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# Divisor Cordial Labeling Of Cycle Related Graphs

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**Abstract-** A divisor cordial labeling of a graph  $G$  is a bijection  $f$  from  $V(G)$  to  $\{1, 2, \dots, |V(G)|\}$  such that an edge  $uv$  is assigned the label 1 if  $f(u)|f(v)$  or  $f(v)|f(u)$  and the label 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a divisor cordial labeling is called a divisor cordial graph. In this paper we prove that cycle with one chord, cycle with twin chords, barycentric subdivision of cycle with one chord, barycentric subdivision of cycle with twin chords, vertex switching of cycle with one chord and vertex switching of cycle with twin chords are divisor cordial graphs.

**Keywords—** Divisor cordial labeling, Barycentric subdivision, Vertex switching. AMS Subject classification number: 05C78.

## I. INTRODUCTION

The graphs considered here are finite, connected, undirected and simple. The vertex set and edge set of a graph  $G$  are denoted by  $V(G)$  and  $E(G)$  respectively. For various graph theoretic notations and terminology we follow Gross and Yellen[2]. The concept of divisor cordial labeling was introduced by R. Varatharajan, S. Navanaeethakrishnan, K. Nagarajan[4] and they proved that paths, cycles, wheels, stars are divisor cordial. They also proved that complete graphs  $K_n$ ,  $n \geq 4$  are not divisor cordial. In addition they also proved that dragons, coronas, wheels, and complete binary trees are divisor cordial. Vaidya and Shah[3] proved that the splitting graphs of stars and bistars are divisor cordial. They also proved that the shadow graphs and the squares of bistars are divisor cordial. A dynamic survey of graph labeling is published and updated every year by Gallian[1]. In this paper we prove that cycle with one chord, cycle with twin chords, vertex switching of cycle with one chord, vertex switching of cycle with twin chords, barycentric subdivision of cycle with one chord, barycentric subdivision of cycle with twin chords are divisor cordial graphs.

**Note:** Fundamental Theorem of Arithmetic

Any natural number (except for 1) can be expressed as the product of primes. For each natural number such an expression is unique.

## II. MAIN RESULTS

**Theorem 1.** Cycle with one chord is divisor cordial, where chord forms a triangle with two edges of the cycle.

**Proof.** Let  $v_1, v_2, \dots, v_n$  be the successive vertices of cycle  $C_n$  and  $e = v_2 v_n$  be the chord of cycle  $C_n$ , where the edges  $e_1 = v_1 v_2, e_2 = v_1 v_n, e = v_2 v_n$  form a triangle. Our aim is to generate  $\lceil (n+1)/2 \rceil$  edges having label 1 and edges having label 0. Label the vertex  $v_1$  by 1 which will generate 2 edges having label 1.

Now it remains to generate  $k = \lceil (n+1)/2 \rceil - 2$  edges with label 1. For the vertices  $v_2, v_3, \dots, v_n$ , assign the vertex labels as per the following ordered pattern up to it generate  $k$  edges with label 1.

$$\begin{aligned} &2, 2^2, 2^3, \dots, 2^{k_1}, \\ &3, 3 \times 2, 3 \times 2^2, \dots, 3 \times 2^{k_2}, \\ &5, 5 \times 2, 5 \times 2^2, \dots, 5 \times 2^{k_3}, \\ &\dots, \dots, \dots, \dots, \\ &\dots, \dots, \dots, \dots, \end{aligned}$$

where  $(2m - 1) 2^{k_m} \leq n$ ,  $m \in \mathbb{N}$  and  $k_m$  is a non-negative integer.

Observe that  $(2m - 1) 2^a | (2m - 1) 2^{a+1}$  and  $(2m - 1) 2^{k_i}$  does not divide  $2m + 1$ .

In view of above labeling pattern we have number of edges with label 0 =  $\lfloor (n+1)/2 \rfloor$  and number of edges with label 1 =  $\lceil (n+1)/2 \rceil$ .

Hence, cycle with one chord is divisor cordial graph.

**Example 1.** Divisor cordial labeling of cycle  $C_8$  with one chord is shown in Fig. 1.

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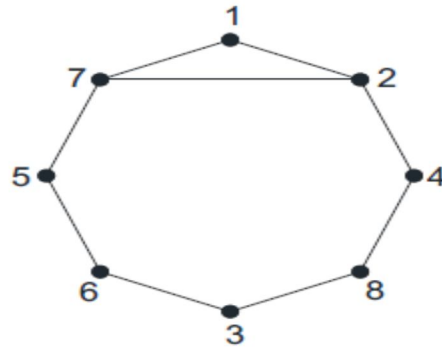


Fig. 1

**Theorem 2.** Cycle with twin chords  $C_{n,3}$  is divisor cordial for all  $n$ , where chords form two triangles and one cycle  $C_{n-2}$ .

**Proof.** Let  $v_1, v_2, \dots, v_n$  be the successive vertices of cycle  $C_n$  and let  $e_1 = v_2v_n$  and  $e_2 = v_3v_n$  be the chords of  $C_n$ .

Our aim is to generate  $\lfloor (n+2)/2 \rfloor$  edges having label 1 and  $\lceil (n+2)/2 \rceil$  edges having label 0. Label the vertex  $v_1$  by 1 which generates 2 edges having label 1. Now it remains to generate  $k = \lceil (n+2)/2 \rceil - 2$  edges with label 1.

For the vertices  $v_2, v_3, \dots, v_n$  assign the vertex labels as per the following ordered pattern up to it generate  $k$  edges with label 1.

- $2, 2^2, 2^3, \dots, 2^{k_1}$ ,
- $3, 3 \times 2, 3 \times 2^2, \dots, 3 \times 2^{k_2}$ ,
- $5, 5 \times 2, 5 \times 2^2, \dots, 5 \times 2^{k_3}$ ,

.....  
 .....

where  $(2m - 1) 2^{k_m} \leq n$ ,  $m \in \mathbb{N}$  and  $k_m$  is a non-negative integer.

Observe that  $(2m - 1) 2^a \mid (2m - 1) 2^{a+1}$  and  $(2m - 1) 2^{k_i}$  does not divide  $2m + 1$ .

In view of above labeling pattern we have, number of edges with label 0 =  $\lceil (n+2)/2 \rceil$  and number of edges with label 1 =  $\lfloor (n+2)/2 \rfloor$ .

Hence, cycle with twin chords is divisor cordial graph.

**Example 2.** Divisor cordial labeling of cycle  $C_9$  with twin chords is shown in Fig. 2.

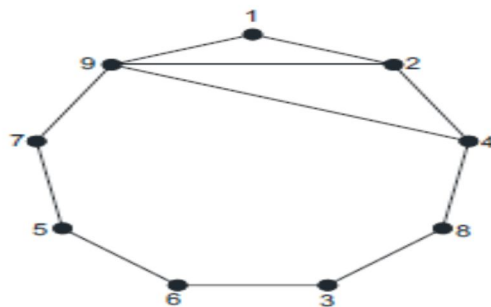


Fig. 2

**Theorem 3.** Vertex switching of cycle  $C_n$  ( $n \geq 4$ ) with one chord admits divisor cordial labeling, where chord forms a triangle with two edges of cycle  $C_n$ .

**Proof .** Let  $G$  be the cycle  $C_n$  with one chord. Let  $v_1, v_2, \dots, v_n$  be the successive vertices of cycle  $C_n$  and  $e = v_2v_n$  be a chord of cycle  $C_n$ . The edges  $e = v_2v_n, e_1 = v_1v_2, e_2 = v_1v_n$  form a triangle.

Without loss of generality let the switched vertex be  $v_1$  (of either degree 2 or degree 3) and let  $Gv_1$  denote the vertex switching of  $G$  with respect to vertex  $v_1$ .

To define labeling function  $f : V(Gv_1) \rightarrow \{1, 2, \dots, n\}$  we consider the following cases.

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Case 1: The vertex  $v_1$  is of degree 2. i.e.  $\deg(v_1) = 2$ .

{Here the number of vertices is  $n$  and number of edges is  $2n - 4$ .}

Subcase 1:  $n \equiv 0(\text{mod}4)$ ,  $n \neq 4$ .

$f(v_n) = 4$ ,  $f(v_{n-1}) = n - 1$ ,  $f(v_{n-2}) = n$ ,

$f(v_i) = i$ ,  $1 \leq i \leq 3$ .

$f(v_i) = i + 1$ ,  $4 \leq i \leq n - 3$ .

In view of the above labeling pattern we have  $e_f(0) = e_f(1) = n - 2$ .

Subcase 2:  $n = 4$ .

$f(v_i) = i$ ,  $1 \leq i \leq n$ .

In view of the above defined labeling pattern,  $e_f(0) = e_f(1) = 2$ .

Subcase 3:  $n \equiv 1, 2, 3(\text{mod}4)$ .

$f(v_n) = 4$

$f(v_i) = i$ ,  $1 \leq i \leq 3$ .

$f(v_i) = i + 1$ ,  $4 \leq i \leq n - 1$ .

In view of the above defined labeling pattern,  $e_f(0) = e_f(1) = n - 2$ .

Case 2: The vertex  $v_1$  is of degree 3. i.e.  $\deg(v_1) = 3$ .

{Here the number of vertices is  $n$  and number of edges is  $2n - 5$ .}

$f(v_3) = 4$ ,  $f(v_4) = 3$

$f(v_i) = i$ ,  $1 \leq i \leq n$ ,  $i \neq 3, i \neq 4$ .

In view of the above labeling pattern we have  $e_f(0) = \lfloor (2n-5)/2 \rfloor$ ,  $e_f(1) = \lceil (2n-5) / 2 \rceil$ .

Thus,  $|e_f(0) - e_f(1)| \leq 1$ .

Hence vertex switching of cycle  $C_n$  with one chord is a divisor cordial graph.

Example 3.

(a) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 2 in cycle  $C_7$  is shown in Fig. 3(a).

(b) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 3 in cycle  $C_7$  is shown in Fig. 3(b).

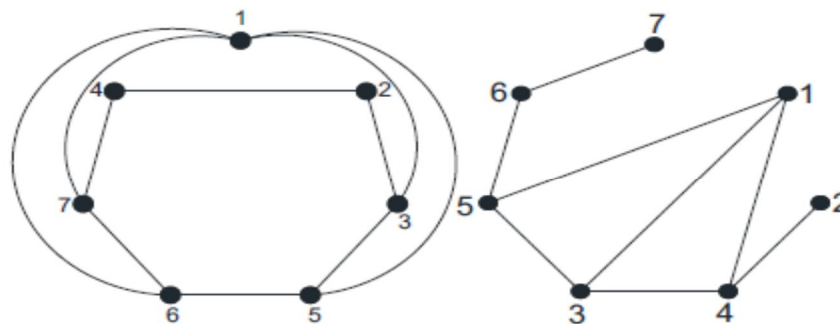


Fig. 3(a)

Fig. 3(b)

Theorem 4. Vertex switching of cycle  $C_n$  with twin chords  $C_{n,3}$  admits divisor cordial labeling, where chords form two triangles and one cycle  $C_{n-2}$ .

Proof. Let  $G$  be the cycle  $C_n$  with twin chords. Let  $v_1, v_2, \dots, v_n$  be the successive vertices of  $G$ . Let  $e_1 = v_n v_2$  and  $e_2 = v_n v_3$  be the chords of cycle  $C_n$  which form two triangles and one cycle  $C_{n-2}$ .

Without loss of generality let  $v_1$  (of either degree 2 or degree 3 or degree 4) be the switched vertex and let  $Gv_1$  denote the vertex

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switching of  $G$  with respect to vertex  $v_1$ .

To define labeling function  $f: V(G_{v_1}) \rightarrow \{1, 2, \dots, n\}$  we consider the following cases.

Case 1: The vertex  $v_1$  is of degree 2. i.e.  $\deg(v_1) = 2$   
 {Here the number of vertices is  $n$  and number of edges is  $2n - 3$ .}  
 $f(v_1) = 4,$   
 $f(v_i) = i, 1 \leq i \leq 3.$   
 $f(v_i) = i + 1, 4 \leq i \leq n - 1.$

In view of above labeling pattern:

When  $n \equiv 0 \pmod{4}, e_f(0) = n - 2, e_f(1) = n - 1.$

Otherwise,  $e_f(0) = n - 1, e_f(1) = n - 2.$

Case 2: The vertex  $v_1$  is of degree 3. i.e.  $\deg(v_1) = 3$   
 {Here the number of vertices is  $n$  and number of edges is  $2n - 5$ .}  
 $f(v_3) = 4, f(v_4) = 3$   
 $f(v_i) = i, 1 \leq i \leq n, i \neq 3, i \neq 4.$

In view of the above labeling pattern we have  $e_f(0) = \lceil (2n-5) / 2 \rceil, e_f(1) = \lfloor (2n-5) / 2 \rfloor.$

Case 3: The vertex  $v_1$  is of degree 4. i.e.  $\deg(v_1) = 4$   
 {Here the number of vertices is  $n$  and number of edges is  $2n - 7$ .}  
 $f(v_3) = 4, f(v_4) = 3$   
 $f(v_i) = i, 1 \leq i \leq n, i \neq 3, i \neq 4.$

In view of the above labeling pattern we have  $e_f(0) = \lfloor (2n-7) / 2 \rfloor, e_f(1) = \lceil (2n-7) / 2 \rceil.$

Thus,  $|e_f(0) - e_f(1)| \leq 1.$

Hence vertex switching of cycle  $C_n$  with twin chords is a divisor cordial graph.

### Example 4.

- (a) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 2 in cycle  $C_8$  with twin chords is shown in Fig. 4(a).
- (b) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 3 in cycle  $C_8$  with twin chords is shown in Fig. 4(b).
- (c) Divisor cordial labeling of the graph obtained by switching of a vertex of degree 4 in cycle  $C_8$  with twin chords is shown in Fig. 4(c).

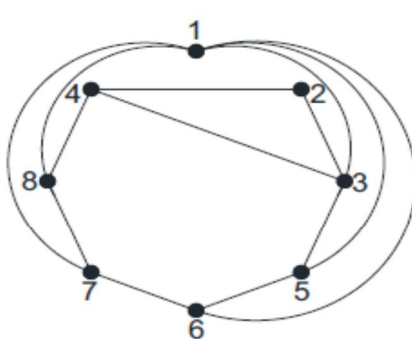


Fig. 4(a)

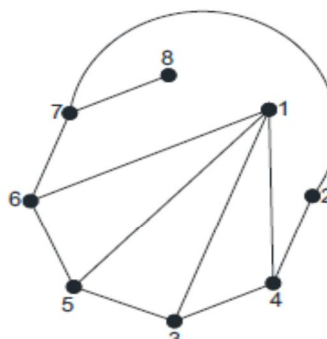


Fig. 4(b)

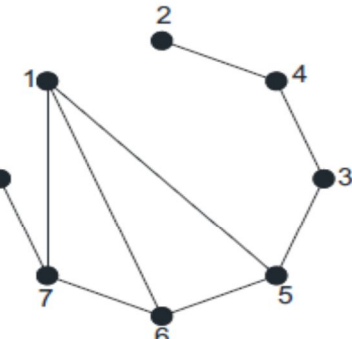


Fig. 4(c)

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Theorem 5. The barycentric subdivision of cycle with one chord admits divisor cordial labeling, where chord forms triangle with two edges of the cycle.

Proof. Let  $G$  be the cycle  $C_n$  with one chord and let  $S(G)$  denote the barycentric subdivision of  $G$ . Here  $|V(S(G))| = 2n + 1$  and  $|E(S(G))| = 2n + 2$ . Let  $v_1, v_2, \dots, v_{2n+1}$  be the successive vertices of  $S(G)$ , where  $v_1, v_3, \dots, v_{2n-1}$  denote the vertices of cycle  $C_n$  and  $e = v_3 v_{2n-1}$  be the chord of  $C_n$ . The vertices  $v_2, v_4, \dots, v_{2n}$  are newly inserted vertices due to barycentric subdivision of  $C_n$  and  $v_{2n+1}$  is the newly inserted vertex due to barycentric subdivision of the chord  $e$  of  $C_n$ .

We define labeling function  $f : V(S(G)) \rightarrow \{1, 2, \dots, 2n\}$  as follows.

Our aim is to generate  $n + 1$  edges with label 1 and  $n + 1$  edges with label 0.

$f(v) = 1$ , which generates 2 edges having label 1. Now it remains to generate  $k = n - 1$  edges with label 1.

For the vertices  $v_1, v_2, \dots, v_{2n}$  assign the vertex label as per following ordered pattern up to it generate  $k$  edges with label 1.

$2, 2^2, 2^3, \dots, 2^{k_1}$ ,  
 $3, 3 \times 2, 3 \times 2^2, \dots, 3 \times 2^{k_2}$ ,  
 $5, 5 \times 2, 5 \times 2^2, \dots, 5 \times 2^{k_3}$ ,

.....  
 .....

where  $(2m - 1) 2^{k_m} \leq 2n+1, m \geq 1$  and  $k_m \geq 0$ .

Observe that  $(2m - 1) 2^a \mid (2m - 1) 2^{a+1}$  and  $(2m - 1) 2^{k_i}$  does not divide  $2m + 1$ .

In view of the above labeling pattern we have  $e_f(0) = e_f(1) = n+1$ .

Thus,  $|e_f(0) - e_f(1)| \leq 1$ .

Hence, the barycentric subdivision of cycle  $C_n$  with one chord admits divisor cordial labeling.

Example 5. Divisor cordial labeling of the graph obtained by the barycentric subdivision of cycle  $C_{10}$  with twin chords is shown in Fig. 5.

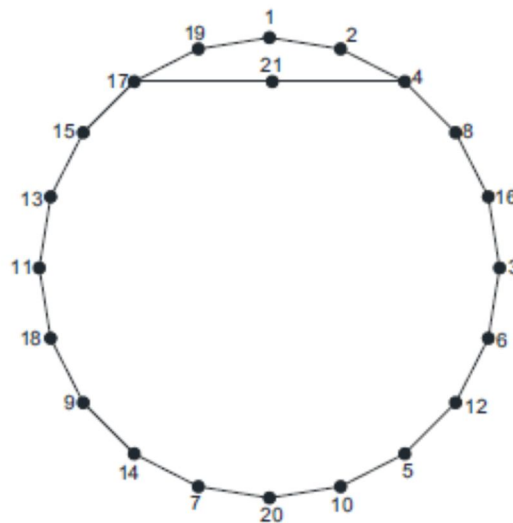


Fig. 5

Theorem 6. The barycentric subdivision of cycle  $C_n$  with twin chords  $C_{n,3}$  admits divisor cordial labeling, where chords form two triangles and one cycle  $C_{n-2}$ .

Proof. Let  $G$  be the cycle  $C_n$  with twin chords and let  $S(G)$  denote the barycentric subdivision of  $G$ . Here  $|V(S(G))| = 2n + 2$  and  $|E(S(G))| = 2n + 4$ . Let  $v_1, v_2, \dots, v_{2n+2}$  be the successive vertices of  $G$ .

Let  $e_1 = v_n v_2$  and  $e_2 = v_n v_3$  be the chords of cycle  $C_n$ . The vertices  $v_2, v_4, \dots, v_{2n}$  are newly inserted vertices due to barycentric subdivision of  $C_n$ ,  $v_{2n+1}$  is the newly inserted vertex due to barycentric subdivision of the chord  $e_1$  and  $v_{2n+2}$  is the newly inserted vertex due to barycentric subdivision of the chord  $e_2$ .

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We define labeling function  $f : V(G) \rightarrow \{1, 2, \dots, 2n\}$  as follows. Our aim is to generate  $n + 2$  edges with label 0 and  $n + 2$  edges with label 1.  $f(v) = 1$ , which generates 2 edges with label 1. Now it remains to generate  $k = n$  edges with label 1. For the vertices  $v_1, v_2, \dots, v_{2n}$  assign the vertex label as per following ordered pattern up to it generate  $k$  edges with label 1.

- $2, 2^2, 2^3, \dots, 2^{k_1}$ ,
- $3, 3 \times 2, 3 \times 2^2, \dots, 3 \times 2^{k_2}$ ,
- $5, 5 \times 2, 5 \times 2^2, \dots, 5 \times 2^{k_3}$ ,
- .....
- .....

where  $(2m - 1) 2^{k_m} \leq n$ ,  $m \in \mathbb{N}$  and  $k_m$  is a non-negative integer.

Observe that  $(2m - 1) 2^a \mid (2m - 1) 2^{a+1}$  and  $(2m - 1) 2^{k_i}$  does not divide  $2m + 1$ .

In view of above labeling pattern, we have  $e_f(0) = e_f(1) = n+2$ .

Thus,  $|e_f(0) - e_f(1)| \leq 1$ .

Hence, the barycentric subdivision of cycle  $C_n$  with twin chords is divisor cordial.

Example 6. Divisor cordial labeling of the graph obtained by barycentric subdivision of cycle  $C_{11}$  with twin chords is shown in Fig. 6.

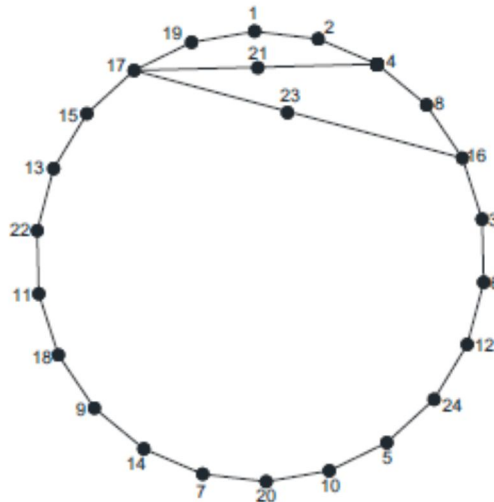


Fig. 6

### III. CONCLUSION

We have contributed six new results in the theory of divisor cordial graphs. We have proved that cycle with one chord and cycle with twin chords are divisor cordial graphs. For these graphs divisor cordial labeling remains invariant under the operation vertex switching and barycentric subdivision.

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