iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 9 **Issue:** III **Month of publication:** March 2021

DOI: https://doi.org/10.22214/ijraset.2021.33311

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue III Mar 2021- Available at www.ijraset.com

Isothermal Strong Spherical Shock Waves in Uniform Medium

Dharmender Singh[1], Harminder Singh[2], Akhlesh Kumar[3],
[1]*Deptt. of Physics, Gram Swablambi Vidhyalaya Inter College, Chhatikara, Mathura (U.P.)*
[2]*Deptt. of Physics Govt. College, Bareilly (U.P.)*
[3]*Principal, Bharat Inter College Bhojipura, Bareilly (UP)*

Abstract: Propagation of isothermal strong spherical shock in uniform medium has been studied by Whitham Method, taking the effect of overtaking disturbances. The result obtained are compared with those obtained by Chisnell's method. The effect of radiation heat flux plays an important role has also been includes in this study.
Keywords: Shock waves, Converging, Radiation heat flux.

I. INTRODUCTION

The study of propagation of spherical shock wave is of great interest due to its application in many practical situations in nuclear and space research. In (1979), Singh studied the converging shocks with heat addition. He gave the propagation law for converging detonation waves and modify Chisnell's method for explosive media. Using Conger's method of folded co-ordinates, he found that detonation wave become stronger and stronger as the wave front approaches the centre. In (1986), Crighton discussed about the Taylor's internal structure of weak shock waves. In (1995), Chen et al. studied about stability of imploding spherical shock waves. In (1998), Rai studied about the problem of converging spherical detonation waves propagating through a gas with varying density.
In (1998), Steiner et al. obtained the numerical solution for spherical laser-driven shock waves. Here they considered that the total laser-energy absorbed by the blast was assumed to vary proportionally to some power of time. In (1999), Singh et al. self-similar solutions investigated behind the magneto radiative spherical shock wave propagating in uniform atmosphere. An idealized azimuthal magnetic field and radiative heat flux had been taken into consideration but radiation pressure and energy had been ignored. In (2003), Gretler and Regenfelder discussed about similarity solution for variable energy shock waves in a dusty gas under isothermal flow-field conditions. In (2007), Singh and Vishwakarma studied the propagation of spherical shock waves in a dusty gas with radiation heat-flux. The effects of an increase in (i) the mass concentration of solid particles in the mixture and (ii) of the ratio of the density of solid particles to the initial density of gas on the flow variables in the region behind the shock had been investigated. The aim of the present chapter is to study the effect of overtaking disturbances on the results of CCW theory applied to spherical isothermal shock in uniform medium.
Here we have derived for both strong and weak shock, the analytical relations for shock velocity and shock strength for two cases (i) when shock moves freely and (ii) when effect of overtaking disturbances is taking into account (the condition □=1 and □□= -2.5 fulfill the condition of uniformity of the medium) and computed the expressions to show the results graphically.
Finally, the flow variables (non-dimensional particle velocity, non-dimensional pressure and non-dimensional radiative heat flux) have been obtained and discussed through tables. For strong shock, shock velocity and shock strength decreases with propagation distance r and specific heat index □ in both cases FP and EOD. Flow variables also decreases with propagation distance r as well as with specific heat index □ in both freely propagation and in presence of overtaking disturbances.

II. BASIC EQUATIONS AND BOUNDARY CONDITIONS

The equations governing the flow of the gas enclosed by the shock front for spherical symmetry are

$$\frac{\partial \rho}{\partial t}+\rho\frac{\partial u}{\partial r}+u\frac{\partial \rho}{\partial r}+\frac{2\rho u}{r}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0 \qquad (2)$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)e+p\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\frac{1}{\rho}+\frac{1}{\rho}\left(\frac{\partial}{\partial r}+\frac{2}{r}\right)F=0 \qquad (3)$$

where, $e=\dfrac{p}{\rho(\gamma-1)}$ and p = $\Gamma\rho T$ (4)

where, r is the radial co-ordinate, u, p, ρ, γ, Γ and T are the particle velocity, pressure, density, specific heat index, gas constant and temperature, respectively.

The variation in the radiative heat flux (F) in transparent limit is (Helliwell-1966).

$$\frac{\partial F}{\partial r}+\frac{2F}{r}=4\pi KB \qquad (5)$$

where, B is the Planck's radiation function and K is the local volumetric absorption coefficient are given by

$$B=\frac{\sigma T^4}{\pi}, K=k\rho^{\alpha}T^{\beta} \qquad (6)$$

where, σ is the Stefan's constant and α, β are constants.

III. BOUNDARY CONDITIONS

Let p_0 and ρ_0 denote the undisturbed values of pressure and density in front of the shock wave, and u, p, and ρ the values of the respective quantities at any point immediately, after the passage of the shock, then the well known Bhowmick conditions are given below

The magneto - hydrodynamic shock conditions (Bhowmick - 1980) can be written in terms of mach number ($M=U/a_0$) as

$$u=a_0\left[M-\frac{1}{\gamma M}\right]$$

$$\rho=\rho_0\gamma M^2$$

$$p=\rho_0 a_0^2 M^2 \qquad (7)$$

$$F=\frac{\rho_0 a_0^3}{2}\left[\frac{1}{\gamma^2 M}-M^3\right]$$

where, the subscript '0' stands for the state just ahead of the shock, a_0 and U are the sound velocity $(=\sqrt{\gamma p_0/\rho_0}$ and shock velocity, respectively.

A. *For Strong shock*

$M=\dfrac{U}{a_0}, M>>1,$ the general conditions (7) reduce to

$$u = U - \frac{a_0^2}{\gamma U}$$

$$\rho = \frac{\rho_0 \gamma}{a_0^2} U^2 \qquad (8)$$

$$p = \rho_0 U^2$$

$$F = -\frac{\rho_0}{2} U^3$$

Theory

The characteristic form of the system of equations (1-4) in only one direction in (r, t) plane is given by

$$dp + \rho a du + \left[\frac{2ua^2\rho}{r} + \frac{(\gamma - 1)}{r^2} \frac{\partial}{\partial r}(r^2 F) \right] \frac{dr}{(u+a)} = 0 \qquad (9)$$

In case of uniform atmosphere

$p_0 = \rho_0 =$ constant (10)

B. For Strong Shock

The freely propagation of strong spherical shock wave in uniform medium

For uniform medium, ρ_0= constant

using equation (8), we get

$$\left[\because p = \rho_0 U^2 \right]$$

$$dp = 2\rho_0 U dU \qquad \text{(i)}$$

$$du = dU + \frac{a_0^2}{\gamma} \frac{1}{U^2} dU \qquad \left[\because u = U - \frac{a_0^2}{\gamma U} \right]$$

$$du = \left\{ \frac{\gamma U^2 + a_0^2}{\gamma U^2} \right\} dU$$

now, $a\rho du = \frac{a_0 \rho_0 \gamma U^2}{a_0^2} \left\{ \frac{\gamma U^2 + a_0^2}{\gamma U^2} \right\} dU$

$$\rho = \frac{\rho_0 \gamma U^2}{a_0^2}$$

$$a\rho du = \frac{\rho_0}{a_0} \left\{ \gamma U^2 + a_0^2 \right\} dU \qquad \text{(ii)}$$

$$u + a = U - \frac{a_0^2}{\gamma U} + a_0 = U\left[1 + \frac{a_0}{U}\right] \qquad \text{(iii)}$$

$$\frac{2u\rho a^2}{r} = \frac{2}{r}\left\{U - \frac{a_0^2}{\gamma U}\right\}\left\{\rho_0 \gamma \frac{U^2}{a_0^2}\right\}a_0^2$$

$$\frac{2u\rho a^2}{r(u+a)} = \frac{2}{r}\left\{U - \frac{a_0^2}{\gamma U}\right\}\left\{\rho_0 \gamma U^2\right\}/U\left(1 + \frac{a_0}{U}\right)$$

$$= \frac{2}{r}\rho_0 \gamma U^2\left\{U - \frac{a_0^2}{\gamma U}\right\}\frac{1}{U}\left\{1 + \frac{a_0}{U}\right\}^{-1}$$

$$= \frac{2}{r}\rho_0 \gamma U\left\{U - \frac{a_0^2}{\gamma U}\right\}\left\{1 - \frac{a_0}{U}\right\}$$

neglecting higher power terms, of a_0 / U because $U / a_0 >> 1$

$$\therefore \quad \frac{2u\rho a^2}{r(u+a)} = \frac{2}{r}\rho_0 \gamma U\left\{U - a_0 - \frac{a_0^2}{\gamma U} + \frac{a_0^3}{\gamma U^2}\right\}$$

$$= \frac{2}{r}\rho_0\left\{\gamma U^2 - a_0 \gamma U - a_0^2 + \frac{a_0^3}{U}\right\} \qquad \text{(iv)}$$

using equations (4), (5) and (6), we get

$$(\gamma - 1)\left(\frac{\partial F}{\partial r} + \frac{2F}{r}\right) = (\gamma - 1)4k\rho^{\alpha} T^{\beta} \sigma T^4$$

$$\frac{(\gamma - 1)}{r^2}\frac{\partial}{\partial r}(r^2 F) = N\rho^{\alpha - \beta - 4} p^{\beta + 4} \qquad \text{(v)}$$

where $N = \dfrac{4k(\gamma - 1)\sigma}{\Gamma^{\beta + 4}}$ (vi)

using equation (8), we get

$$\frac{(\gamma - 1)}{r^2(u+a)}\frac{\partial}{\partial r}(r^2 F) = N\left\{\frac{\rho_0 \gamma U^2}{a_0^2}\right\}^{\alpha - \beta - 4}\left\{\rho_0 U^2\right\}^{\beta + 4}/U\left\{1 + \frac{a_0}{U}\right\} \qquad \text{(vii)}$$

$$\frac{(\gamma - 1)}{r^2(u+a)}\frac{\partial}{\partial r}(r^2 F) = N\frac{\rho_0^{\alpha}}{a_0^{2(\alpha - \beta - 4)}}\gamma^{\alpha - \beta - 4}\left\{U^{2\alpha - 1} - a_0 U^{2\alpha - 2}\right\} \qquad \text{(viii)}$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue III Mar 2021- Available at www.ijraset.com

using all these values in equation (10), we get

$$\left(2U+\frac{\gamma U^2}{a_0}+a_0\right)dU+\left\{\frac{2}{r}\left(\gamma U^2-a_0\gamma U-a_0^2+\frac{a_0^3}{U}\right)+\frac{N\rho_0^{\alpha-1}\gamma^{\alpha-\beta-4}}{a_0^{2(\alpha-\beta-4)}}\left(U^{2\alpha-1}-a_0U^{2\alpha-2}\right)\right\}dr=0$$

The atmosphere will be uniform, if α=1.5 and β=-2.5, we get

$$\left(2U+\frac{\gamma U^2}{a_0}+a_0\right)dU+\left\{\frac{2}{r}\left(\gamma U^2-a_0\gamma U-a_0^2+\frac{a_0^3}{U}\right)+N\sqrt{\rho_0}\left(U^2-a_0U\right)\right\}dr=0$$

dividing this equation by U^2 and simplifying, we get

$$\left(\frac{2}{U}+\frac{\gamma}{a_0}+\frac{a_0}{U^2}\right)dU+\left\{\frac{2}{r}\left(\gamma-\frac{a_0\gamma}{U}-\frac{a_0^2}{U^2}+\frac{a_0^3}{U^3}\right)+N\sqrt{\rho_0}\left(1-\frac{a_0}{U}\right)\right\}dr=0$$

for strong shock $\frac{U}{a_0}>>1,$ the above equation becomes

$$\left(\frac{2}{U}+\frac{\gamma}{a_0}\right)dU+\left\{\frac{2}{r}\gamma+N\sqrt{\rho_0}\right\}dr=0 \qquad (12)$$

integrating equation (12), we get

$$2\log_e U+\frac{\gamma}{a_0}U+2\gamma\log_e r+N\sqrt{\rho_0}\;r=K_1'$$

where, K_1' is a constant of integration.

i.e. $$-2\log_e\left[1+\left(\frac{1}{U}-1\right)\right]+\frac{\gamma}{a_0}U+2\gamma\log_e r+N\sqrt{\rho_0}\;r=K_1'$$

$$-2\left(\frac{1}{U}-1\right)+\frac{\gamma}{a_0}U+2\gamma\log_e r+N\sqrt{\rho_0}\;r=K_1'$$

multiplying by U, we get

$$-2+2U+\frac{\gamma}{a_0}U^2+2\gamma U\log_e r+N\sqrt{\rho_0}\;rU=K_1'U$$

$$\frac{\gamma}{a_0}U^2+\left\{2+2\gamma\log_e r+N\sqrt{\rho_0}\;r-K_1'\right\}U-2=0$$

simplifying this equation, we get the expression for **Shock velocity**

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue III Mar 2021- Available at www.ijraset.com

$$U_{+}=\frac{a_0}{2\gamma}\left\{-\left(2+2\gamma\log_e r+N\sqrt{\rho_0}r-K_1^{'}\right)+\sqrt{\left(2+2\gamma\log_e r+N\sqrt{\rho_0}r-K_1^{'}\right)^2+\frac{8\gamma}{a_0}}\right\} \quad (13)$$

and **Shock strength**

$$\frac{U_{+}}{a_a}=\frac{1}{2\gamma}\left\{-\left(2+2\gamma\log_e r+N\sqrt{\rho_0}r-K_1^{'}\right)+\sqrt{\left(2+2\gamma\log_e r+N\sqrt{\rho_0}r-K_1^{'}\right)^2+\frac{8\gamma}{a_0}}\right\} \quad (14)$$

C. Propagation Of Spherical Shock Wave In Presence Of Overtaking Disturbances

To estimate the effect of overtaking disturbances, we are using the differential equation.

$$dp-a\rho du+\left[\frac{2ua^2\rho}{r}+\frac{(\gamma-1)}{r^2}\frac{\partial}{\partial r}(r^2F)\right]\frac{dr}{(u-a)}=0 \quad (15)$$

from equations (i), (ii), (v) and (vi), we have

$$dp=2\rho_0 UdU \quad (i)$$

$$du=\left[\frac{\gamma U^2+a_0^2}{\gamma U^2}\right]dU$$

$$a\rho du=\frac{\rho_0}{a_0}(\gamma U^2+a_0^2)dU \quad (ii)$$

using equations (4), (5) and (6), we get

$$(\gamma-1)\left(\frac{\partial F}{\partial r}+\frac{2F}{r}\right)=(\gamma-1)4\pi k\rho^{\alpha}T^{\beta}\sigma\frac{T^4}{\pi}$$

$$\frac{(\gamma-1)}{r^2}\left\{\frac{\partial}{\partial r}(r^2F)\right\}=N\rho^{(\alpha-\beta-4)}p^{\beta+4} \quad (v)$$

where, $N=\dfrac{4k\sigma(\gamma-1)}{\Gamma^{\beta+4}}$ (vi)

so, $u-a=U\left(1-\dfrac{a_0}{U}\right)$ (ix)

and $\dfrac{2u\rho a^2}{r}=\dfrac{2}{r}\left\{U-\dfrac{a_0^2}{\gamma U}\right\}\left\{\dfrac{\rho_0\gamma U^2}{a_0^2}\right\}a_0^2$

$$\frac{2u\rho a^2}{r(u-a)}=\frac{2}{r}\left\{U-\frac{a_0^2}{\gamma U}\right\}\left\{\rho_0\gamma U^2\right\}/U\left(1-\frac{a_0}{U}\right)$$

neglecting higher power terms of a_0/U because $U/a_0>>1$

$$\frac{2u\rho a^2}{r(u-a)}=\frac{2}{r}\rho_0\gamma U\left\{U+a_0-\frac{a_0^3}{\gamma U^2}-\frac{a_0^2}{\gamma U}\right\}$$

$$\frac{2u\rho a^2}{r(u-a)}=\frac{2}{r}\rho_0\left\{\gamma U^2+a_0\gamma U-\frac{a_0^3}{U}-a_0^2\right\} \tag{x}$$

hence

$$\frac{(\gamma-1)}{r^2(u-a)}\frac{\partial}{\partial r}(r^2F)=\frac{N\rho_0^\alpha\gamma^{(\alpha-\beta-4)}}{a_0^{2(\alpha-\beta-4)}}\left(U^{2\alpha-1}+a_0U^{2\alpha-2}\right) \tag{xi}$$

using all these values equation (15) gives

$$2\rho_0 UdU-\frac{\rho_0}{a_0}\left(\gamma U^2+a_0^2\right)dU+\left[\frac{2}{r}\rho_0\left(\gamma U^2+a_0\gamma U-a_0^2-\frac{a_0^3}{U}\right)+\frac{N\rho_0^\alpha\gamma^{(\alpha-\beta-4)}}{a_0^{2(\alpha-\beta-4)}}\left(U^{2\alpha-1)}+a_0U^{2\alpha-2}\right)\right]dr=0$$

The atmosphere will be uniform if α=1.5 and β= –2.5, we get,

$$\left(\frac{2}{U}-\frac{\gamma}{a_0}-\frac{a_0^2}{U^2}\right)dU+\left[\frac{2}{r}\left(\gamma+\frac{a_0}{U}\gamma-\frac{a_0^2}{U^2}-\frac{a_0^3}{U^3}\right)\right.$$

$$\left.+N\sqrt{\rho_0}\left(1+\frac{a_0}{U}\right)\right]dr=0$$

for strong shock ,$U/a_0>>1$, the above equation becomes

$$\left(\frac{2}{U}-\frac{\gamma}{a_0}\right)dU+\left[\frac{2\gamma}{r}+N\sqrt{\rho_0}\right]dr=0$$

simplifying this equation, we get

$$-\frac{\gamma}{a_0}dU+\left[\frac{2\gamma}{r}+N\sqrt{\rho_0}\right]dr=0$$

integrating this equation, we get

$$U=U_-=\frac{a_0}{\gamma}\left(2\gamma\log_e r+N\sqrt{\rho_0}r-K_2^{'}\right) \tag{16}$$

where $K_2^{'}$ is a constant of integration.

In presence of overtaking disturbances, the resultant pressure increment for strong shock is written as

$$dp=dp_++dp_-$$

using equation (8), we get

$$d\left\{\rho_0U^{*2}\right\}=d\left\{\rho_0U_+^2\right\}+d\left\{\rho_0U_-^2\right\} \tag{17}$$

Here U_+ is used for freely propagation and U_- is used for overtaking disturbances.
simplifying equation (17), we get

$$U^{*2} = U_+^2 + U_-^2 + \frac{K'}{\rho_0}$$

where, K′ is a constant of integration.
using equation (13) and (16), we get

$$U^{*2} = \frac{a_0^2}{4\gamma^2}\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right) + \sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right)^2 + \frac{8\gamma}{a_0}}\right\}^2$$

$$+\frac{a_0^2}{\gamma^2}\left(2\gamma\log r + N\sqrt{\rho_0}r - K_2'\right)^2 + \frac{K'}{\rho_0}$$

therefore, the expression for Modified shock velocity

$$U^{*} = \frac{a_0}{2\gamma}\left[\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right) + \sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right)^2 + \frac{8\gamma}{a_0}}\right\}^2\right.$$

$$\left. + 4\left(2\gamma\log r + N\sqrt{\rho_0}r - K_2'\right)^2 + \frac{4K'\gamma^2}{\rho_0 a_0^2}\right]^{1/2} \tag{18}$$

and the expression for Modified shock strength

$$\frac{U^{*}}{a_0} = \frac{1}{2\gamma}\left[\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right) + \sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right)^2 + \frac{8\gamma}{a_0}}\right\}^2\right.$$

$$\left. + 4\left(2\gamma\log_e r + N\sqrt{\rho_0}r - K_2'\right)^2 + \frac{4K'\gamma^2}{\rho_0 a_0^2}\right]^{1/2} \tag{19}$$

The expressions for non-dimensional particle velocity, non-dimensional pressure and non-dimensional radiative heat flux immediately behind the shock for freely propagation (u/a_0, p/p_0, $F/\rho_0 a_0^3$) and under the influence of overtaking disturbances (u^*/a_0, p^*/p_0, $F^*/\rho_0 a_0^3$) are

Non-dimensional particle velocity

$$\frac{u}{a_0} = \frac{1}{2\gamma}\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right) + \sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right)^2 + \frac{8\gamma}{a_0}}\right\}$$

$$-2\left[\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right) + \sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1'\right) + \frac{8\gamma}{a_0}}\right\}\right]^{-1} \tag{20}$$

Non-dimensional pressure

$$\frac{p}{p_0} = +\frac{1}{4\gamma}\left[-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)+\sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)+\frac{8\gamma}{a_0}}\right]^2 \qquad (21)$$

Non-dimensional radiative heat flux

$$\frac{F}{\rho_0 a_0^3} = -\frac{1}{16\gamma^3}\left[-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)+\sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)^2+\frac{8\gamma}{a_0}}\right]^3 \qquad (22)$$

Modified non-dimensional particle velocity

$$\frac{u^*}{a_0} = \frac{1}{2\gamma}\left[\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)+\sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)^2+\frac{8\gamma}{a_0}}\right\}^2\right.$$

$$\left.+4\left(2\gamma\log_e r + N\sqrt{\rho_0}r - K_2^{'}\right)^2+\frac{4K^{'}\gamma^2}{\rho_0 a_0^2}\right]^{1/2}$$

$$-2\left[\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)+\sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)^2+\frac{8\gamma}{a_0}}\right\}^2\right.$$

$$\left.+4\left(2\gamma\log_e r + N\sqrt{\rho_0}r - K_2^{'}\right)^2+\frac{4K^{'}\gamma^2}{\rho_0 a_0^2}\right]^{-1/2} \qquad (23)$$

Modified non-dimensional pressure

$$\frac{p^*}{p_0} = \frac{1}{4\gamma}\left[\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)+\sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)^2+\frac{8\gamma}{a_0}}\right\}^2\right.$$

$$\left.+4\left(2\gamma\log_e r + N\sqrt{\rho_0}r - K_2^{'}\right)^2+\frac{4K^{'}\gamma^2}{\rho_0 a_0^2}\right] \qquad (24)$$

Modified non-dimensional particle velocity

$$\frac{F^*}{\rho_0 a_0^3} = \frac{-1}{16\gamma^3}\left[\left\{-\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)+\sqrt{\left(2+2\gamma\log_e r + N\sqrt{\rho_0}r - K_1^{'}\right)^2+\frac{8\gamma}{a_0}}\right\}^2\right.$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue III Mar 2021- Available at www.ijraset.com

$$+4\left(2\gamma\log_e r+N\sqrt{\rho_0}r-K_2^{'}\right)^2+\frac{4K^{'}\gamma^2}{\rho_0 a_0^2}\Bigg]^{3/2} \tag{25}$$

IV. RESULT AND DISCUSSION

A. *For strong shock*

1) *Shock Velocity and Shock Strength:* The expressions (13) and (18) are obtained for the shock velocity of strong spherical isothermal shock propagating freely and under the influence of overtaking disturbances, respectively. The expressions (14) and (19) are the corresponding relations for shock strength for both the cases. Initially taking U/a_0=20 at r =5 for ρ_0=1.29 kg/m^3, p_0=1.01 x 10^5 N/m^2, N=5 and γ=1.4, the shock velocity and shock strength are numerically computed for various situations and presented in tables (1)and (3). From tables (1) and (3), it is found that when strong spherical diverging shock propagates isothermally in uniform density region, the shock velocity and shock strength for both freely propagation as well as with inclusion of overtaking disturbances decreases with propagation distance and with specific heat index. Table (1) shows that as shock propagates from 5.0 to 9.0, the shock velocity decreases from 6621.560 to 860.948 for freely propagation and from 9704.460 to 1587.180 in presence of overtaking disturbances and also shock strength decreases from 20.0 to 2.60 for freely propagation and from 29.312 to 4.794 in presence of overtaking disturbances, respectively. Table (3) shows that as specific heat index increases from 1.40 to 1.50, the shock velocity decreases from 6621.560 to 6323.500 for freely propagation and from 9704.460 to 9271.460 in presence of overtaking disturbances and also shock strength decreases from 20.0 to 18.452 for freely propagation and from 29.312 to 27.054 in presence of overtaking disturbances, respectively. The variation of shock velocity with propagation distance r, specific heat index γ are also shown in figures(1 and 6). Shock velocity decreases with propagation distance r and specific heat index γ in both the cases FP and EOD as shown in figures (1 and 6). Similarly the variation in shock strength with propagation distance r and specific heat index γ is shown in figures (2 and 7). These results are agreed with the observations obtained by Gangwar (2006).
2) *Non-dimensional Particle Velocity, non-dimensional Pressure and non-dimensional Radiative heat flux:* The expressions for non-dimensional particle velocity, non-dimensional pressure and non-dimensional radiative heat flux immediately behind the shock for freely propagation (u/a_0, p/p_0, $F/\rho_0 a_0^3$) are given by equations (20), (21) and (22) and under the influence of overtaking disturbances (u^*/a_0, p^*/p_0, $F^*/\rho_0 a_0^3$) are given by equations (23),(24) and (25) respectively. The variation of these variables with different parameters are shown in tables (1-4) respectively. The variations of freely and modified non-dimensional particle velocity, non-dimensional pressure and non-dimensional radiative heat flux with propagation distance r and specific heat index γ are shown in the tables(1-4) respectively. Tables (1), (2), (3) and (4) show that when strong isothermal spherical diverging shock propagates in uniform density region, the non-dimensional particle velocity, non-dimensional pressure and non-dimensional radiative heat flux decreases with propagation distance and with specific heat index for both freely as well as with inclusion of overtaking disturbances. From tables (1) and (2), it is found that, the non-dimensional particle velocity decreases from 19.964 to 2.326, non-dimensional pressure from 560.0010 to 9.4672 and non-dimensional radiative heat flux from -4000.0200 to -8.7925 for freely propagation. The non-dimensional particle velocity decreases from 29.287 to 4.645, non-dimensional pressure from 1202.8500 to 32.1753 and non-dimensional radiative heat flux from -12592.000 to -55.0885 in presence of overtaking disturbances as shock propagates from 5.0 to 9.0. From tables (3) and (4), we concluded that as the specific heat index increases from 1.40 to 1.50, the non-dimensional particle velocity decreases from 19.964 to 18.416, non-dimensional pressure from 560.0010 to 510.7200 and non-dimensional radiative heat flux from -4000.0200 to -3141.2900 for freely propagation. The non-dimensional particle velocity decreases from 29.287 to 27.030, non-dimensional pressure from 1202.8500 to 1097.9000 and non-dimensional radiative heat flux from -12592.000 to -9901.010 in presence of overtaking disturbances. Non-dimensional particle velocity, non-dimensional pressure and non-dimensional radiative heat flux decreases with propagation distance r and specific heat index γ for both the cases FP and EOD are also shown in figures (3, 4, 5, 8, 9 and 10) respectively.

Table 1 : Variation of shock velocity, shock strength and non-dimensional particle velocity with propagation distance when strong spherical diverging shock propagates isothermally in uniform density region (initially taken $U/a_0=20$ at r=5 and N=5, $\alpha=1.5$, $=\beta-2.5$, $\rho_0 = 1.29$ kg/m^3, $p_0=1.01 \times 10^5$N/m^2 and $\gamma=1.4$)

Propagation distance	Shock velocity	Modified shock velocity	Shock strength	Modified shock strength	Non-dimensional particle velocity	Modified non-dimensional particle velocity
r	U	U*	U/a_0	U^*/a_0	u/a_0	u^*/a_0
5.0	6621.560	9704.460	20.000	29.312	19.964	29.287
5.5	5886.970	8666.280	17.781	26.176	17.741	26.149
6.0	5157.890	7636.050	15.579	23.064	15.533	23.033
6.5	4433.410	6612.620	13.391	19.973	13.338	19.937
7.0	3712.880	5595.150	11.215	16.900	11.151	16.858
7.5	2995.740	4583.160	9.048	13.843	8.970	13.792
8.0	2281.570	3576.580	6.891	10.803	6.788	10.737
8.5	1570.030	2576.370	4.742	7.782	4.592	7.690
9.0	860.948	1587.180	2.600	4.794	2.326	4.645

Table 2 : Variation of non-dimensional pressure and non-dimensional heat flux with propagation distance when strong spherical diverging shock propagation isothermally in uniform density region (initially taken $U/a_0=20$ at r=5 and N=5, $\alpha=1.5$, $=\beta-2.5$, $\rho_0 = 1.29$ kg/m^3, $p_0=1.01 \times 10^5$N/m^2 and $\gamma=1.4$)

Propagation distance	Non-dimensional pressure	Modified non-dimensional pressure	Non-dimensional heat flux	Modified Non-dimensional heat flux
r	p/p_0	p^*/p_0	$F/\rho_0 a_0^3$	$F^*/\rho_0 a_0^3$
5.0	560.0010	1202.8500	-4000.0200	-1292.0000
5.5	442.6420	959.2540	-2810.9700	-8967.6400
6.0	339.7910	744.7430	-1890.5800	-6134.6100
6.5	251.0410	558.4900	-1200.5900	-3983.8300
7.0	176.0720	399.8460	-705.2000	-2413.3300
7.5	114.6240	268.2860	-370.4180	-1326.4000
8.0	66.4868	163.3820	-163.6370	-630.3540
8.5	31.4837	84.7783	-53.3221	-235.6160
9.0	9.4672	32.1753	-8.7925	-55.0885

Table 3 : Variation of shock velocity, shock strength and non-dimensional particle velocity with specific heat index when strong spherical diverging shock propagates isothermally in uniform density region (initially taken U/a_0=20 at r=5 for N=5, α=1.5, =β-2.5, ρ_0 =1.29 kg/m^3, p_0=1.01 x 10^{5}N/m^2 and γ=1.4)

Specific heat index	Shock velocity	Modified shock velocity	Shock strength	Modified shock strength	Non-dimensional particle velocity	Modified non-dimensional particle velocity
γ	U	U*	U/a_0	U^*/a_0	u/a_0	u^*/a_0
1.40	6621.560	9704.460	20.000	29.312	19.964	29.287
1.41	6590.450	9659.260	19.835	29.072	19.800	29.047
1.42	6559.650	9614.510	19.673	28.835	19.637	28.810
1.43	6529.140	9570.190	19.513	28.601	19.477	28.577
1.44	6498.930	9526.290	19.355	28.371	19.319	28.347
1.45	6469.000	9482.810	19.199	28.144	19.164	28.120
1.46	6439.350	9439.750	19.046	27.920	19.010	27.896
1.47	6409.980	9397.080	18.894	27.699	18.858	27.675
1.48	6380.890	9354.820	18.745	27.481	18.709	27.457
1.49	6352.060	9312.950	18.598	27.266	18.562	27.242
1.50	6323.500	9271.460	18.452	27.054	18.416	27.030

Table 4 : Variation of non-dimensional pressure and non-dimensional heat flux with specific heat index with strong spherical diverging shock propagation isothermally in uniform density region (initially taken U/a_0=20 at r=5 and N=5, α=1.5, =β-2.5, ρ_0 =1.29 kg/m^3, p_0=1.01 x 10^{5}N/m^2 and γ=1.4)

Specific heat index	Non-dimensional pressure	Modified non-dimensional pressure	Non-dimensional heat flux	Modified Non-dimensional heat flux
γ	p/p_0	p^*/p_0	$F/\rho_0 a_0^3$	$F^*/\rho_0 a_0^3$
1.40	560.0010	1202.8500	-4000.0200	-1292.0000
1.41	554.7520	1191.6700	-3902.0200	-12285.000
1.42	549.5780	1180.6500	-3806.9900	-11987.300
1.43	544.4780	1169.7900	-3714.8100	-11698.5000
1.44	539.4500	1159.0900	-3625.3800	-11418.3000
1.45	534.4930	1148.5300	-3538.6000	-11146.3000
1.46	529.6060	1138.1200	-3454.3800	-10882.4000
1.47	524.7860	1127.8600	-3372.6200	-10626.1000
1.48	520.0330	1117.7400	-3293.2400	-10377.3000
1.49	515.3450	1107.7500	-3216.1600	-10135.7000
1.50	510.7200	1097.9000	-3141.2900	-9901.0100

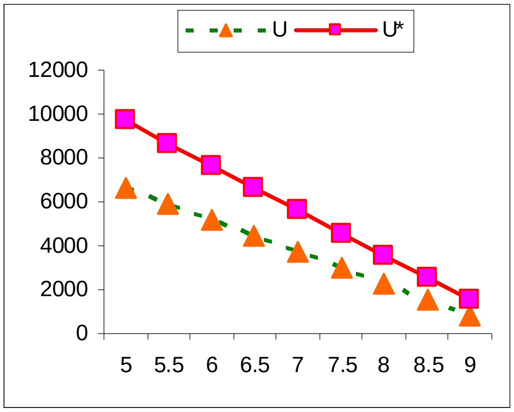


Fig.1: Shock velocity U and U^* v/s propagation distance (r).

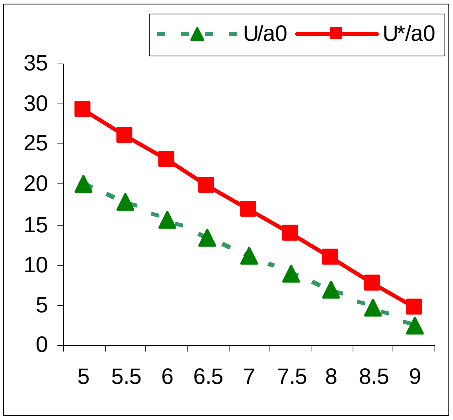


Fig.2: Shock strength U/a_0 and U^*/a_0 v/s propagation distance (r).

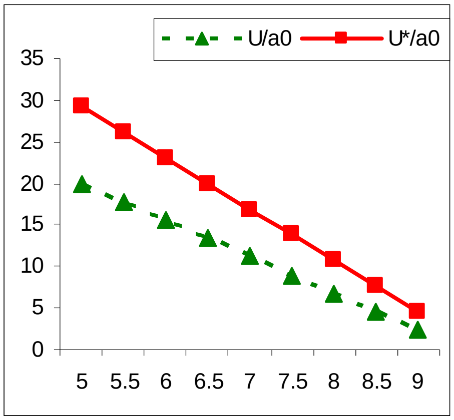


Fig.3: Non dimensional particle velocity u/a_0 and u^*/a_0 v/s propagation distance (r).

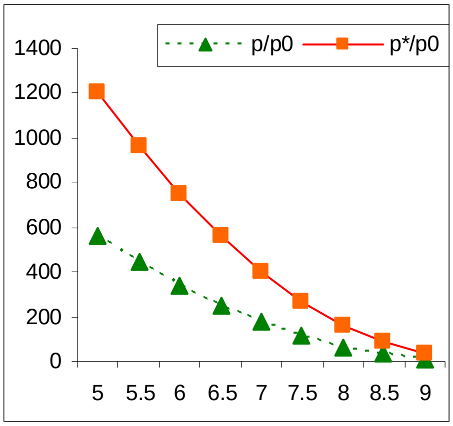


Fig.4: Non dimensional pressure p/p_0 and p^*/p_0 v/s Propagation distance (r).

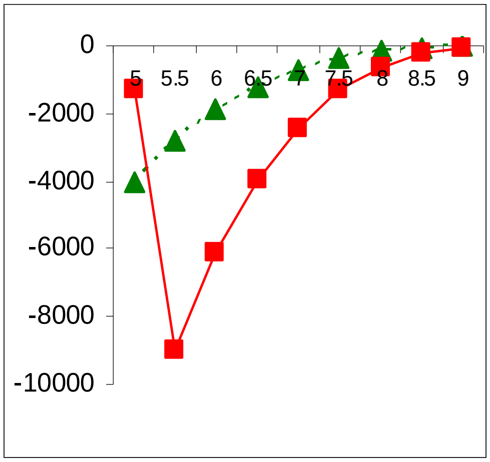


Fig.5: Non dimensional heat flux $F/\rho_0 a_0^3$ and $F^*/\rho_0 a_0^3$ v/s Propagation distance (r).

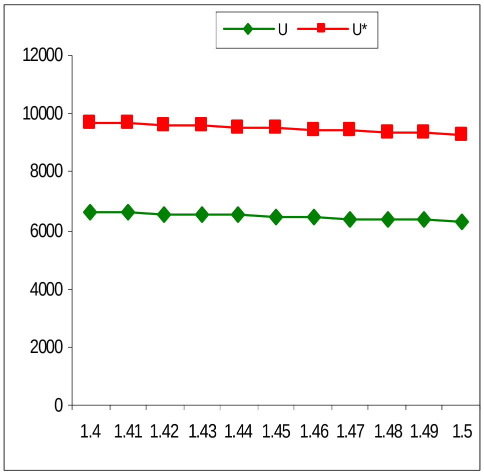


Fig.6: Shock velocity U and U^* v/s Specific heat index (γ).

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue III Mar 2021- Available at www.ijraset.com

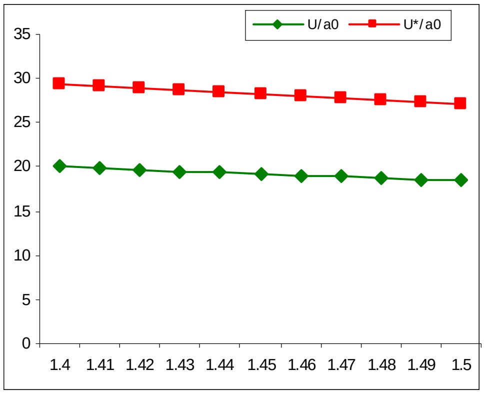


Fig.7: Shock strength U/a_0 and U^*/a_0 v/s Specific heat index (γ).

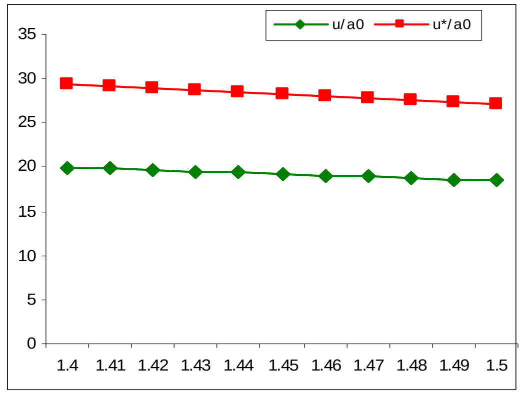


Fig.8: Non-dimensional particle velocity u/a_0 and u^*/a_0 v/s Specific heat index (γ).

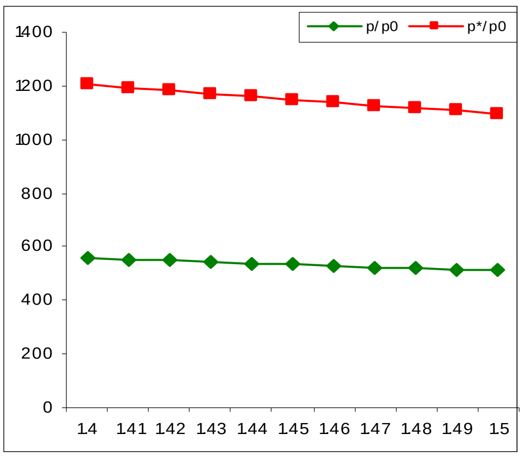


Fig.9: Non-dimensional pressure p/p_0 and p^*/p_0 v/s Specific heat index (γ).

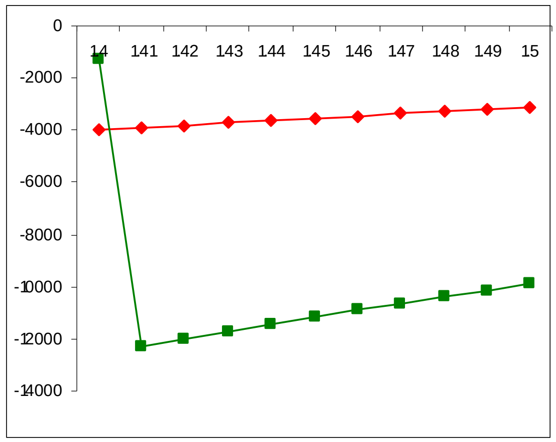


Fig.10: Non-dimensional heat flux $F/\rho_0 a_0^3$ and $F^*/\rho_0 a_0^3$ v/s Specific heat index (γ).

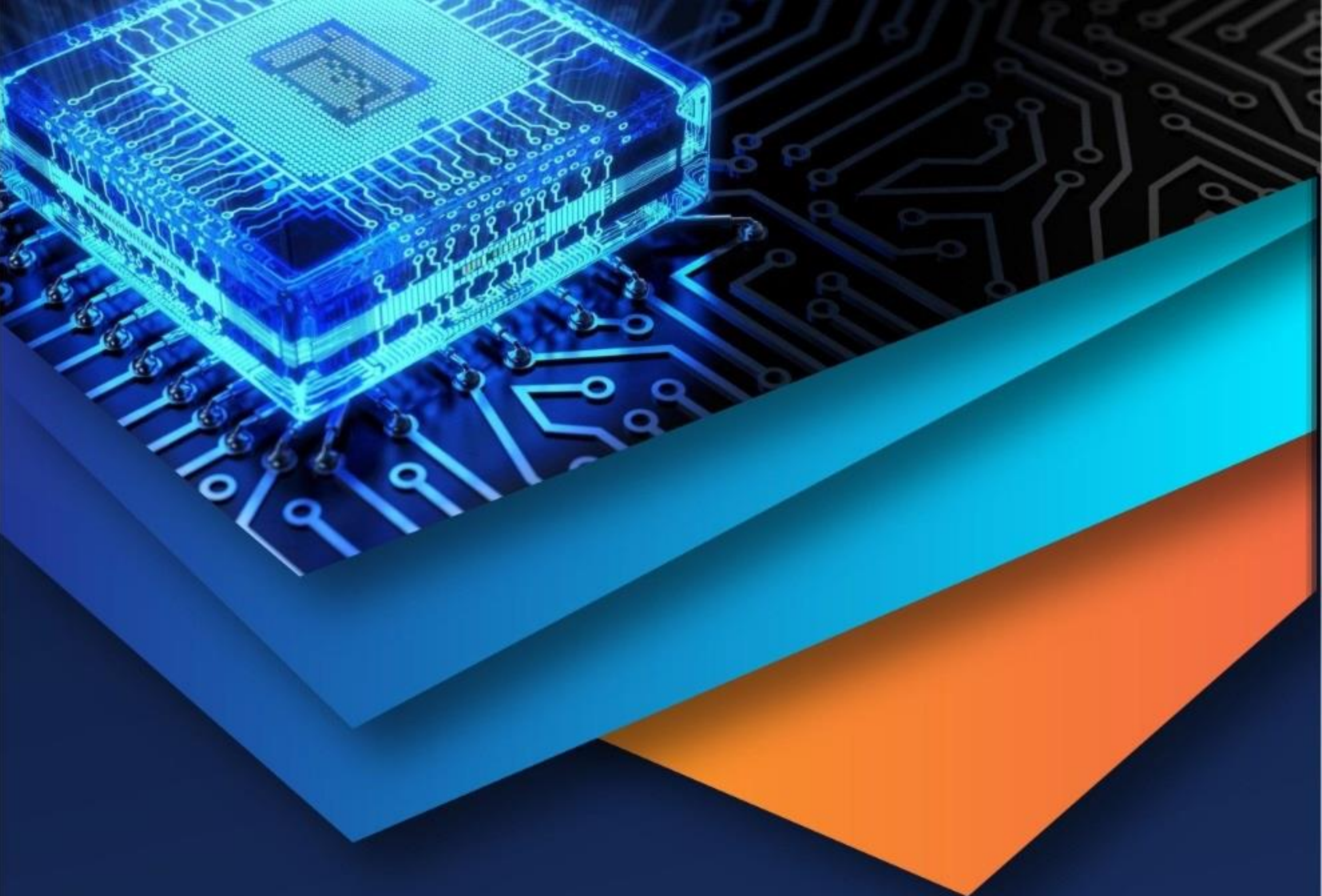
V. CONCLUSION

The propagation of strong spherical shock waves in uniform self-gravitation gas atmosphere has astrophysically significance specially in context to star formation, in many practical situations in nuclear physics and space science. When shock propagates in the atmosphere, perturbation of the medium takes place either adiabatically or isothermally. These are the two extreme and ideal situations occurs in the medium, all the actual phenomena lie in between these two situations. In the present study flow variables of the perturbed medium are investigated for these two extreme situations by Chester-Chisnell-Whitham method and improved Yadav approach for overtaking disturbances. It is concluded that strengthening and weaking of the shock depends on the specific heat index γ and propagation distance r. The change in radiation heat flux contributes very significantly in the propagation of shock waves.

REFERENCES

[1] Bhowmick,J.B. : A study of the expansion of the solar corona with radiation heat flux, Astrophys. and Space Science,Vol.70,433-439 (1980).
[2] Chen, H.B., Zhang, L., and Panarella, E.: Stability of imploding spherical shock waves, Journal of Fusion Energy, Vol. 14, No. 4, 389-392 (1995).
[3] Chester, W. : The quasi-cylindrical shock tube, Phil. Mag., Sec. 7, Vol. 45, 1293-1301(1954).
[4] Chisnell, R.F. : The normal motion of a shock wave through a non-uniform one-dimensional medium, Proc. Roy. Soc. A, 232, 350-370 (1955).
[5] Chisnell, R.F. : The motion of the shock wave in a channel, with applications to cylindrical and spherical shock waves, J. Fluid Mech. 2, 286-298 (1957).
[6] Chisnell, R.F. : An analytic description of converging shock waves, Journal of Fluid Mechanics, 354, 357-375 (1998).
[7] Crighton, D.G. : The Taylor internal structure of weak shock waves, Journal of Fluid Mechnics, 173, 625-642 (1986).
[8] Gretler, W. and Regenfelder, R. : Similarity solution for variable energy shock waves in a dusty gas under isothermal flow-field condition, Fluid Dynamics Research, Vol. 32, issue 3, 69-84 (2003).

[9] Helliwell, J.B. : Differential approximation for the flux of thermal radiation, Phys. Fluids, 9, 1869-1871 (1966).
[10] Helliwell, J.B. : Self-similar piston problems with radiative heat transfer, J. Fluid Mech., vol. 37, part 3, 497-512 (1969).
[11] Rai, A. : Converging spherical detonation waves, Defence Science Journal, Vol. 48, No.-2, 205-210 (1998).
[12] Singh, J.B., Singh, P.S. and Dwivedi, A.K. : A model of magnetoradiative strong shock wave in uniform atmosphere, Journal of Natural & Physical Sciences, Vol. 13, No. 2, 159-172 (1999).
[13] Singh, K. K. and Vishwakarma, J.P. : Propagation of spherical shock waves in a dusty gas with radiation heat-flux, Journal of Theoretical and Applied Mechanics, Vol. 45, No.4, 801-817 (2007).
[14] Singh, V.P. : Converging detonation waves, Journal of the Indian Math. Soc., Vol. 42, 381-386 (1978).
[15] Steiner, H., Gretler, W. and Hirschler, T.: Numerical solution for spherical laser-driven shock waves, Shock waves, Vol. 8, No.3, 139-147 (1998).
[16] Whitham, G.B. : On the propagation of shock waves through regions of non-uniform area or flow, Journal of Fluid Mechanics 4, 337-360 (1958).
[17] Whitham, G.B. : The propagation of weak spherical shocks in stars, Pure and Applied Mathematics, Vol. VI, 397-414 (1953).



doi crossref

10.22214/IJRASET

INDEX COPERNICUS

45.98

ISRA JIF

IMPACT FACTOR: 7.129

SJIFactor
TOGETHER WE REACH THE GOAL

IMPACT FACTOR: 7.429

ISI

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY