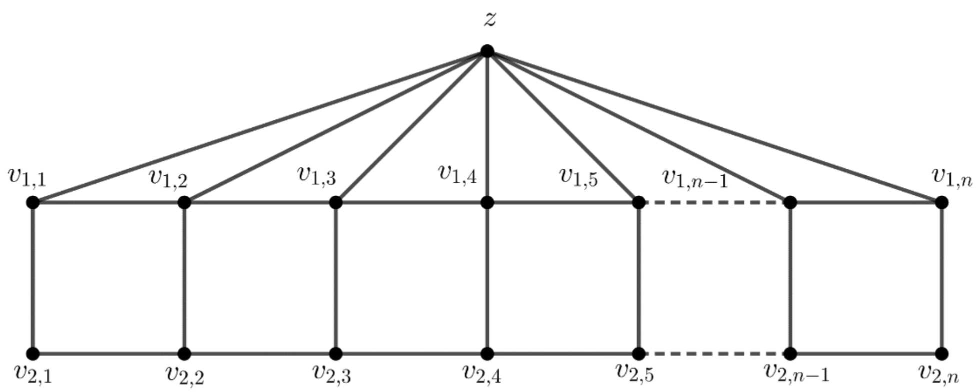

Fig. 1. $MT(n)$

a) *Remark 1.* For our convenience, the vertices $v_{1,j}, 1 \le j \le n$ are considered to form the top row R_1 and the vertices $v_{2,j}, 1 \le j \le n$ forms the bottom row R_2 of $MT(n)$.

- *Lemma 2.1.* The radial radio mean labeling of Mongolian tent graph will be at least $2n + 1$

The Mongolian tent graph has $2n + 1$ vertices and $4n - 2$ edges. The radius of $MT(n)$ will be 2. Since the radial mean labeling function is an bijective function, each vertex of $MT(n)$ will be labelled with distinct integers. As there are $2n + 1$ vertices in $MT(n)$, we will need at least $2n + 1$ numbers to label all the vertices of $MT(n)$.

Therefore, $rrmn(MT(n)) \ge 2n + 1$.

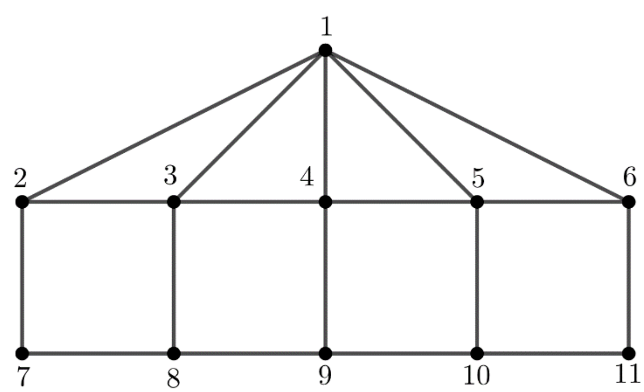

Fig. 2. radial radio mean labeling of $MT(5)$

- *Theorem 2.1.* The radial radio mean number of Mongolian tent graph will be at most $2n + 1$.

Proof. Let $\{z, v_{1,1}, v_{1,2}, \ldots, v_{1,n}, v_{2,1}, \ldots, v_{2,n}\}$ be the vertex set of $MT(n)$.

Label the top single vertex z as 1.

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The vertices in the top row of $MT(n)$ are labeled by the mapping,

$$f(v_{1,j}) = j+1, 1 \leq j \leq n \quad (1)$$

And the vertices in the bottom row are labelled by the mapping,

$$f(v_{2,j}) = n+j+1, 1 \leq j \leq n \quad (2)$$

b) Claim. The mapping (1) and (2) are valid radial radio mean labeling

Let u, v be any two vertices of $MT(n)$.

- *Case 1.* If the vertices u, v lies in top row

✓ *Case 1.1.* Suppose if the vertices u and v are adjacent, then $d(u,v) = 1$. In this case, the vertex u and v will be of the form $u = v_{1,k}$ and $v = v_{1,k+1}$, $1 \leq k \leq n$.

Therefore, by mapping (1), $f(v_{1,k}) = k+1$ and $f(v_{1,k+1}) = k+2$ and $\left\lceil \frac{f(v_{1,k})+f(v_{1,k+1})}{2} \right\rceil = \frac{2k+3}{2}$.

Hence, $d(u,v) + \left\lceil \frac{f(v_{1,k})+f(v_{1,k+1})}{2} \right\rceil \geq 1 + \left\lceil \frac{2k+3}{2} \right\rceil > 3$.

✓ *Case 1.2.* If the vertices u and v are non adjacent, then $d(u,v) > 1$. Here the vertices u and v will be of the form $u = v_{1,k}$ and $v = v_{1,k+a}$, $a \neq 1, 1 \leq a, k \leq n$.

By mapping (1), $\left\lceil \frac{f(v_{1,k})+f(v_{1,k+a})}{2} \right\rceil \geq \left\lceil \frac{2k+2+a}{2} \right\rceil, 1 \leq k, a \leq n$

Therefore, in this case we have $d(u,v) + \left\lceil \frac{f(v_k)+f(v_{k+1})}{2} \right\rceil \geq \left\lceil \frac{2k+2+a}{2} \right\rceil > 3$.

- *Case 2.* Let the vertices u, v lies in the bottom row.

✓ *Case 2.1.* Suppose if the vertices u and v are adjacent, then the distance between them will be 1. In this case, the vertex u and v will be of the form $u = v_{1,k}$ and $v = v_{1,k+1}$, $1 \leq k \leq n$.

By mapping (2), $f(v_{2,k}) = n+k+1$ and $f(v_{2,k+1}) = n+k+2$ and $\left\lceil \frac{f(v_{2,k})+f(v_{2,k+1})}{2} \right\rceil = \frac{n+2k+3}{2}$.

Hence, $d(u,v) + \left\lceil \frac{f(v_k)+f(v_{k+1})}{2} \right\rceil \geq 1 + \left\lceil \frac{n+2k+3}{2} \right\rceil > 3$.

✓ *Case 2.2.* If the vertices u and v are not adjacent, then the distance between these vertices will be more than 1. Here the vertices u and v will be of the form $u = v_{1,k}$ and $v = v_{1,k+a}$, $a \neq 1, 1 \leq a, k \leq n$.

By mapping (2), $\left\lceil \frac{f(v_{1,k})+f(v_{1,k+a})}{2} \right\rceil \geq \left\lceil \frac{2k+2+a}{2} \right\rceil, 1 \leq k, a \leq n$

Therefore, in this case we have $d(u,v) + \left\lceil \frac{f(v_k)+f(v_{k+1})}{2} \right\rceil \geq \left\lceil \frac{2k+2+a}{2} \right\rceil > 3$.

- *Case 3.* If the vertex $u \in z$ and the vertex v is any one of the two rows, we will have the following sub cases.

✓ *Case 3.1.* If $u \in z$ and v is in top row, then the distance between u and v will be 1 by mapping (1) we have $\left\lceil \frac{f(z)+f(v_{1,k})}{2} \right\rceil \geq 3$.

Therefore, $d(u,v) + \left\lceil \frac{f(z)+f(v_{1,k})}{2} \right\rceil \geq 1+3 > 3$.

✓ *Case 3.2.* If $u \in z$ and $v \in R_2$, then the distance between u and v will be at least 2 and by mapping (2) we have, $\left\lceil \frac{f(z)+f(v_{2,k})}{2} \right\rceil \geq 3$.

Therefore, $d(u,v) + \left\lceil \frac{f(z)+f(v_{2,k})}{2} \right\rceil \geq 1+3 > 3$.

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- *Case 4.* Suppose the vertex $u \in R_1$ and $v \in R_2$
- ✓ *Case 4.1.* In this case let the vertices u and v be adjacent. Then the distance between these vertices will be 1 and by mapping (1) and (2), $\left\lceil \frac{f(v_{1,k})+f(v_{2,k})}{2} \right\rceil \geq 2$ and therefore,

$$d(u,v) + \left\lceil \frac{f(v_{1,k})+f(v_{2,k})}{2} \right\rceil \geq 3.$$

- ✓ *Case 4.2.* If the vertices u and v are not adjacent, then the distance between these two vertices will be at least 2 and by mapping (1) and (2), we have

$$d(u,v) + \left\lceil \frac{f(v_{1,k})+f(v_{2,m})}{2} \right\rceil \geq 3, 1 \leq k,m \leq n, k \neq m.$$

Hence, in all the cases it can be seen that $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + r(G)$.

Therefore, mapping (1) and (2) are valid radial radio mean labeling.

By mapping (2), the vertex v_{2n} receives the maximum labeling and it is given by $f(v_{2,n}) = 2n + 1$.

Therefore, $rrmn(MT(n)) \leq 2n + 1$.

Theorem 2.2. The radial radio mean number of $MT(n), rrmn(MT(n)) = 2n + 1$.

Proof. The proof is obvious from Lemma 2.1 and Theorem 2.1.

Remark 2. As $|V(MT(n))| = 2n + 1$ and $rrmn(MT(n)) = 2n + 1$, $MT(n)$ is said to be radial radio mean graceful labeling.

III.RADIAL RADIO MEAN NUMBER OF DIAMOND GRAPH

1) *Definition 3.1.* [1]. *Diamond graph* denoted by $D(n)$ is the graph obtained from Mongolian tent graph $MT(n)$ by adding a new vertex z_1 and joining each vertex $v_{2,j}, 1 \leq j \leq n$ with z_1. See Figure 3.

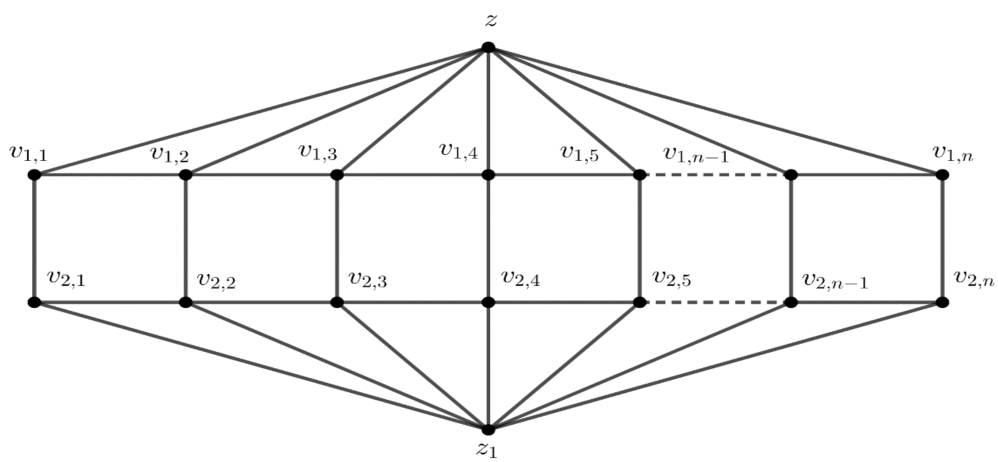

Fig. 3. $D(n)$

a) *Remark 3.* For our convenience, the vertices of the form $v_{1,j}, 1 \leq j \leq n$ are considered to be in the top row R_1 and the vertices of the form $v_{2,j}, 1 \leq j \leq n$ forms the bottom row R_2 of $D(n)$.

b) *Lemma 3.1.* The radial radio mean number of diamond graph will be at least $2n + 2$.

Proof. The diamond graph $D(n)$ has $2n + 2$ vertices and $5n - 2$ edges. The radius of $D(n)$ is 3. We first label the top single vertex z as 2. Then we label the vertex z_1 as 1. Now, we have to label the remaining $2n$ vertices of $D(n)$. Therefore the remaining $2n$ vertices of $D(n)$ can be labelled by consecutive numbers. This implies that we need at least $2n$ numbers to label these vertices. Together with the other two numbers, we will need at least $2n + 2$ numbers to label all the vertices of $D(n)$.

Therefore, $rrmn(D(n)) \geq 2n + 2$.

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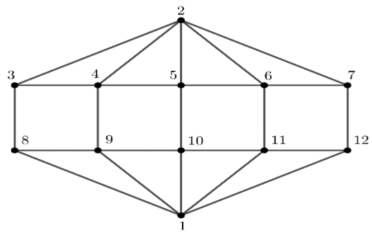

Fig. 4. radial radio mean number of $D(5)$

c) *Theorem 3.1.* The radial radio mean number of diamond graph $D(n)$, $rrmn(D(n)) \leq 2n+2$.

Proof. Let $\{z, z_1, v_{1,1}, v_{1,2}, \dots, v_{1,n}, v_{2,1}, \dots, v_{2,n}\}$ be the vertex set of $D(n)$.

Label the vertex z as 2 and z_1 as 1.

The vertices in the top row R_1 of $D(n)$ is labeled by the mapping,

$$f(v_{1,j}) = j+2, 1 \leq j \leq n \quad (3)$$

And the vertices in the bottom row are labelled by the mapping,

$$f(v_{2,j}) = n+j+2, 1 \leq j \leq n \quad (4)$$

d) *Claim.* The mapping (3) and (4) are valid radial radio mean labeling.

Let u, v be any two vertices of $D(n)$.

- *Case 1.* If the vertices u, v lies in top row

✓ *Case 1.1.* If $d(u,v) = 1$.

In this case, the vertex u and v will be of the form $u = v_{1,k}$ and $v = v_{1,k+1}$, $1 \leq k \leq n$.

Therefore, by mapping (3), $f(v_{1,k}) = k+2$ and $f(v_{1,k+1}) = k+3$ and $\left\lceil \frac{f(v_{1,k})+f(v_{1,k+1})}{2} \right\rceil = \frac{2k+5}{2}$.

Hence, $d(u,v) + \left\lceil \frac{f(v_{1,k})+f(v_{1,k+1})}{2} \right\rceil \geq 1 + \left\lceil \frac{2k+5}{2} \right\rceil \geq 4$.

✓ *Case 1.2.* If $d(u,v) > 1$.

Here the vertices u and v will be of the form $u = v_{1,k}$ and $v = v_{1,k+a}$, $a \neq 1, 1 \leq a, k \leq n$.

By mapping (3), $\left\lceil \frac{f(v_{1,k})+f(v_{1,k+a})}{2} \right\rceil \geq \left\lceil \frac{2k+4+a}{2} \right\rceil, 1 \leq k, a \leq n$

Therefore, in this case we have $d(u,v) + \left\lceil \frac{f(v_k)+f(v_{k+1})}{2} \right\rceil \geq \left\lceil \frac{2k+2+a}{2} \right\rceil \geq 4$.

- *Case 2.* Let the vertices u, v lies in the bottom row.

✓ *Case 2.1.* When $d(u,v) = 1$.

In this case, the vertex u and v will be of the form $u = v_{2,k}$ and $v = v_{2,k+1}$, $1 \leq k \leq n$.

By mapping (4), $f(v_{2,k}) = n+k+2$ and $f(v_{2,k+1}) = n+k+3$ and $\left\lceil \frac{f(v_{2,k})+f(v_{2,k+1})}{2} \right\rceil = \frac{2n+2k+5}{2}$.

Therefore, $d(u,v) + \left\lceil \frac{f(v_k)+f(v_{k+1})}{2} \right\rceil \geq 1 + \left\lceil \frac{2n+2k+5}{2} \right\rceil > 4$.

✓ *Case 2.2.* If $d(u,v) > 1$.

Here the vertices u and v will be of the form $u = v_{2,k}$ and $v = v_{2,k+a}$, $a \neq 1, 1 \leq a, k \leq n$.

By mapping (4), $\left\lceil \frac{f(v_{1,k})+f(v_{1,k+a})}{2} \right\rceil \geq \left\lceil \frac{2k+2n+a}{2} \right\rceil, 1 \leq k, a \leq n$

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Hence, in this case we have $d(u,v)+\left\lceil\frac{f(v_{2,k})+f(v_{2,k+1})}{2}\right\rceil \geq \left\lceil\frac{2k+2n+a}{2}\right\rceil > 4$.

- *Case 3.* If the vertex $u \in z$ and the vertex v is any one of the two rows, we will have the following sub cases.

✓ *Case 3.1.* If $u \in z$ and v is in top row, then the distance between u and v will be 1 by mapping (3) we have $\left\lceil\frac{f(z)+f(v_{1,k})}{2}\right\rceil \geq 3$.

Therefore, $d(u,v)+\left\lceil\frac{f(z)+f(v_{1,k})}{2}\right\rceil \geq 1+3 \geq 4$.

✓ *Case 3.2.* If $u \in z$ and $v \in R_2$,

Then $d(u,v) \geq 2$ and by mapping (4) we have, $\left\lceil\frac{f(z)+f(v_{2,k})}{2}\right\rceil \geq 3$.

Therefore, $d(u,v)+\left\lceil\frac{f(z)+f(v_{2,k})}{2}\right\rceil \geq 2+3 > 4$.

- *Case 4.* If the vertex $u \in z_1$ and the vertex v is any one of the two row

The proof is similar to case 3.

- *Case 5.* Suppose the vertex $u \in R_1$ and $v \in R_2$

✓ *Case 5.1.* If $d(u,v)=1$

In this case, by mapping (3) and (4), $\left\lceil\frac{f(v_{1,k})+f(v_{2,k})}{2}\right\rceil \geq 4$ and therefore,

$d(u,v)+\left\lceil\frac{f(v_{1,k})+f(v_{2,k})}{2}\right\rceil > 4$.

✓ *Case 5.2.* If $d(u,v) > 1$

Then by mapping (3) and (4), we have

$d(u,v)+\left\lceil\frac{f(v_{1,k})+f(v_{2,m})}{2}\right\rceil > 4, 1 \leq k,m \leq n, k \neq m$.

Hence, in all the cases it can be seen that $d(u,v)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 1+r(G)$.

Therefore, mapping (3) and (4) are valid radial radio mean labeling.

By mapping (4), the vertex v_{2n} receives the maximum label and it is given by $f(v_{2,n})=2n+2$.

Hence, $rrmn(D(n)) \leq 2n+2$.

2) Theorem 3.2. The radial radio mean number of $D(n), rrmn(D(n)) = 2n+2$.

Proof. The proof is obvious from Lemma 3.1 and Theorem 3.1.

a) Remark 4. As $|V(D(n))| = 2n+2$ and $rrmn(D(n)) = 2n+2$, $D(n)$ is said to be radial radio mean graceful labeling.

IV. CONCLUSIONS

In this paper, we have introduced a new graph labeling and found out the exact radial radio mean number of Mongolian tent and diamond graphs. Also we have proved that these graphs are radial radio mean graceful labeling. This work can be further extended to other networks also.

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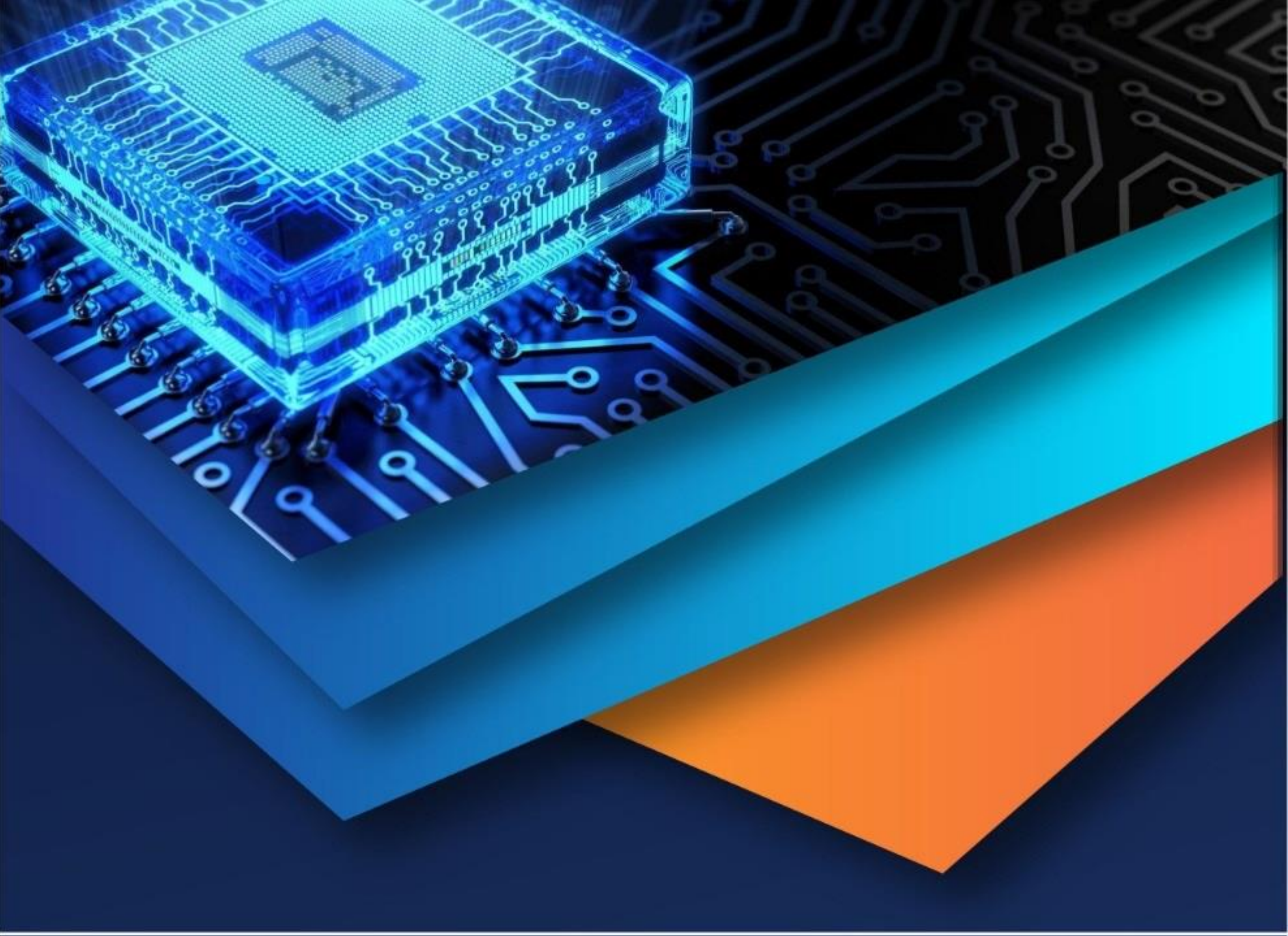

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