iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 9 **Issue:** VI **Month of publication:** June 2021

DOI: https://doi.org/10.22214/ijraset.2021.34849

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue VI Jun 2021- Available at www.ijraset.com

Inverse Sum Status Energy of a Graph

K. N. Prakasha[1], Padmanava Samanta[2], Niyati Misra[3]
[1]Department of Mathematics, Vidyavardhaka College Engineering, Mysuru-570002, INDIA
[2]Department of Mathematics, Berhampur University, Odisha, INDIA

Abstract: Motivated by the inverse sum status index, we introduce the inverse sum status matrix $ISS(G)$ *as* $ISS(G) = (ISS)_{n\times n}$

$$ISS = \begin{cases} \dfrac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} & if\ u_i \sim v_j, \\ 0 & otherwise \end{cases}$$

Thus we also obtained the results for well known graphs.

Keywords: Inverse sum status energy, Inverse Sum Indeg matrix. 2010 AMS Subject Classification: 05C50.

I. INTRODUCTION

Status of the vertex *'u'* is defined as the sum of distance of all other vertices from *'u'*... Inverse Sum Status index is introduced by V. R. Kulli [35]. He defined it as

$$ISS(G) = \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)}$$

Where σ_u represents the status of the vertex *u*.

Motivated by the Inverse Sum Status index, We introduce the Inverse Sum Status matrix $ISS(G)\ as\ ISS(G) = (ISS)_{n\times n}$

$$ISS = \begin{cases} \dfrac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)} & if\ v_i \sim v_j \\ 0 & otherwise \end{cases}$$

The Inverse Sum Status energy is given by

$$ISSE(G) = \sum_{i=1}^{n} |\beta_i| \tag{1}$$

The inverse sum status energy is given by

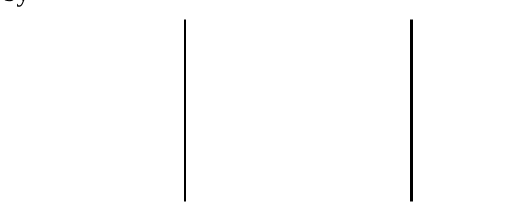


Figure 1. Graph with 6 vertices

$$ISSE(G) = \sum_{i=1}^{n} |\beta_i|$$

II. INVERSE SUM STATUS ENERGY OF SPECIFIC GRAPH STRUCTURES

1) *Theorem-2.1.* Inverse sum status energy of complete graph K_n is $ISSE(K_n) = ISIE(K_n) = 2\sqrt{2n-4}$. [?]

Proof. For each and every vertex u in K_n , $\sigma(u) = (n-1)$. Then the every ij^{th} – entry of the Inverse Sum Status matrix will be $n-1$.

$$ISS\left(K_n\right) = \begin{bmatrix} 0 & n-1 & n-1 & \cdots & n-1 & n-1 \\ n-1 & 0 & n-1 & \cdots & n-1 & n-1 \\ n-1 & n-1 & 0 & \cdots & n-1 & n-1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-1 & n-1 & \cdots & n-1 & 0 & n-1 \\ n-1 & n-1 & \cdots & n-1 & n-1 & 0 \end{bmatrix}$$

This matrix will be same as Inverse Sum Indeg Matrix. Then the energy will be also as same as Inverse Sum indeg energy. Therefore, $ISSE(K_n) = (n-1)^2$.

2) *Theorem-2.2.* The inverse sum status energy of the star $K_{1,n-1}$

$$ISSE(K_{1,n-1}) = \frac{\sqrt{n-1}}{2^{n-1}},$$

Proof. The status of the central vertex in $n-1$ and the pendant vertices is $2n-3$. Thus the inverse sum status matrix is

$$\left(K_{1,n-1}\right) = \begin{bmatrix} 0 & \frac{2n^2-5n+3}{3n-4} \,\&\, \frac{2n^2-5n+3}{3n-4} \,\&\, \ldots & \frac{2n^2-5n+3}{3n-4} \,\&\, \frac{2n^2-5n+3}{3n-4} & & & \\ \frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & 0 \\ \frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & 0 \\ \frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Clearly, the characteristic equation is $\left|\beta I - ISS(K_{1,n-1})\right| = 0$

$$\begin{vmatrix} \beta & -\frac{2n^2-5n+3}{3n-4} \,\&\, -\frac{2n^2-5n+3}{3n-4} \,\&\, \ldots & -\frac{2n^2-5n+3}{3n-4} \,\&\, -\frac{2n^2-5n+3}{3n-4} & & & \\ -\frac{2n^2-5n+3}{3n-4} & \beta & 0 & \cdots & 0 & 0 \\ -\frac{2n^2-5n+3}{3n-4} & 0 & \beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & \beta & 0 \\ -\frac{2n^2-5n+3}{3n-4} & 0 & 0 & \cdots & 0 & \beta \end{vmatrix} = 0$$

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue VI Jun 2021- Available at www.ijraset.com

$\beta^{n-2}\left(\beta+\frac{\left(\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4}\right)\left(\beta-\frac{\left(\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4}\right)=0$. Therefore, the spectrum would be

$$Spec_{ISS}\left(K_{1,n-1}\right)=\begin{pmatrix} \frac{\left(\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4} & -\frac{\left(\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4} & 0 \\ 1 & 1 & n-1 \end{pmatrix}$$

Therefore,

$$ISSE\left(K_{1,n-1}\right)=\frac{\left(2\sqrt{n-1}\right)\left(2n^2-5n+3\right)}{3n-4}.$$

3) *Theorem-2.3.* The inverse sum status energy of crown graph S_n^0 is

$$ISSE\left(S_n^0\right)=6n(n-1).$$

Proof. The inverse sum status matrix is

$$\begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 & \frac{3n}{2} & \cdots & \frac{3n}{2} & \frac{3n}{2} \\
0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & \frac{3n}{2} \\
0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & 0 & \frac{3n}{2} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & \frac{3n}{2} & 0 \\
0 & \frac{3n}{2} & \frac{3n}{2} & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0 \\
\frac{3n}{2} & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\frac{3n}{2} & \frac{3n}{2} & 0 & \cdots & \frac{3n}{2} & 0 & 0 & \cdots & 0 & 0 \\
\frac{3n}{2} & \frac{3n}{2} & \frac{3n}{2} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0
\end{bmatrix}$$

$\left|\beta I - RTZ_3\left(K_n\right)\right|=$

$$\begin{bmatrix} \beta & 0 & 0 & \cdots & 0 & 0 & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & -\frac{3n}{2} \\ 0 & \beta & 0 & \cdots & 0 & -\frac{3n}{2} & 0 & \cdots & -\frac{3n}{2} & -\frac{3n}{2} \\ 0 & 0 & \beta & \cdots & 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & 0 & -\frac{3n}{2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & 0 \\ 0 & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & \beta & 0 & \cdots & 0 & 0 \\ -\frac{3n}{2} & 0 & -\frac{3n}{2} & \cdots & -\frac{3n}{2} & 0 & \beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{3n}{2} & -\frac{3n}{2} & 0 & \cdots & -\frac{3n}{2} & 0 & 0 & \cdots & \beta & 0 \\ -\frac{3n}{2} & -\frac{3n}{2} & -\frac{3n}{2} & \cdots & 0 & 0 & 0 & \cdots & 0 & \beta \end{bmatrix} = 0$$

Characteristic equation is

$$\left(\beta - \frac{3n}{2}\right)^{n+1} \left(\beta + \frac{3n}{3}\right)^{n-1} \left(\beta(n-1) - \frac{3n}{2}\right)\left(\beta - (n-1) - \frac{3n}{2}\right) = 0$$

spectrum is $Spec_{ISS}\left(S_n^0\right)$

$$= \begin{pmatrix} \frac{3n}{2}(n-1) & -\frac{3n}{2}(n-1) & -\frac{3n}{2} & \frac{3n}{2} \\ 1 & 1 & n-1 & n-1 \end{pmatrix}$$

Therefore, $ISSE\left(S_n^0\right) = 6n(n-1)$.

4) *Theorem-2.4.* For complete bipartite graph, the Inverse Sum Status energy is

$$ISSE\left(K_{m,n}\right) = 2(mn)^{\frac{5}{2}}.$$

Proof. $ISS\left(K_{m,n}\right) = ISS\left(K_{m,n}\right) = (mn)^2 \begin{pmatrix} 0_{m\times m} & J_{m\times n} \\ J_{n\times m} & 0_{n\times n} \end{pmatrix}$

$$Spec_{ISS}\left(K_{m,n}\right) = \begin{pmatrix} 0 & (mn)^{\frac{5}{2}} & -(mn)^{\frac{5}{2}} \\ m+n-2 & 1 & 1 \end{pmatrix}$$

Therefore, $ISSE\left(K_{m,n}\right) = 2(mn)^{\frac{5}{2}}$.

5) *Theorem-2.5.* The Inverse Sum Status energy of $K_{n\times 2}$ is

$$ISSE(K_{n\times 2}) = 2n(n-1).$$

Proof. Here the status of each vertex is $2n$. hence Inverse Sum Status matrix is

$$\begin{bmatrix} 0 & 0 & n & n & \cdots & n & n & n & n \\ 0 & 0 & n & n & \cdots & n & n & n & n \\ n & n & 0 & 0 & \cdots & n & n & n & n \\ n & n & 0 & 0 & \cdots & n & n & n & n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ n & n & n & n & \cdots & 0 & 0 & n & n \\ n & n & n & n & \cdots & 0 & 0 & n & n \\ n & n & n & n & \cdots & n & n & 0 & 0 \\ n & n & n & n & \cdots & n & n & 0 & 0 \end{bmatrix}$$

$$Spec_{ISS}(K_{n\times 2}) = \begin{pmatrix} n(n-1) & 0 & -n \\ 1 & n & n-1 \end{pmatrix}$$

$$ISSE(K_{n\times 2}) = 2n(n-1).$$

III. PROPERTIES OF INVERSE SUM STATUS ENERGY

The following results will give the initial coefficients of the Inverse Sum Status characteristic polynomial.

1) *Proposition-3.1.* In the Inverse Sum Status characteristic polynomial $\phi_{ISS}(G,\beta)$, the first three coefficients are 1,0 and $-\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2$ respectively.

2) *Proposition-3.2.*

$$\sum_{i=1}^{n}\beta_i^2 = 2\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2$$

(Where If β_i represents Inverse sum status eigenvalues.)

Proof. We know that

$$\sum_{i=1}^{n}\beta_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\,a_{ji}$$

$$= 2\sum_{i<j} a_{ij}^2 + \sum_{i=1}^{n}a_{ii}^2$$

$$= 2\sum_{i<j}a_{ij}^2$$

$$= 2\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2.$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue VI Jun 2021- Available at www.ijraset.com

3) Theorem-1.3. Let G be a graph with n vertices. Then

$$ISSE(G) \le \sqrt{2n\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2}$$

Proof. We have Cauchy-Schwartz inequality as

$$\left(\sum_{i=1}^{n} x_i y_i\right)^2 \le \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right)$$

Substitute $x_i = 1 \; and \; y_i = \beta_i$. Then

$$\left(\sum_{i=1}^{n}|\beta_i|\right)^2 \le \left(\sum_{i=1}^{n}1\right)\left(\sum_{i=1}^{n}|\beta_i|^2\right)$$

Which implies that

$$ISSE(G) \le \sqrt{2n\left[\sum_{i=1(\sigma_u+\sigma_v)}^{n}\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2}$$

4) Theorem-1.4.

$$ISSE(G) \ge \sqrt{2\sum_{i=1}^{n}\left[\frac{\sigma_u\sigma_v}{(\sigma_u+\sigma_v)}\right]^2 + n(n-1)\left[Det(ISS(G))\right]^{\frac{2}{n}}}$$

Proof. By definition,

$$(ISSE(G))^2 = \left(\sum_{i=1}^{n}|\beta_i|\right)^2$$

$$= \sum_{i=1}^{n}|\beta_i|\sum_{j=1}^{n}|\beta_j|$$

$$= \left(\sum_{i=1}^{n}|\beta_i|^2\right) + \sum_{i\ne j}|\beta_i||\beta_j|.$$

Using arithmetic mean and geometric mean inequality, we have

$$\frac{1}{n(n-1)}\sum_{i\ne j}|\beta_i||\beta_j| \ge \left(\prod_{i\ne j}|\beta_i||\beta_j|\right)^{\frac{1}{n(n-1)}}.$$

Therefore,

$$(ISSE(G))^2 \geq \sum_{i=1}^{n} |\beta_i|^2 + n(n+1)\left(\prod_{i \neq j} |\beta_i||\beta_j|\right)^{\frac{1}{n(n-1)}}$$

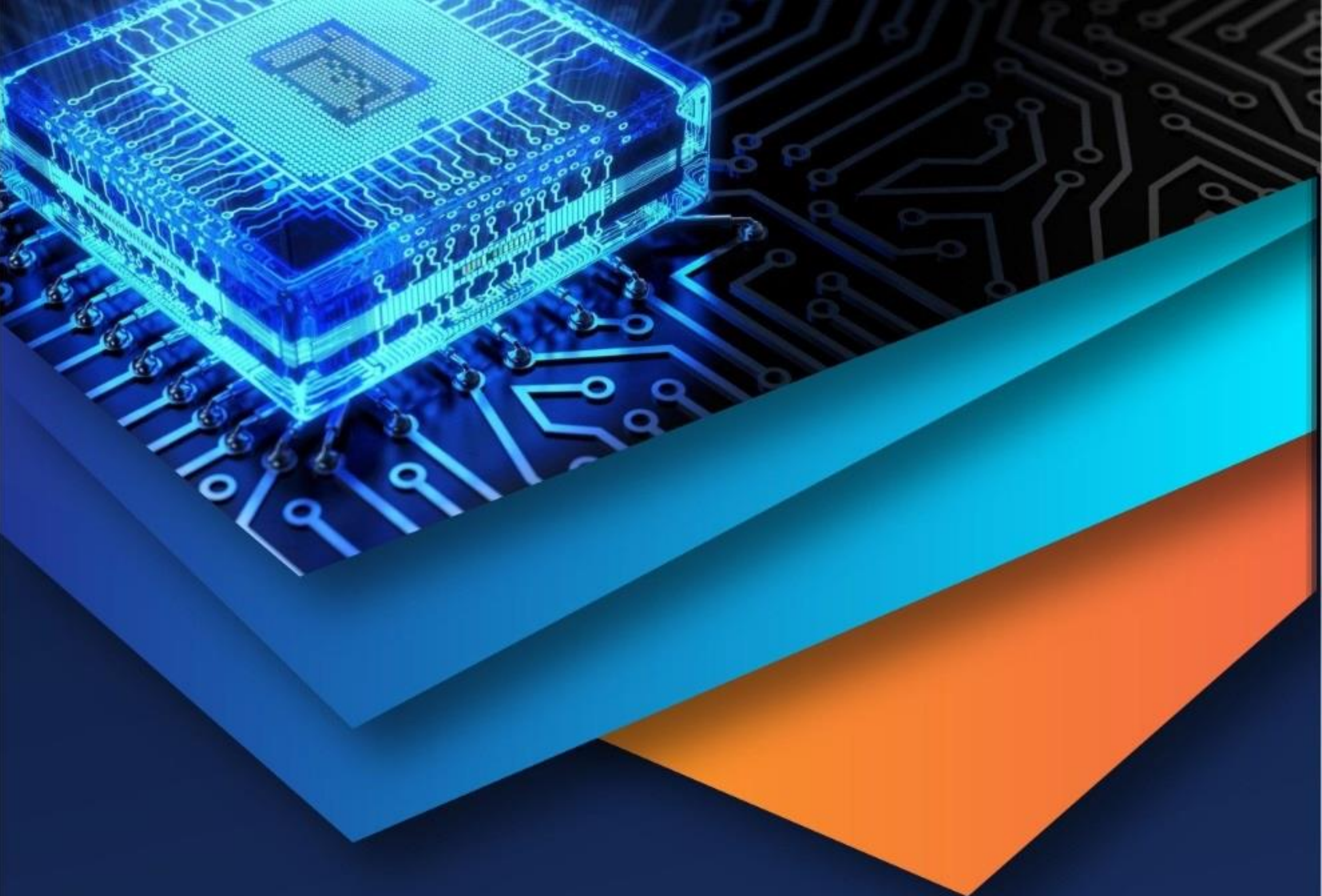
$$\geq \sum_{i=1}^{n} |\beta_i|^2 + n(n+1)\left(\prod_{i=1}^{n} |\beta_i|^{2(n-1)}\right)^{\frac{1}{n(n-1)}}$$

$$= 2\left[\sum_{i=1}^{n} \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)}\right] + n(n-1)\,[[ISS]]^{\frac{2}{n}}.$$

$$ISSE(G) \geq \sqrt{2\left[\sum_{i=1}^{n} \frac{\sigma_u \sigma_v}{(\sigma_u + \sigma_v)}\right] + n(n-1)[[ISS]]^{\frac{2}{n}}} \;\ldots$$

REFERENCES

[1] C.Adiga and C.S.Shivakumaraswamy, Bounds on the largest minimum degree eigenvalues of graphs, Int. Math. Forum, 5(37)(2010), 1823-1831.
[2] I. Gutman, The energy of a graph: old and new results, Combinarotics and applications, A. Betten, A. Khoner, R. Laue and A. Wassermann, eds., Sp[ringer, Berlin, 2001, 196-211.
[3] F. Harry, On the notion of balance of a signed graph, Michigan Math. J., 2(1953-54), 143-146 and addendum proceeding, p.1.
[4] F. Harary, On local balance and N-balance in signed graphs, Michigan Math. J., 3(1955), 37-41.
[5] Kinkar Ch. Das, Bounds on the greatest eigenvalue of a graphs, Indian J. Pure appl. Math., 334(6), June 2003, 917-925.
[6] Mohsen Molahajiaghaei, The eigenvalues and energy of integral circulant graph, Transactions on Combinatorics, 1(2012), No.3, 47-56.
[7] V. Nikiforov, Graphs and matrices with maximal energy, J. Math. Anal. Appl., 327(2007), 735-738.
[8] H.S. Ramane, H.B. Walikar, S.B.Rao, B.D. Acharya, P.R. Hampiholi, S.R., Jog, I. Gutman, Spctra and energies of iterated line graphs of regular graphs, Applied Mathematical Letters, 18(2005), 679-682.
[9] E.Sampathkumar, P.Siva Kota Reddy and M.S. Subramanya, (4,d)-sigraph and its applications, Advan. Stud. Contemp. Math., 20(1)(2010), 115-124.
[10] E.Sampathkumar, P.Siva Kota Reddy and M.S. Subramanya, Directonally n-signed Graphs-II, International. J. Math. And Combin, 4,89-98, (2009).
[11] E.Sampathkumar and M.A.Sriraj, Vertex Labeled / Colored Graphs, Matrices and Signed Graphs, J. of Combinatorics, Inforamtion and System Sciences, to appear.
[12] D.Stevanovic and G.l Indulal, The distance spectrum and energy of the composition of regular graphs, Appl. Math. Lett., 22(2009), 1136-1140.
[13] H.B.Walikar, H.S.Ramane, and P.R.Hampiholi, On the energy of graph, Proc. Of the Conference on Graph connections (R. Balakrishnan, H.M.Mulder and A. Vijaykumar, ed.) Allied Pub. Ltd., New Delhi, 1999, 120-123.
[14] T. Zaslavsky, Signed Graphs, Discrete Appl. Math., 4(1)(1982), 47-74.
[15] T.Zaslavsky, A Mathematical Bibliography of Signed and Gain Graphs and Allied Areas, VII Edition, Electronic J. Combinatorics, 8(1)(1998), Dynamic Surveys # 8, 124 pp. (Also, see VIII Preliminary Edition, March 2008).
[16] T.Zaslavsky, Matrices in the Theory of Signed Simple Graphs, RMS-Lecture Notes Series 13, (2010), 207-229.



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY