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An EOQ Model for Deteriorating and Ameliorating Items under Exponentially Increasing Demand and Partial Backlogging

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Abstract: The effect of deterioration for items cannot be disregarded in many inventory system and it is a real phenomenon in our life. Along with, few papers considered inventory system under amelioration environment where amelioration occurs when the value or utility of a product increases over time. This paper discussed the development of an inventory model of deteriorating items in presence of ameliorating environment. Here the demand rate is considered as an exponentially increasing over a fixed time horizon and shortages which is partially backlogged. Finally the model is illustrated with the help of a numerical example and the sensitivity of the optimal solution towards the changes in the values of different parameters is also studied.
Keywords: Inventory, EOQ, deteriorating, ameliorating, exponential demand, shortages, and partial backlogging.
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I. INTRODUCTION

Several researchers have addressed the importance of the deterioration phenomenon in their field of applications, as a result, many inventory models with deteriorating items have been developed. But due to lack of considering the influence of demand, the ameliorating items for the amount of inventory is increasing gradually. Amelioration is a natural phenomenon observing in much life stock models. A few researchers have focused on ameliorating system. The vast majority of the traditional stock models depend on the rule that the estimation of stock stays consistent after some time. This is an uncommon sort of stock model where scientists have concentrated on for both ameliorating and deteriorating things, where items ameliorate when stay at breeding yard and deteriorates when in the distribution systems. Hwang [1] developed an inventory model for ameliorating items only. Again Hwang [2] added to a stock model for both ameliorating and deteriorating things independently. Mallick et al. [3] has considered a creation inventory model for both ameliorating and deteriorating items. Professionals did not give much attention for fast growing animals like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in berry (pond) which are known as ameliorating items. When these items are in storage, the stock increases (in weight) due to growth of the items and also decrease due to death, various diseases or some other factors. At the point when these things are away, the stock increments (in weight) because of development of the things. Furthermore the stock diminishes because of death, different illnesses or due to some different components. Many researchers like Moon et al[4], Law et al [5], L-Q ji[6], Valliathal et al[7], Chen [8], Nodoust [9] are few noteworthy.

In the competitive market, the demand of some product may increase due to the consumer's preference on some eye-catching product. Therefore, the demand of the product at the time of its growth and the phase of declination may be approached by continuous-time-dependent function. These continuous-time-dependent functions may be a function of exponential or linear type. Ritchie [10] discussed the solution of a linear increasing time-dependent demand, which is obtained by Donaldson [11]. Silver and Meal [12] developed a model for deterministic time-varying demand, which also gives an approximate solution procedure termed as Silver-Meal Heuristic. Exponential demand has been developed by Aggarwal and Bahari-Kashani [13], Wee [14] and many other researchers.

In fact an inventory policy that allows for shortage is always less expensive to operate than a policy without shortage. Many authors like Goyal et al [15], Benkheronf [16] assumed that shortages are completely backlogged.. In practice, some customers would like to wait for backlogging during that shortage period, but other would not. Consequently, the opportunity cost due to lost sales should be considered in the modelling. Zhao[17], Giri et al. [18]. Biswaranjan Mandal [19] etc assumed that the backlogging rate was a fixed fraction of demand rate during this period. However, in some inventory system, for some fashionable commodities , the length of waiting time for next replenishment become main factor for determining whether the backlogging will be accepted or not. The longer the waiting time is, the smaller the backlogging rate would be. Therefore backlogging rate is variable and is dependent on the waiting time for the next replenishment.

For these sort of situations, efforts have been made to develop an EOQ inventory model in presence of both ameliorating and deteriorating items. The demand rate is considered as exponentially increasing demand over a fixed time horizon and shortages which is partially backlogged Finally the model is illustrated with the help of a numerical example and the sensitivity of the optimal solution towards the changes in the values of different parameters is also studied.

II. NOTATIONS AND ASSUMPTIONS

The present inventory model is developed under the following notations and assumptions:

A. Notations

1) *I(t) :* On hand inventory at time t.
2) *R(t) :* Demand rate.
3) *Q* : On-hand inventory.
4) θ : The constant deterioration rate where $0 \le \theta < 1$
5) *A* : The constant ameliorating rate.
6) *T* : The fixed length of each production cycle.
7) A_0 : The ordering cost per order during the cycle period.
8) p_c : The purchasing cost per unit item.
9) h_c : The holding cost per unit item.
10) d_c : The deterioration cost per unit item.
11) a_c : The cost of amelioration per unit item.
12) c_s : The shortage cost per unit item.
13) o_c : The opportunity cost per unit item.
14) TC : Average total cost per unit time.

B. Assumptions

1) Lead time is zero.
2) Replenishment rate is infinite but size is finite.
3) The time horizon is finite.
4) There is no repair of deteriorated items occurring during the cycle.
5) Amelioration and deterioration occur when the item is effectively in stock.
6) The demand rate is a time dependent exponentially increasing function

$$R(t) = e^{\lambda t}, \lambda > 0.$$

7) Shortages are allowed and they adopt the notation used in Abad[20], where the unsatisfied demand is backlogged and the fraction of shortages backordered is $e^{-\delta t}$, where δ is a positive constant and t is the waiting time for the next replenishment. We also assume that $te^{-\delta t}$ is an increasing function used in Skouri et al. [21].

III. MATHEMATICAL FORMULATION AND SOLUTION

In this model, we consider an EOQ model starting with no shortage. Replenishment occurs at time t = 0 and the inventory level attains its maximum. From t = 0 to t = t_1 the stock will be diminished due to the effect of amelioration, deterioration and demand, and ultimately falls to zero at t = t_1. The shortages occur during time period [t_1, T] which are partially backlogged. The behaviour of the model at any time t can be described by the following differential equations:

$$\frac{dI(t)}{dt} + (\theta - A)I(t) = -e^{\lambda t}, 0 \le t \le t_1 \qquad (1)$$

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And $$\frac{dI(t)}{dt} = -e^{\lambda t} e^{-\delta(T-t)}, t_1 \le t \le T \qquad (2)$$

The initial condition is $I(0) = Q$ and $I(t_1) = 0$ (3)

The solutions of the equations (1) and (2) using (3) are given by the following

$$I(t) = \frac{1}{\theta - A + \lambda} e^{\lambda t} \{e^{(\theta - A + \lambda)(t_1 - t)} - 1\},\ 0 \le t \le t_1 \qquad (4)$$

And $$I(t) = \frac{1}{\lambda + \delta} e^{\lambda t} e^{-\delta(T-t)} \{e^{(\lambda+\delta)(t_1 - t)} - 1\},\ t_1 \le t \le T \qquad (5)$$

Since $I(t_1) = 0$, we get the following expression of on-hand inventory from the equation (4)

$$Q = \frac{1}{\theta - A + \lambda} \{e^{(\theta - A + \lambda)t_1} - 1\} \qquad (6)$$

The total inventory holding during the time interval [0, t_1] is given by

$$I_T = \int_0^{t_1} I(t)dt = \int_0^{t_1} \frac{1}{\theta - A + \lambda} e^{\lambda t} \{e^{(\theta - A + \lambda)(t_1 - t)} - 1\} dt$$

$$= \frac{1}{\theta - A + \lambda} [\frac{e^{\lambda t_1}}{\theta - A} \{e^{(\theta - A)t_1} - 1\} - \frac{1}{\lambda} \{e^{\lambda t_1} - 1\}] \qquad (7)$$

The total number of deteriorated units during the inventory cycle is given by

$$D_T = \theta \int_0^{t_1} I(t)dt$$

$$= \frac{\theta}{\theta - A + \lambda} [\frac{e^{\lambda t_1}}{\theta - A} \{e^{(\theta - A)t_1} - 1\} - \frac{1}{\lambda} \{e^{\lambda t_1} - 1\}] \qquad (8)$$

The total number of ameliorating units during the inventory cycle is given by

$$A_T = A \int_0^{t_1} I(t)dt$$

$$= \frac{A}{\theta - A + \lambda} [\frac{e^{\lambda t_1}}{\theta - A} \{e^{(\theta - A)t_1} - 1\} - \frac{1}{\lambda} \{e^{\lambda t_1} - 1\}] \qquad (9)$$

The total number of shortages during the period [t_1, T] is given by

$$S_T = \int_{t_1}^{T} -I(t)dt$$

$$= -\frac{1}{\lambda + \delta} e^{-\delta T} e^{(\lambda+\delta)t_1} [T - t_1 - \frac{1}{\lambda + \delta} \{e^{(\lambda+\delta)(T - t_1)} - 1\}] \qquad (10)$$

The amount of lost sales during the period [t_1,T] is given by

$$L_T = \int_{t_1}^{T} R(t)\{1-e^{-\delta(T-t)}\}dt = \int_{t_1}^{T} e^{\lambda t}\{1-e^{-\delta(T-t)}\}dt$$

$$= \frac{1}{\lambda}e^{\lambda t_1}\{e^{\lambda(T-t_1)}-1\} - \frac{1}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}\{e^{(\lambda+\delta)(T-t_1)}-1\} \qquad (11)$$

IV. COST COMPONENTS

The total cost over the period [0, T] consists of the following cost components:

1) Ordering cost **(OC)** over the period [0,T] = A_0 (fixed)

2) Purchasing cost **(PC)** over the period [0,T] **=** p_c I(0) = p_c Q

$$= p_c\left[\frac{1}{\theta-A+\lambda}\{e^{(\theta-A+\lambda)t_1}-1\}\right]$$

3) Holding cost for carrying inventory **(HC)** over the period [0,T] = $h_c I_T$

$$= \frac{h_c}{\theta-A+\lambda}\left[\frac{e^{\lambda t_1}}{\theta-A}\{e^{(\theta-A)t_1}-1\} - \frac{1}{\lambda}\{e^{\lambda t_1}-1\}\right]$$

4) Cost due to deterioration **(CD)** over the period [0,T] = $d_c D_T$

$$= \frac{d_c\theta}{\theta-A+\lambda}\left[\frac{e^{\lambda t_1}}{\theta-A}\{e^{(\theta-A)t_1}-1\} - \frac{1}{\lambda}\{e^{\lambda t_1}-1\}\right]$$

5) The amelioration cost **(AMC)** over the period [0,T] = $a_c A_T$

$$= \frac{a_c A}{\theta-A+\lambda}\left[\frac{e^{\lambda t_1}}{\theta-A}\{e^{(\theta-A)t_1}-1\} - \frac{1}{\lambda}\{e^{\lambda t_1}-1\}\right]$$

6) Cost due to shortage **(CS)** over the period [0,T] = $c_s S_T$

$$= -\frac{c_s}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}\left[T-t_1-\frac{1}{\lambda+\delta}\{e^{(\lambda+\delta)(T-t_1)}-1\}\right]$$

7) Opportunity Cost due to lost sales **(OPC)** over the period [0,T] = $o_c\, L_T$

$$= o_c\left[\frac{1}{\lambda}e^{\lambda t_1}\{e^{\lambda(T-t_1)}-1\} - \frac{1}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}\{e^{(\lambda+\delta)(T-t_1)}-1\}\right]$$

The average total cost per unit time of the system during the cycle [0,T] will be

$$TC(t_1) = \frac{1}{T}\,[\,\mathbf{OC+PC+HC+CD+AMC+CS+OPC}\,]$$

$$= \frac{1}{T}\Big[A_0 + p_c\left[\frac{1}{\theta-A+\lambda}\{e^{(\theta-A+\lambda)t_1}-1\}\right] + \frac{(h_c+\theta d_c+Aa_c)}{\theta-A+\lambda}\left[\frac{e^{\lambda t_1}}{\theta-A}\{e^{(\theta-A)t_1}-1\} - \frac{1}{\lambda}\{e^{\lambda t_1}-1\}\right]$$

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$$-\frac{c_s}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}[T-t_1-\frac{1}{\lambda+\delta}\{e^{(\lambda+\delta)(T-t_1)}-1\}]$$

$$+\ o_c[\frac{1}{\lambda}e^{\lambda t_1}\{e^{\lambda(T-t_1)}-1\}-\frac{1}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}\{e^{(\lambda+\delta)(T-t_1)}-1\}]] \quad (12)$$

For minimum, the necessary condition is $\frac{dTC(t_1)}{dt_1}=0$

This gives

$$p_c e^{(\theta-A)t_1} + (h_c+\theta d_c+Aa_c)\frac{1}{\theta-A}\{e^{(\theta-A)t_1}-1\} - c_s e^{-\delta(T-t_1)}(T-t_1) + o_c\{e^{-\delta(T-t_1)}-1\} = 0 \quad (13)$$

For minimum the sufficient condition $\frac{d^2TC(t_1)}{dt_1^{\,2}}>0$ would be satisfied.

Let $t_1=t_1^*$ be the optimum value of t_1.

The optimal values Q^* of Q and TC^* of TC are obtained by putting the value $t_1=t_1^*$ from the expressions (6) and (12).

V. SOME PARTICULAR CASES

A. *Absence of Deterioration*

If the deterioration of items is switched off i.e. $\theta=0$, then the expressions (6) and (12) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q=\frac{1}{\lambda-A}\{e^{(\lambda-A)t_1}-1\} \quad (14)$$

$$\text{And } TC(t_1)=\frac{1}{T}[\ A_0\ +\ p_c\,[\frac{1}{-A+\lambda}\{e^{(-A+\lambda)t_1}-1\}\,]+\frac{(h_c+Aa_c)}{-A+\lambda}[\frac{e^{\lambda t_1}}{-A}\{e^{-At_1}-1\}-\frac{1}{\lambda}\{e^{\lambda t_1}-1\}]$$

$$-\frac{c_s}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}[T-t_1-\frac{1}{\lambda+\delta}\{e^{(\lambda+\delta)(T-t_1)}-1\}]$$

$$+\ o_c[\frac{1}{\lambda}e^{\lambda t_1}\{e^{\lambda(T-t_1)}-1\}-\frac{1}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}\{e^{(\lambda+\delta)(T-t_1)}-1\}]] \quad (15)$$

The equation (13) becomes

$$p_c e^{-At_1} - (h_c+Aa_c)\frac{1}{A}(e^{-At_1}-1) - c_s e^{-\delta(T-t_1)}(T-t_1) + o_c\{e^{-\delta(T-t_1)}-1\} = 0 \quad (16)$$

This gives the optimum value of t_1.

B. *Absence of Amelioration*

If the amelioration of items is switched off i.e. $A=0$, then the expressions (6) and (12) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q=\frac{1}{\theta+\lambda}\{e^{(\theta+\lambda)t_1}-1\} \quad (17)$$

$$\text{And } TC\,(t_1)=\frac{1}{T}[\ A_0\ +\ p_c\,[\frac{1}{\theta+\lambda}\{e^{(\theta+\lambda)t_1}-1\}\,]+\frac{(h_c+\theta d_c)}{\theta+\lambda}[\frac{e^{\lambda t_1}}{\theta}\{e^{\theta t_1}-1\}-\frac{1}{\lambda}\{e^{\lambda t_1}-1\}]$$

$$-\frac{c_s}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}[T-t_1-\frac{1}{\lambda+\delta}\{e^{(\lambda+\delta)(T-t_1)}-1\}]$$

$$+\ o_c[\frac{1}{\lambda}e^{\lambda t_1}\{e^{\lambda(T-t_1)}-1\}-\frac{1}{\lambda+\delta}e^{-\delta T}e^{(\lambda+\delta)t_1}\{e^{(\lambda+\delta)(T-t_1)}-1\}]] \quad (18)$$

The equation (13) becomes

$$p_c e^{\theta t_1}\ +\ (h_c+\theta d_c)\frac{1}{\theta}(e^{\theta t_1}-1)\ -\ c_s e^{-\delta(T-t_1)}(T-t_1)+\ o_c\{e^{-\delta(T-t_1)}-1\}\ =0 \quad (19)$$

This gives the optimum value of t_1.

VI. NUMERICAL EXAMPLE

To illustrate the developed inventory model, let the values of parameters be as follows:

A_0 = \$500 per order ; λ = 0.1 ; θ = 0.8; A= 0.01; δ = 10; p_c = \$ 5 per unit, h_c = \$ 4 per unit; d_c = \$9 per unit; a_c = \$ 6 per unit; c_s = \$10 per unit; o_c = \$12 per unit; T = 1 year

Solving the equation (13) with the help of computer using the above values of parameters, we find the following optimum outputs t_1^* = 0.40 year; Q^* = 0.48 units and TC^* = Rs. 551.01

It is checked that this solution satisfies the sufficient condition for optimality.

VII. SENSITIVITY ANALYSIS AND DISCUSSION.

We now study the effects of changes in the system parameters $A_0, p_c, h_c, d_c, a_c, c_s, o_c, \theta, A, \delta$ and λ on the optimal on-hand inventory quantity (Q^*) and the optimal total cost (TC^*) in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A.

Table A: Effect of changes in the parameters on the model

Changing parameter	% change in the system parameter	% change in	
		Q^*	TC^*
A_0	-50	No effect	- 45.38
	-20		- 18.15
	+20		18.14
	+50		45.37
p_c	-50	55.27	0.11
	-20	19.60	0.05
	+20	-16.07	- 0.05
	+50	-37.05	- 0.13
h_c	-50	13.30	0.06
	-20	4.91	0.02
	+20	- 5.58	- 0.03
	+50	- 9.78	- 0.05
d_c	-50	30.09	0.14
	-20	9.11	0.04
	+20	- 7.68	- 0.03
	+50	-16.07	- 0.05
a_c	-50	0.17	- 0.004
	-20	0.08	- 0.005
	+20	- 0.06	0.002
	+50	- 0.17	0.006

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c_s	-50 -20 +20 +50	- 0.03 - 0.01 0.02 0.04	- 0.01 - .009 0.008 0.005
o_c	-50 -20 +20 +50	- 7.04 - 29.83 32.29 85.58	- 0.48 - 0.22 0.25 0.73
θ	-50 -20 +20 +50	26.23 15.69 - 10.47 - 20.97	0.18 0.09 - 0.07 - 0.13
A	-50 -20 +20 +50	0.06 0.02 - 0.02 - 0.04	- 0.005 - 0.004 0.002 0.004
δ	-50 -20 +20 +50	- 2.31 - 0.15 0.10 0.15	- 0.08 - 0.05 0.03 0.06
λ	-50 -20 +20 +50	- 1.05 - 0.42 0.42 2.20	2.62 1.99 - 1.33 - 2.65

Analyzing the results of table A, the following observations may be made:

1) TC^* increases or decreases with the increase or decrease in the values of the system parameters A_0, a_c, c_s, o_c, A and δ. On the other hand TC^* increases or decreases with the decrease or increase in the values of the system parameters p_c, h_c, d_c, θ and λ. The results obtained show that TC^* is highly sensitive towards changes of ordering cost A_0, moderate sensitive towards the changes of p_c, d_c, o_c, θ and λ. It is less sensitive towards the changes of h_c, a_c, c_s, A and δ.

2) Q^* increases or decreases with the increase or decrease in the values of the system parameters c_s, o_c, δ and λ. On the other hand Q^* increases or decreases with the decrease or increase in the values of the system parameters $p_c, h_c, d_c, a_c, \theta$ and A. The results obtained show that Q^* is highly sensitive towards changes of p_c, h_c, d_c, o_c and θ, moderate sensitive towards the changes of a_c, δ and λ. It is less sensitive towards the changes of c_s and A.

From the above analysis, it is seen that the ordering cost A_0 is a highly sensitive and critical parameter in the sense that any error in the estimation of A_0 results in significant errors in the optimal cost. Hence, proper adequate attention must be taken to estimate the ordering cost parameter A_0.

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