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The Logistic NHE Distribution with Properties and Applications

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Abstract: In this paper, we have presented a univariate three-parameter continuous distribution called Logistic NHE distribution. We have discussed some mathematical and statistical characteristics of the distribution like the probability density function, cumulative distribution function and hazard rate function, survival function, quantile function, the skewness, and kurtosis measures. The proposed distribution's model parameters are estimated with the help of three well-accepted estimation methods which are maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods. The proposed distribution's goodness of fit, is also evaluated by fitting it in comparison with some other existing distributions using a real data set.
Keywords: Logistic distribution, Exponential distribution, Hazard function, NHE, MLE

I. INTRODUCTION

Lifetime distributions are generally used to study the lifespan of components of a system, and in general, reliability and survival analysis. Frequently we have seen the applications of lifetime distributions in fields like life science, medicine, biology, engineering, insurance, etc. Many continuous probability distributions like Cauchy, gamma, Weibull, exponential have been commonly used in for the evaluation of lifetime data. For a few years, most of the researchers are attracted towards one parameter Logistic distribution in its ability in modeling lifetime data, and excellent performance of this distribution has been seen in many fields.

In probability theory and statistics, the exponential distribution plays a significant role in evaluation of survival data. Exponential distribution is particular case of the gamma distribution which shows memoryless property. It is the probability distribution of the time between events in a Poisson point process, where events occur independently and continuously at a steady average rate. It is the continuous analog of the geometric distribution which also shows the property of being memoryless.

For a few decades, it is found that the exponential distribution is taken as base distribution for generating a new family of distribution. The modifications of the exponential distribution were introduced by different researchers, some of them are, beta exponential (Nadarajah and Kotz, 2006)[27], Gupta and Kundu (2007) have presented the generalized exponential (GE) with some development[9], Abouammoh & Alshingiti (2009) has introduced the reliability estimation of the generalized inverted exponential distribution[2], beta GE (Barreto-Souza et al., 2010)[5], Exponential Extension (EE) distribution (Kumar, 2010)[15], KW (Kumaraswamy) exponential (Cordeiro and de Castro, 2011)[8], Nadarajah & Haghighi (2011) have presented an extension of the exponential distribution[26], gamma EE by (Ristic and Balakrishnan, 2012)[30], Transmuted EE distribution by (Merovci, 2013)[24], Lemonte, A. J. (2013) has introduced a new exponential-type distribution with constant, decreasing, increasing, upside-down bathtub and bathtub-shaped failure rate function[19]. Gomez et al. (2014) have presented a new extension of the exponential distribution[10]. The exponentiated exponential geometric (Louzada et al., 2014)[20] and Kumaraswamy transmuted exponential (Afify et al., 2016) distributions[3]. Mahdavi & Kundu (2017) have presented a new method for generating distributions with an application to the exponential distribution[21]. Recently, the Alpha power transformed extended exponential distribution have introduced by (Hassan et al., 2018)[11]. Almarashi et al. (2019) have presented a new extension of exponential distribution with some statistical properties[4]. Abdulkabir & Ipinyomi, (2020) have introduced the Type II half-logistic exponentiated exponential distribution[1]. Joshi and Kumar (2020) have created half-logistic NHE distribution and studied its various statistical and mathematical characteristics[13].

Logistic distribution is a continuous univariate distribution and in various different fields, like logistic regression, neural networks and logistic models, both its cumulative distribution function and probability density functions have been used. It has been used in the demographics, sports modeling, physical sciences, and in both finance and insurance recently. There are broader tails of the logistic distribution than a normal distribution, so we see better consistency with the underlying data and offers more insight into the extreme event's likelihood.

Let X be a non negative random variable follows the logistic distribution with shape parameter $\theta > 0$, and its cumulative distribution function is given by

$$G(x;\theta)=\frac{1}{1+e^{-\theta x}};\quad \theta>0, x\in\Re \tag{1.3}$$

and its corresponding PDF is

$$g(x;\theta)=\frac{\theta e^{-\theta x}}{\left(1+e^{-\theta x}\right)^{2}};\quad \theta>0, x\in\Re \tag{1.4}$$

A continuous distributions' new generating family developed from a logistic random variable known as logistic-X family has been described by Tahir et al. (2016)[33]. Its density function can be left-skewed, right-skewed, and symmetrical and reversed-J shaped, and can have decreasing, increasing, bathtub and upside-down bathtub hazard rates shaped. Mandouh (2018) has introduced Logistic-modified Weibull distribution which is flexible for survival analysis as compared to modified Weibull distribution[23]. Joshi & Kumar (2020) have introduced the Lindley exponential power distribution having a more flexible hazard rate function[12]. Chaudhary & Kumar (2020) have presented the half logistic exponential extension distribution using the parent distribution as exponential extension distribution[7]. Lan and Leemis (2008) has presented an approach to define the logistic compounded model and introduced the logistic–exponential survival distribution[17]. This has several useful probabilistic properties for lifetime modeling. Unlike most distributions in the bathtub and upside down bathtub classes, the logistic–exponential distribution exhibit closed-form density, hazard, cumulative hazard, and survival functions. The survival function of the logistic–exponential distribution is

$$S(x;\lambda)=\frac{1}{1+\left(e^{\lambda x}-1\right)^{\alpha}};\quad \alpha>0,\lambda>0, x\geq 0 \tag{1.5}$$

Using the same approach used by (Lan & Leemis, 2008) we have introduced the new distribution called Logistic NHE distribution[17]. The main objective of this study is to introduce a more flexible distribution by inserting just one extra parameter to the exponential extension distribution to attain a better fit to the lifetime data sets. We have discussed some distributional properties and its applicability. The different sections of the proposed study are arranged as follows. In Section 2 we present the Logistic NHE distribution and its various mathematical and statistical properties. We have make use of three well-known estimation methods for estimating the model parameters namely the maximum likelihood estimation (MLE), least-square estimation (LSE) and Cramer-Von-Mises estimation (CVME) methods. For the maximum likelihood (ML) estimate, we have constructed the asymptotic confidence intervals using the observed information matrix are described in Section 3. In Section 4, a real data set has been analyzed to explore the applications and capability of the proposed distribution. In this section, we present the estimated value of the parameters and log-likelihood, AIC, BIC and AICC criterion for ML, LSE, and CVME also the goodness of fit of the proposed distribution is also evaluated by fitting it in comparison with some other existing distributions using a real data set. In Section 5 conclusion to the study has been presented

II. THE LOGISTIC NHE(L-NHE) DISTRIBUTION

Nadarajah & Haghighi (2011) has defined the extension of the exponential distribution we have called this as NHE distribution[26]. The CDF of NHE is defined as

$$F(x;\alpha,\beta)=1-e^{\{1-(1+\alpha x)^{\beta}\}};\quad \alpha,\beta>0,\ x>0 \tag{2.1}$$

The corresponding PDF can be written as

$$f(x;\alpha,\beta)=\alpha\beta(1+\alpha x)^{\beta-1}e^{\{1-(1+\alpha x)^{\beta}\}};\quad \alpha,\beta>0,\ x>0 \tag{2.2}$$

Using the same approach used by (Lan & Leemis, 2008) we have defined the new distribution called logistic NHE distribution[17]. In this study we have taken the NHE as baseline distribution. Let X be a non negative random variable with a positive shape parameters α and β and a positive scale parameter λ then CDF of logistic NHE distribution can be defined as

$$F(x)=1-\frac{1}{1+\left[\exp\left\{(1+\lambda x)^{\beta}-1\right\}-1\right]^{\alpha}};\quad (\alpha,\beta,\lambda)>0,\ x>0 \tag{2.3}$$

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The PDF of Logistic- NH distribution is

$$f(x)=\frac{\alpha\beta\lambda(1+\lambda x)^{\beta-1}\exp\left\{(1+\lambda x)^{\beta}-1\right\}\left[\exp\left\{(1+\lambda x)^{\beta}-1\right\}-1\right]^{\alpha-1}}{\left\{1+\left[\exp\left\{(1+\lambda x)^{\beta}-1\right\}-1\right]^{\alpha}\right\}^{2}};\ (\alpha,\beta,\lambda)>0,\ x>0 \quad (2.4)$$

This CDF function can be compared to log logistic CDF function with the second term of the denominator being changed in its base to NHE function, and hence we named it logistic NHE distribution.

A. Reliability Function

The reliability function of Logistic- Rayleigh distribution can be defined as,

$$R(x)=1-F(x)$$

$$=\frac{1}{1+\left[\exp\left\{(1+\lambda x)^{\beta}-1\right\}-1\right]^{\alpha}};\ (\alpha,\beta,\lambda)>0,\ x>0 \qquad (2.5)$$

B. Hazard Function

The hazard rate function of LEE distribution is,

$$h(x)=\frac{f(x)}{R(x)}$$

$$=\frac{\alpha\beta\lambda(1+\lambda x)^{\beta-1}\exp\left\{(1+\lambda x)^{\beta}-1\right\}\left[\exp\left\{(1+\lambda x)^{\beta}-1\right\}-1\right]^{\alpha-1}}{\left\{1+\left[\exp\left\{(1+\lambda x)^{\beta}-1\right\}-1\right]^{\alpha}\right\}^{2}} \qquad (2.6)$$

In Figure 1, we have displayed the graphs of the PDF and hazard rate function of L-NHE distribution for different values of α, β and λ.

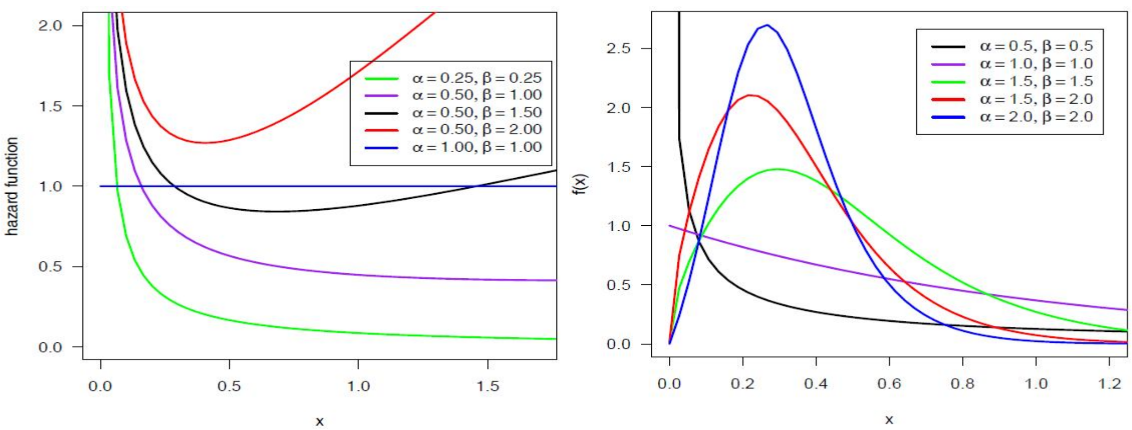

Figure 1. For different values of α, β and λ, graphs of hazard function (left panel) and PDF (right panel)

C. Quantile function

The Quantile function of Logistic NHE distribution can be expressed as

$$Q(u)=\frac{1}{\lambda}\left[\left[\ln\left\{\left(\frac{u}{1-u}\right)^{1/\alpha}+1\right\}+1\right]^{1/\beta}-1\right];\quad 0<u<1 \qquad (2.7)$$

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III. METHODS OF ESTIMATION

Here, the parameters of the proposed distribution are estimated with application some well-known estimation methods which are as follows

A. Maximum Likelihood Estimates

For the estimation of the parameter, the maximum likelihood method is the most commonly used method introduced by (Casella & Berger, 1990)[6]. Let, $x_1, x_2, ..., x_n$ is a random sample from $L-NHE(\alpha,\beta,\lambda)$ and the likelihood function, $L(\alpha,\beta,\lambda)$ is given by,

$$L(\psi; x_1, x_2 ... x_n) = f(x_1, x_2, ... x_n / \psi) = \prod_{i=1}^{n} f(x_i / \psi)$$

$$L(\alpha,\beta,\lambda) = \alpha\beta\lambda \prod_{i=1}^{n} \frac{(1+\lambda x_i)^{\beta-1} \exp\left\{(1+\lambda x_i)^{\beta}-1\right\}\left[\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}-1\right]^{\alpha-1}}{\left\{1+\left[\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}-1\right]^{\alpha}\right\}^{2}}; \ (\alpha,\beta,\lambda)>0, \ x>0$$

Now log-likelihood density is

$$\begin{aligned}\ell(\alpha,\beta,\lambda \mid \underline{x}) &= n\ln(\alpha\beta\lambda) - n + (\beta-1)\sum_{i=1}^{n}\ln(1+\lambda x_i) + \sum_{i=1}^{n}(1+\lambda x_i)^{\beta} \\ &+ (\alpha-1)\ln\sum_{i=1}^{n}\left[\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}-1\right] - 2\sum_{i=1}^{n}\ln\left\{1+\left[\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}\right]^{\alpha}\right\}\end{aligned} \tag{3.1.1}$$

Differentiating (3.1.1) with respect to α, β and λ we get,

$$\frac{\partial l}{\partial \alpha} = \frac{1}{\alpha} + \ln\sum_{i=1}^{n} A(x_i) - 2\sum_{i=1}^{n}\frac{\ln[A(x_i)][A(x_i)]^{\alpha}}{1+[A(x_i)]^{\alpha}}$$

$$\frac{\partial l}{\partial \beta} = \frac{1}{\beta} + \sum_{i=1}^{n}\ln(1+\lambda x_i) + \frac{\lambda}{\beta^2}\left\{1+(\alpha-1)\sum_{i=1}^{n}\frac{\{A(x_i)+1\}}{A(x_i)} - 2\alpha\sum_{i=1}^{n}\frac{[A(x_i)+1][A(x_i)]^{\alpha-1}}{1+[A(x_i)]^{\alpha}}\right\}$$

$$\sum_{i=1}^{n}(1+\lambda x_i)^{\beta}\ln(1+\lambda x_i)$$

$$\begin{aligned}\frac{\partial l}{\partial \lambda} = \frac{1}{\lambda} + (\beta-1)\sum_{i=1}^{n}\frac{x_i}{(1+\lambda x_i)} + \beta\sum_{i=1}^{n}x_i(1+\lambda x_i)^{\beta-1}[1+(\alpha-1)\frac{\{A(x_i)+1\}}{A(x_i)} \\ -2\alpha\frac{[A(x_i)+1][A(x_i)]^{\alpha-1}}{1+[A(x_i)]^{\alpha}}]\end{aligned}$$

Where $A(x_i) = \exp\left\{(1+\lambda x_i)^{\beta}-1\right\}-1$

Equating above three non linear equations to zero and solving simultaneously for α, β and λ, we obtain the maximum likelihood estimate $\hat{\alpha}, \hat{\beta}$ *and* $\hat{\lambda}$ of the parameters α, β and λ. By using computer software like R, Matlab, Mathematica etc for maximization of (3.1.1) we can obtain the estimated value of α, β and λ. For the confidence interval estimation of α, β and λ and testing of the hypothesis, we have to calculate the observed information matrix. The observed information matrix for α, β and λ can be obtained as,

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix}$$

Where

$$U_{11} = \frac{\partial^2 l}{\partial \alpha^2},\ U_{12} = \frac{\partial^2 l}{\partial \alpha \partial \beta},\ U_{13} = \frac{\partial^2 l}{\partial \alpha \lambda}$$

$$U_{21} = \frac{\partial^2 l}{\partial \beta \partial \alpha},\ U_{22} = \frac{\partial^2 l}{\partial \beta^2},\ U_{23} = \frac{\partial^2 l}{\partial \beta \partial \lambda}$$

$$U_{31} = \frac{\partial^2 l}{\partial \lambda \partial \alpha},\ U_{32} = \frac{\partial^2 l}{\partial \beta \partial \lambda},\ U_{33} = \frac{\partial^2 l}{\partial \lambda^2}$$

Let $\Omega = (\alpha, \beta, \lambda)$ represent the parameter space and the corresponding MLE of Ω as $\hat{\Omega} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$, then $\left(\hat{\Omega} - \Omega\right) \to N_3\left[0, \left(U(\Omega)\right)^{-1}\right]$ where $U(\Omega)$ is the information matrix of Fisher. Using the Newton-Raphson algorithm for maximizing the likelihood creates the observed information matrix and hence the variance-covariance matrix is obtained as,

$$\left[U(\Omega)\right]^{-1} = \begin{pmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha}, \hat{\beta}) & \mathrm{cov}(\hat{\alpha}, \hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha}, \hat{\beta}) & \mathrm{var}(\hat{\beta}) & \mathrm{cov}(\hat{\beta}, \hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha}, \hat{\lambda}) & \mathrm{cov}(\hat{\beta}, \hat{\lambda}) & \mathrm{var}(\hat{\lambda}) \end{pmatrix} \tag{3.1.2}$$

Thus, with the help of asymptotic normality of Maximum Likelihood Estimates, approximate 100(1-α) % confidence intervals for α, β and λ can be built as,

$$\hat{\alpha} \pm Z_{\alpha/2} SE(\hat{\alpha}),\ \hat{\beta} \pm Z_{\alpha/2} SE(\hat{\beta}) \text{ and, } \hat{\lambda} \pm Z_{\alpha/2} SE(\hat{\lambda})$$

where upper percentile of standard normal variate is denoted by $Z_{\alpha/2}$

B. Method of Least-Square Estimation (LSE)

Swain et al. (1988) have introduced the ordinary least square estimators and weighted least square estimators for estimating the parameters of Beta distributions[32]. Here we use the same technique for the estimation of the parameters of the logistic NHE distribution. The least-square estimators of the unknown parameters α, β and λ of L-NHE distribution can be calculated by minimizing

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^{n} \left[F(X_i) - \frac{i}{n+1} \right]^2 \tag{3.2.1}$$

with respect to unknown parameters α, β and λ.

Consider $F(X_i)$ represent the distribution function of the ordered random variables $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ where $\{X_1, X_2, \ldots, X_n\}$ is a random sample of size n from a distribution function F(.). The least-square estimators of α, β and λ say $\hat{\alpha}, \hat{\beta}, and\, \hat{\lambda}$ respectively, can be obtained by minimizing

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^{n} \left[1 - \frac{1}{1 + \left[\exp\left\{(1+\lambda x_i)^{\beta} - 1\right\} - 1\right]^{\alpha}} - \frac{i}{n+1} \right]^2 ; x \geq 0, (\alpha, \beta, \lambda) > 0. \tag{3.2.2}$$

with respect to α, β and λ.

Differentiating (3.2.2) with respect to α, β and λ we get,

$$\frac{\partial M}{\partial \alpha}=-2\sum_{i=1}^{n}\left[1-\frac{1}{1+\left[A_i(x)\right]^{\alpha}}-\frac{i}{n+1}\right]\frac{\left[A_i(x)\right]^{\alpha}\ln\left[A_i(x)\right]}{\left\{1+\left[A_i(x)\right]^{\alpha}\right\}^{2}}$$

$$\frac{\partial M}{\partial \beta}=-2\alpha\sum_{i=1}^{n}\left[1-\frac{1}{1+\left[A_i(x)\right]^{\alpha}}-\frac{i}{n+1}\right]\frac{\left[A_i(x)\right]^{\alpha-1}(1+\lambda x_i)^{\beta}\ln(1+\lambda x_i)\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}}{\left\{1+\left[A_i(x)\right]^{\alpha}\right\}^{2}}$$

$$\frac{\partial M}{\partial \lambda}=-2\beta\sum_{i=1}^{n}\left[1-\frac{1}{1+\left[A_i(x)\right]^{\alpha}}-\frac{i}{n+1}\right]\frac{\left[A_i(x)\right]^{\alpha-1}x_i(1+\lambda x_i)^{\beta-1}\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}}{\left\{1+\left[A_i(x)\right]^{\alpha}\right\}^{2}}$$

Where $A_i(x)=\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}-1$

Similarly we can get the weighted least square estimators by minimizing

$$M\left(X;\alpha,\beta,\lambda\right)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]^{2}$$

with respect to α, β and λ. The weights w_i are $w_i=\frac{1}{Var(X_{(i)})}=\frac{(n+1)^{2}(n+2)}{i(n-i+1)}$

Hence, we can get weighted least square estimators of α, β and λ respectively by minimizing,

$$M\left(X;\alpha,\beta,\lambda\right)=\sum_{i=1}^{n}\frac{(n+1)^{2}(n+2)}{i(n-i+1)}\left[1-\frac{1}{1+\left[\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}-1\right]^{\alpha}}-\frac{i}{n+1}\right]^{2}\qquad(3.2.3)$$

with respect to α, β and λ.

C. *Method of Cramer-Von-Mises estimation (CVME)*

The Cramer-Von-Mises estimators of α, β and λ are acquired by minimizing the function

$$C\left(X;\alpha,\beta,\lambda\right)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\,|\,\alpha,\beta,\lambda\right)-\frac{2i-1}{2n}\right]^{2}$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[1-\frac{1}{1+\left[\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}-1\right]^{\alpha}}-\frac{2i-1}{2n}\right]^{2}\qquad(3.4.1)$$

Differentiating (3.4.1) with respect to α, β and λ we get,

$$\frac{\partial C}{\partial \alpha}=-2\sum_{i=1}^{n}\left[1-\frac{1}{1+\left[A_i(x)\right]^{\alpha}}-\frac{2i-1}{2n}\right]\frac{\left[A_i(x)\right]^{\alpha}\ln\left[A_i(x)\right]}{\left\{1+\left[A_i(x)\right]^{\alpha}\right\}^{2}}$$

$$\frac{\partial C}{\partial \beta}=-2\alpha\sum_{i=1}^{n}\left[1-\frac{1}{1+\left[A_i(x)\right]^{\alpha}}-\frac{2i-1}{2n}\right]\frac{\left[A_i(x)\right]^{\alpha-1}(1+\lambda x_i)^{\beta}\ln(1+\lambda x_i)\exp\left\{(1+\lambda x_i)^{\beta}-1\right\}}{\left\{1+\left[A_i(x)\right]^{\alpha}\right\}^{2}}$$

$$\frac{\partial C}{\partial \lambda} = -2\beta \sum_{i=1}^{n} \left[1 - \frac{1}{1+[A_i(x)]^{\alpha}} - \frac{2i-1}{2n} \right] \frac{[A_i(x)]^{\alpha-1} x_i (1+\lambda x_i)^{\beta-1} \exp\left\{(1+\lambda x_i)^{\beta}-1\right\}}{\left\{1+[A_i(x)]^{\alpha}\right\}^2}$$

Where $A_i(x) = \exp\left\{(1+\lambda x_i)^{\beta}-1\right\}-1$

By solving $\frac{\partial C}{\partial \alpha} = 0, \frac{\partial C}{\partial \beta} = 0$ *and* $\frac{\partial C}{\partial \lambda} = 0$ simultaneously we will get the CVM estimators.

IV. APPLICATION IN REAL DATASET

In this portion, we demonstrate the logistic NHE distribution's real data application using two real datasets used by earlier researchers.

Dataset I

From accelerated life test of 59 conductors, the failure time data in hours with no censored observation provided in this section was derived (Nelson & Doganaksoy, 1995)[28]. Owing to the diffusion of atoms in the coils in the circuit, we may see microcircuit failure; such phenomenon is known as electro-migration.

6.545, 9.289, 7.543, 6.956, 6.492, 5.459, 8.120, 4.706, 8.687, 2.997, 8.591, 6.129, 11.038, 5.381, 6.958, 4.288, 6.522, 4.137, 7.459, 7.495, 6.573, 6.538, 5.589, 6.087, 5.807, 6.725, 8.532, 9.663, 6.369, 7.024, 8.336, 9.218, 7.945, 6.869, 6.352, 4.700, 6.948, 9.254, 5.009, 7.489, 7.398, 6.033, 10.092, 7.496, 4.531, 7.974, 8.799, 7.683, 7.224, 7.365, 6.923, 5.640, 5.434, 7.937, 6.515, 6.476, 6.071, 10.491, 5.923.

The MLEs are calculated by utilizing the optim() function in R software (R Core Team, 2020)[29] and (Mailund, 2017)[22]. by maximizing the likelihood function (3.1). We have obtained Log-Likelihood value is l = -111.2572 and the MLE's with their standard errors (SE) for α, β, and λ are presented in Table 1.

Table 1
MLE and SE for α, β and λ

Parameter	MLE	SE
alpha	4.3908	0.7999
beta	6.9896	24.1900
lambda	0.0113	0.0407

We have displayed the graph of the profile log-likelihood function of α, β, and λ in Fig. 2 (Kumar & Ligges, 2011) and observed that the MLEs are unique[16].

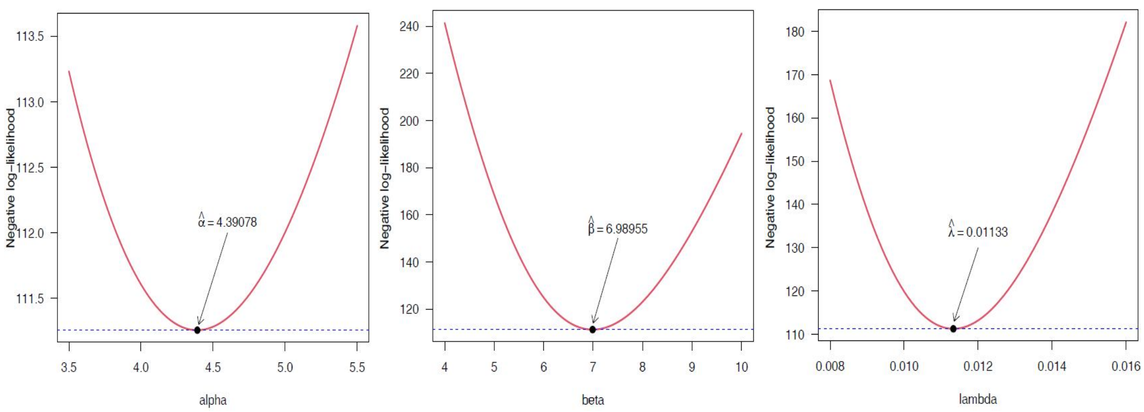

Figure 2. Graph of profile log-likelihood function of α, β, and λ.

In Table 2 we have presented the estimated value of the parameters of L-NHE distribution using MLE, LSE and CVE method and their corresponding negative log-likelihood, AIC and KS criterion.

Table 2
Estimated parameters, log-likelihood, and AIC

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-LL	AIC	KS(p-value)
MLE	4.39078	6.98955	0.01133	111.2572	228.5144	0.0486(0.9979)
LSE	4.86559	1.6381	0.05498	111.4011	228.8023	0.0493(0.9974)
CVE	5.42424	1.07152	0.09209	111.4984	228.9968	0.0465(0.9989)

In Figure 3 we have plotted the histogram and the density function of fitted distributions of estimation methods MLE, LSE and CVM and Q-Q plot and it is observed that the proposed distribution fits the data very well.

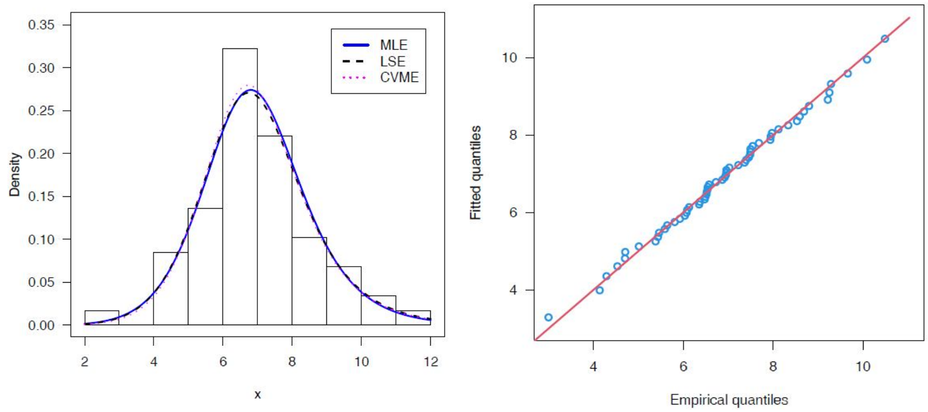

Figure 3. The Histogram and the density function of fitted distributions of estimation methods MLE, LSE and CVM (left panel) and Q-Q plot (right panel) of L-NHE distribution.

Dataset-II (Lee)
The second real data set represents the remission times (in months) of a random sample of 128 bladder cancer patients (Lee & Wang, 2003)[18]: sorted data
0.08, 0.20, 0.40, 0.50, 0.51, 0.81, 0.90, 1.05, 1.19, 1.26, 1.35, 1.40, 1.46, 1.76, 2.02, 2.02, 2.07, 2.09, 2.23, 2.26, 2.46, 2.54, 2.62, 2.64, 2.69, 2.69, 2.75, 2.83, 2.87, 3.02, 3.25, 3.31, 3.36, 3.36, 3.48, 3.52, 3.57, 3.64, 3.70, 3.82, 3.88, 4.18, 4.23, 4.26, 4.33, 4.34, 4.40, 4.50, 4.51, 4.87, 4.98, 5.06, 5.09, 5.17, 5.32, 5.32, 5.34, 5.41, 5.41, 5.49, 5.62, 5.71, 5.85, 6.25, 6.54, 6.76, 6.93, 6.94, 6.97, 7.09, 7.26, 7.28, 7.32, 7.39, 7.59, 7.62, 7.63, 7.66, 7.87, 7.93, 8.26, 8.37, 8.53, 8.65, 8.66, 9.02, 9.22, 9.47, 9.74, 10.06, 10.34, 10.66, 10.75, 11.25, 11.64, 11.79, 11.98, 12.02, 12.03, 12.07, 12.63, 13.11, 13.29, 13.80, 14.24, 14.76, 14.77, 14.83, 15.96, 16.62, 17.12, 17.14, 17.36, 18.10, 19.13, 20.28, 21.73, 22.69, 23.63, 25.74, 25.82, 26.31, 32.15, 34.26, 36.66, 43.01, 46.12, 79.05
We have obtained Log-Likelihood value is l = -409.7288 and the MLE's with their standard errors (SE) and 95% confidence interval for α, β, and λ are presented in Table 3.

Table 3
MLE and SE and 95% confidence interval for α, β and λ

Parameter	MLE	SE	95% ACI
alpha	1.4644	0.2090	(1.0548, 1.8740)
beta	0.5391	0.1463	(0.2524, 0.8259)
lambda	0.2635	0.1212	(0.0260, 0.5011)

.

We have displayed the graph of the profile log-likelihood function of α, β, and λ in Fig. 4 (Kumar & Ligges, 2011) and observed that the MLEs are unique.

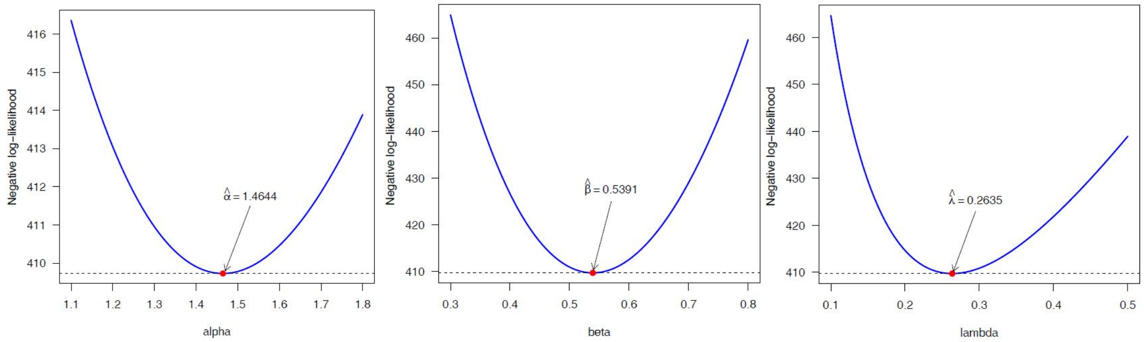

Figure 4. Graph of profile log-likelihood function of α, β, and λ.

In Table 4 we have presented the estimated value of the parameters of L-NHE distribution using MLE, LSE and CVE method and their corresponding negative log-likelihood, AIC and KS criterion.

Table 4
Estimated parameters, log-likelihood, and AIC

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-LL	AIC	KS(p-value)
MLE	1.4644	0.5391	0.2635	409.7288	825.4575	0.0341(0.9984)
LSE	1.6282	0.4488	0.3584	409.9813	825.9626	0.0300(0.9998)
CVE	1.6427	0.4521	0.3539	410.0230	826.0461	0.0305(0.9998)

In Figure 5 we have plotted the histogram and the density function of fitted distributions of estimation methods MLE, LSE and CVM and Q-Q plot and it is seen that the proposed distribution fits the data very well.

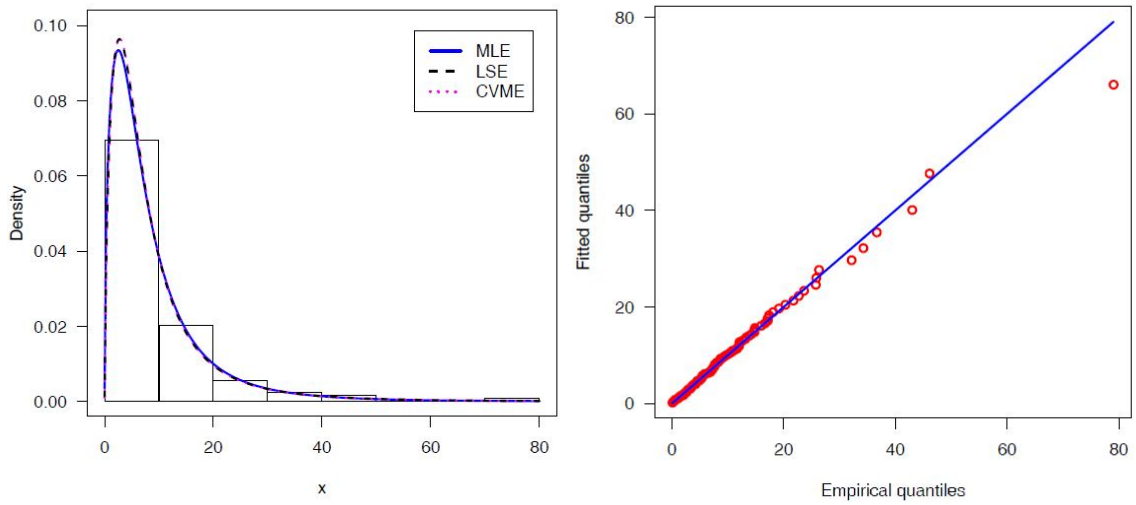

Figure 5. The Histogram and the density function of fitted distributions of estimation methods MLE, LSE and CVM (left panel) and fitted CDF (right panel) of L-NHE distribution.

To demonstrate the goodness of fit of the L-NHE distribution, we have select some well known distribution for comparison purpose which are listed blew,

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A. Generalized Exponential Extension (GEE) Distribution

The probability density function of GEE introduced by (Lemonte, 2013)[19] having upside down bathtub-shaped hazard function distribution with parameters α, β and λ is

$$f_{GEE}(x;\alpha,\beta,\lambda)=\alpha\beta\lambda(1+\lambda x)^{\alpha-1} exp\left\{1-(1+\lambda x)^{\alpha}\right\}$$
$$\left[1-exp\left\{1-(1+\lambda x)^{\alpha}\right\}\right]^{\beta-1} \quad ;\ x\geq 0.$$

B. Logistic-Exponential Distribution

The density of logistic-exponential (LE) distribution given by (Lan & Leemis, 2008)[17] with shape parameter α and scale parameter λ is

$$f_{LE}(x)=\frac{\lambda\,\alpha\, e^{\lambda x}\left(e^{\lambda x}-1\right)^{\alpha-1}}{\left\{1+\left(e^{\lambda x}-1\right)^{\alpha}\right\}^{2}} \quad ;x\geq 0,\alpha>0,\lambda>0.$$

C. Generalized Exponential (GE) Distribution

The PDF of generalized exponential distribution (Gupta & Kundu, 1999)[9] is.

$$f_{GE}(x;\alpha,\lambda)=\alpha\,\lambda\, e^{-\lambda x}\left\{1-e^{-\lambda x}\right\}^{\alpha-1};(\alpha,\lambda)>0,x>0.$$

D. Exponential power (EP) Distribution

The probability density function of Exponential power (EP) distribution (Smith & Bain, 1975)[31] is

$$f_{EP}(x)=\alpha\,\lambda^{\alpha}\, x^{\alpha-1}\, e^{(\lambda x)^{\alpha}}\exp\left\{1-e^{(\lambda x)^{\alpha}}\right\} \quad ;(\alpha,\lambda)>0,\quad x\geq 0.$$

where respectively α and λ denotes shape and scale parameters,.

E. Gompertz distribution (GZ)

The probability density function of Gompertz distribution (Murthy et al., 2003)[25] with parameters α and θ is

$$f_{GZ}(x)=\theta\; e^{\alpha x}\, exp\left\{\frac{\theta}{\alpha}\left(1-e^{\alpha x}\right)\right\} \quad ;x\geq 0,\theta>0,-\infty<\alpha<\infty.$$

For the assessment of potentiality of the proposed model we have calculated the Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC) and Hannan-Quinn information criterion (HQIC) which are presented in Table 5 and Table 6.

Table 5
Log-likelihood (LL), AIC, BIC, CAIC and HQIC (dataset-I)

Distribution	-LL	AIC	BIC	CAIC	HQIC
LNHE	111.2572	228.5144	234.7470	228.9507	230.9473
MW	112.5218	231.0435	237.2761	231.4799	233.4765
GE	114.9473	233.8946	238.0497	234.1089	235.5166
EP	116.5015	237.0029	241.1580	237.2098	238.6249
GZ	117.1740	238.3480	242.5031	238.5623	239.9700

Table 6
Log-likelihood (LL), AIC, BIC, CAIC and HQIC (dataset-II)

Distribution	-LL	AIC	BIC	CAIC	HQIC
LNHE	409.7288	825.4575	834.0136	825.6511	828.9339
GEE	410.6013	827.2026	835.7586	827.3961	830.6789
LE	412.6254	829.2507	834.9548	829.3467	831.5683
GE	413.0776	830.1552	835.8592	830.2512	832.4728
EP	426.6474	857.2948	862.9989	857.3893	859.6124

The Histogram and the density function of fitted distributions and Empirical distribution function with the estimated distribution function of NEEE and some selected distributions are presented in Figure 6.

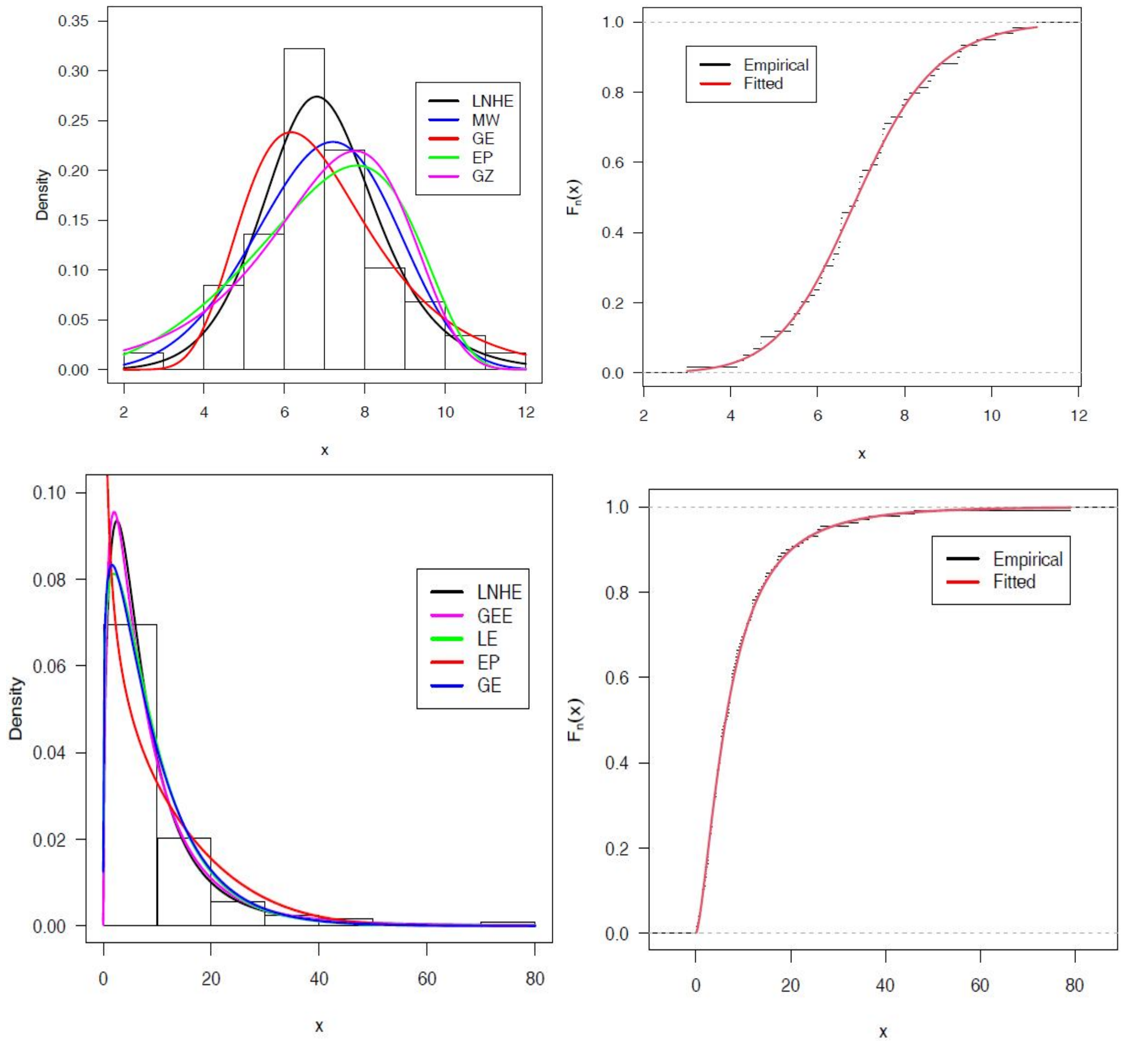

Figure 6. The Histogram and the density function of fitted distributions (first column) and Empirical distribution function with estimated distribution function (second column) respectively of datasets I and II.

For comparing the goodness-of-fit of the NEEE distribution with other competing distributions we have presented the value of Kolmogorov-Simnorov (KS), the Anderson-Darling (W) and the Cramer-Von Mises (A^2) statistics in Table 7 and Table 8. It is observed that the NEEE distribution has the minimum value of the test statistic and higher *p*-value thus we conclude that the NEEE distribution gets quite better fit and more consistent and reliable results from others taken for comparison.

Table 7
The goodness-of-fit statistics and their corresponding p-value (dataset-I)

Distribution	*KS(p-value)*	*W(p-value)*	*A^2(p-value)*
LNHE	0.0486(0.9979)	0.0211(0.9961)	0.1333(0.9995)
MW	0.0914(0.6738)	0.0821(0.6816)	0.4839(0.7626)
GE	0.1042(0.5103)	0.1173(0.5079)	0.7368(0.5282)
EP	0.1365(0.2021)	0.2398(0.2021)	1.3735(0.2098)
GZ	0.1306(0.2464)	0.216(0.2387)	1.3143(0.2277)

Table 8
The goodness-of-fit statistics and their corresponding p-value (dataset-II)

Distribution	*KS(p-value)*	*W(p-value)*	*A^2(p-value)*
LNHE	0.0341(0.9984)	0.0178(0.9987)	0.1282(0.9996)
GEE	0.0442(0.9636)	0.0394(0.9367)	0.2630(0.9631)
LE	0.0691(0.5740)	0.1131(0.5252)	0.6276(0.6219)
GE	0.0725(0.5115)	0.1279(0.4652)	0.7137(0.5472)
EP	0.1199(0.0503)	0.5993(0.0223)	3.6745(0.0126)

V. CONCLUSIONS

In this study, a three-parameter univariate continuous distribution named Logistic NHE distribution have been introduced. Some statistical and mathematical properties of the Logistic NHE distribution are presented like shapes of the probability density, cumulative density and hazard rate functions, survival function, hazard function quantile function, the skewness, and kurtosis measures are derived and established and found that the proposed model is flexible and inverted bathtub shaped hazard function. The model parameters are projected by using three well-known estimation methods namely maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods and we concluded that the MLEs are quite better than LSE, and CVM. A real data set is considered to explore the applicability and suitability of the proposed distribution and found that the proposed model is quite better than other lifetime model taken into consideration. We hope this model may be an alternative in the field of survival analysis, probability theory and applied statistics.

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