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Doubling of Group Velocity near the Coast due to Storm Surge, Tides or Tsunami: An Analytical Study

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Abstract: Tsunami may cause due to storm surge, earthquake at coastal region or at under the sea (obviously Hypocenter of earthquake must be within the earth's crust or below). Each cases a wave packet which develops a set of simple harmonic motion of different wave lengths. These simple harmonic waves have been analyzed analytically. The velocity of propagation of waves depends upon the wave lengths. So the waves of nearly equal wave lengths can be considered as a group. This group of waves will propagate with nearly equal velocity which is known as group velocity. On considering two consecutive simple harmonic waves of same amplitude, we can find two equations describing simple harmonic motions having slightly different wave lengths and time periods. The combination of these two said SHM's we can find another resultant SHM with different amplitude than of the previous two SHM's. This new SHM has a slight variation in wave length and time period than that of that of the said earlier two SHM's. Then using the perturbation technique on this resultant equation of SHM, we can find a new wave velocity (group velocity) in differential notation of wave velocity. This differential notation of wave velocity has been eliminated from the relation between the wave velocity on the surface of water and the depth of the sea. Then we get a relation between group velocity and wave velocity depending with depth of the sea. The application of boundary conditions on depth of deep sea and that of at the sea shore, we can find the group velocity at each region respectively. It is seen from the analysis that the group velocity at the sea shore is as much as double that of at the deep sea. Several recent cases of tsunamis or storm surges have been studied and it is found that the results depicted the same implementation through devastation that established by analytical study.
Keyword: Tsunami; Storm surge; wave packet; Group Velocity; SHM; Hypocenter; perturbation technique.

I. INTRODUCTION

Most of the coastal areas of West Bengal and Bangladesh whose mean sea level heights are very near to sea level are disaster prone due to anomalous unpredictable surge of ocean. The geographical condition of soils are equally dangerous for huts, long trees, human habitations, cattle's etc. to survive from the destructive nature of ocean wave at the Bay Of Bengal (BOB). The soil condition is so sandy up to a great depth that it is very difficult to construct concrete structure here. The repeated pounding of waves through out the year due to different natural phenomena over the ocean are the main hurdle for development. Fortunately, mangrove habitations are able to survived due to their peculiar characteristics of roots. The yearly damage of mud barriers constructed by rural methods are very common. Most of the local people respect severe weather phenomena and ocean surge as almighty. Unfortunately, the region is a permanent rest house of severe weather phenomena and ocean surge whose caterers or treated by all kinds of local habitats by their last respiration.

As per national and international rules, helps through administration reaches to the affected region on the basis of category of atmospheric phenomena where wave velocity is not being countered alongside. For this reason, even if the destruction is higher, categorically help may come at lower scale. Best examples are Aila , May 2009 FANI, April 2020 and AMPHAN May 2020.

The head BOB is a region shallow depth. There are many theories behind it that severe atmospheric phenomena began to lose its energy due to this shallow depth at head BOB. In this piece of work, it has been tried to show fluid dynamically that even if the atmospheric phenomena began to slow down, the ocean wave may increase substantially due to conservation of energy or momentum.

It is well known that any perturbation makes a wave packet. Just like when a stone is thrown into a pond, with full of water, a wave packet is generated. During tsunami such wave packets are generated near the hypocenter of the earth quake region. Or during vigorous storm in the sea the wave packets also be developed. Each wave of the wave packet can be analyzed into a set of simple harmonic waves of different wave lengths.

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It is also well known that velocity of propagation depends upon wave length λ, the waves of slightly different wavelengths will form different groups. The velocity with which a group of waves progress is called the group velocity. Some times waves due to this group velocity has been identified as **swell waves**. The characteristic of swell wave or group velocity is different from wave velocity. It is the observer's primary concurrence of identification of group velocity from wave velocity. Sailors are accustomed with rolling and pitching of the ship. In general, if the wave velocity is faces by the ship, then it is called rolling, and then the pitching is caused by the group velocity (if there be any). Some times the direction of wave velocity and group velocity may be same then it may not be identified by rolling and pitching.

Considering two simple harmonic waves of the same amplitude a (let us say), the resultant disturbance has been calculated. The resultant wave is of the same form as one of the original waves but with a different amplitude. The resulting amplitude of the resulting disturbance varies as a wave velocity which is known as group velocity. This group velocity is different from the original wave velocity. The group velocity in the limiting case with the depths have been calculated. The wave velocity comparing with the group velocity result has been achieved.

II. METHOD

We consider the ocean as of uniform depth h and the propagation of a simple harmonic wave motion is given by

$$\eta = a\, sin(mx - nt). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(1)$$

Inside the water where η is the maximum elevation of free surface from the undisturbed level. The waves are under gravity control. Let us take the equilibrium level as x axis and the y axis is vertically upwards as shown in the following figure (fig-1).

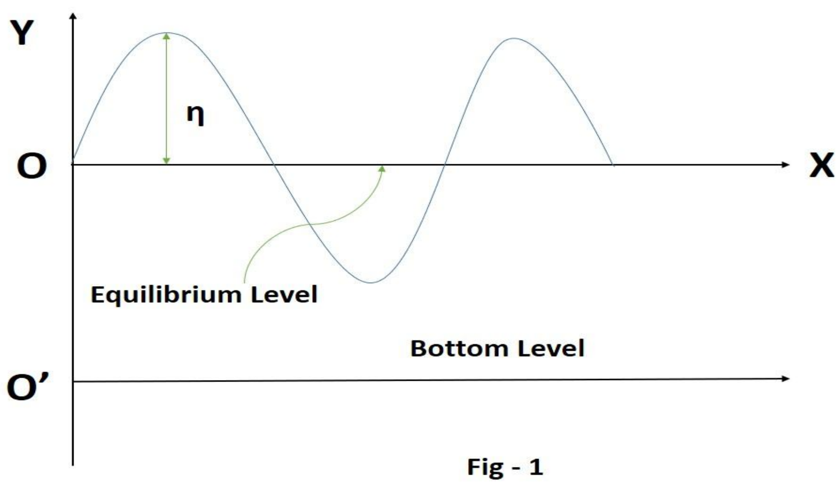

Fig - 1

The velocity potential φ satisfy the following conditions:

$$\frac{\partial \eta}{\partial t} = v = -\frac{\partial \varphi}{\partial y} \quad \text{at} \quad y = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(2)$$

$$\nabla^2 = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(3)$$

$$\frac{\partial \varphi}{\partial y} = 0 \text{ at } y = -\ h \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(4)$$

$$\frac{\partial^2 \varphi}{\partial t^2} + g\,\frac{\partial \varphi}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(5)$$

Where the symbols have their usual meaning.

From (1) and (2)

$$-\frac{\partial\varphi}{\partial t} = \frac{\partial\eta}{\partial t} = an\cos(mx-nt) \quad \text{at } y = 0$$

$$\boldsymbol{\varphi} = f(y)\cos(mx-nt) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Putting in (3)

We get the partial differential equation of the form

$$m^2 f + \frac{d^2 f}{dy^2} = 0$$

whose solution is of the form

$$f = Ae^{my} + Be^{-my}$$

Thus (6) converted to the form

$$\boldsymbol{\varphi} = (Ae^{my} + Be^{-my})\cos(mx-nt)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots....(7)$$

Differentiating (7) with respect to y (partially) and using to (4)

We get

$$0 = m(Ae\text{-}^{mh} - Be^{mh})\cos(mx\text{-}nt)$$

or $\quad Ae\text{-}^{mh} - Be^{mh} = 0$

$$Ae\text{-}^{mh} = Be^{mh} = \frac{1}{2}C \text{ (let us say)}$$

Therefore $\quad \boldsymbol{\varphi} = \frac{1}{2}C\,[e^{m(y+h)} + Be^{-m(y+h)}]\cos(mx\text{-}nt)$

Or $\quad \boldsymbol{\varphi} = C\cosh m(y+h)\cos(mx\text{-}nt)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(8)$

By (1) and (8),

$$\frac{\partial\eta}{\partial t} = an\cos(mx\text{-}nt),$$

$$\frac{\partial\varphi}{\partial y} = mc\sinh m(y+h)\cos(mx\text{-}nt)$$

Putting this in the equation (2) we get

$$-an\cos(mx-nt) = -mC\,sinH\,mh.\,Cos\,(mx-nt)$$

Or

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C = $\frac{an}{m \sinh mh}$ = $\frac{ac}{\sinh mh}$, where c = $\frac{n}{m}$

Now (8) becomes

$\boldsymbol{\varphi}$ = $\frac{ac}{\sinh mh}$ cos m(y+h) cos(mx-nt)……………………….(9)

Putting (9) in (5) we get

- n^2 cosh m(y+h) cos (mx-nt) + gm sinh m(y+h) cos(mx-nt) = 0 at y = 0.

or

- n^2 cosh mh + gm sinh mh = 0

or

c^2 = $\frac{n^2}{m^2}$ = $\frac{g}{m}$ tanh mh is an expression for wave velocity .

let us consider two simple harmonic waves of same amplitudes as follows:

$$\eta_1 = a\, sin(mx - nt).$$

$$\eta_2 = a\, sin\, [(m + \delta m)x - (n + \delta n)t].$$

$$\delta m < < m)\ and\ \delta n << n$$

Symbols having their usual meaning.

The resultant disturbance is given by η = $\eta_1 + \eta_2$,

$$\eta = 2a\, sin\left[\left(m + \frac{\delta m}{2}\right)x - \left(n + \frac{\delta n}{2}\right)t\right] cos\left(\frac{x\, \delta m - t\delta n}{2}\right).$$

$$Since\ \ \delta m < < m)\ and\ \delta n << n.$$

Hence $\eta = A\, Sin(mx - nt)$……………………………………………………..(10)

Where $A = 2a\, cos\left(\frac{x\, \delta m - t\delta n}{2}\right)$.…………………………………………………(11)

Hence the resultant wave is of the same from as one of the original waves but with a different amplitude. From (11), it follows that the amplitude A of the resulting disturbance (11) varies as a wave velocity c_g = $\frac{\delta n}{\delta m}$ known as **Group velocity**. In differential notation,

$c_g = \frac{dn}{dm}$

But $c = \frac{n}{m}$

Hence $c_g = \frac{d(mc)}{dm} = c + m\frac{dc}{dm}$, ………………………………………………….(12).

For the waves on the surface of water of depth h, we have seen the relation

$$c^2 = \frac{n^2}{m^2} = \frac{g}{m}\tanh mh$$

taking logarithm, $2\log c = \log g - \log m + \log \tanh mh.$

Differentiating with respect to m,

$$\frac{2\,dc}{c\,dm} = -\frac{1}{m} + h\frac{\mathrm{sech}^2\, mh}{\tanh mh}$$

or $$\frac{mdc}{dm} = -\frac{c}{2} + c.\frac{mh}{\sinh 2mh}$$

Then (12) becomes

$$c_g = \frac{c}{2}\left(1 + \frac{mh}{\sinh 2mh}\right) \quad \text{…………………………………..(13)}$$

From (13) we see that

$$\frac{\text{Group velocity}}{\text{Wave velocity}} = \frac{1}{2}\left(1 + \frac{mh}{\sinh 2mh}\right)$$

where $m = \frac{2\pi}{\lambda}$

1) *Case I*
For depth at deep sea $h \to \infty$

so that $\frac{2\,mh}{\sinh 2mh} \to 0$

$c_g = \frac{c}{2}[1+0]$ or $c_g = \frac{c}{2}$

This shows that

Group velocity = $\frac{1}{2}$ wave velocity.

2) *Case II*
When near the coast $h \to 0$

$\frac{2\,mh}{\sinh 2mh} \to 1$ Hence $c_g = \frac{c}{2}[1+1]$

Or $c_g = c$

Which shows that near the coast: group velocity = wave velocity.

III. CONCLUSION

It is obvious that the depth of sea is many times at deep sea than near the sea shore. The analytical discussion shows that the group velocity is double at the coast than the deep sea. In fact, the data are not steel available for identification of wave velocity from group velocity. The tide gauges are also not working to measure the surge height. But technically it is possible. The photographs as for evidences from recent cyclones are given bellow. It is seen that a barge p305 of Oil and Natural Gas Commission, Government of India sank near the coast of Gujarat, due to recent cyclone Tauktae. In general, the ships are anchored with the alignment with the wave velocity. So it can face wave velocity as good as pitching of the ship. But when a group velocity of equal magnitude comes from other directions particularly from perpendicular direction, then the ship rolls. This rolling is very difficult to maintain by the wheel(steering) of the ship. Where as the ship well placed in the deep sea and in running condition can easily support the rolling effects due to group velocity which is half the wave velocity there in the deep sea. The recent Severe cyclones Yash and Aila and extreme severe cyclone Amphan had damaged several places of West Bengal as if those were Super Cyclone due to doubling of group velocities. The saline Ocean waves crosses through the low mean sea level areas and stalled permanently there. These permanently stalled saline water causes sustainable disasters on forest, agriculture, sweet water fish industries.

Rough sea during landfall of cyclone Yaas at Digha in East Midnapore district.

PTI

Sea water enters through boundaries of a house during cyclone Yaas.

PTI

Super Cyclone Philine

Pathar Pratim, South 24 parganas, Cyclone Yash; courtesy : Krishnendu Manna, PJB college, Andul, Howrah.

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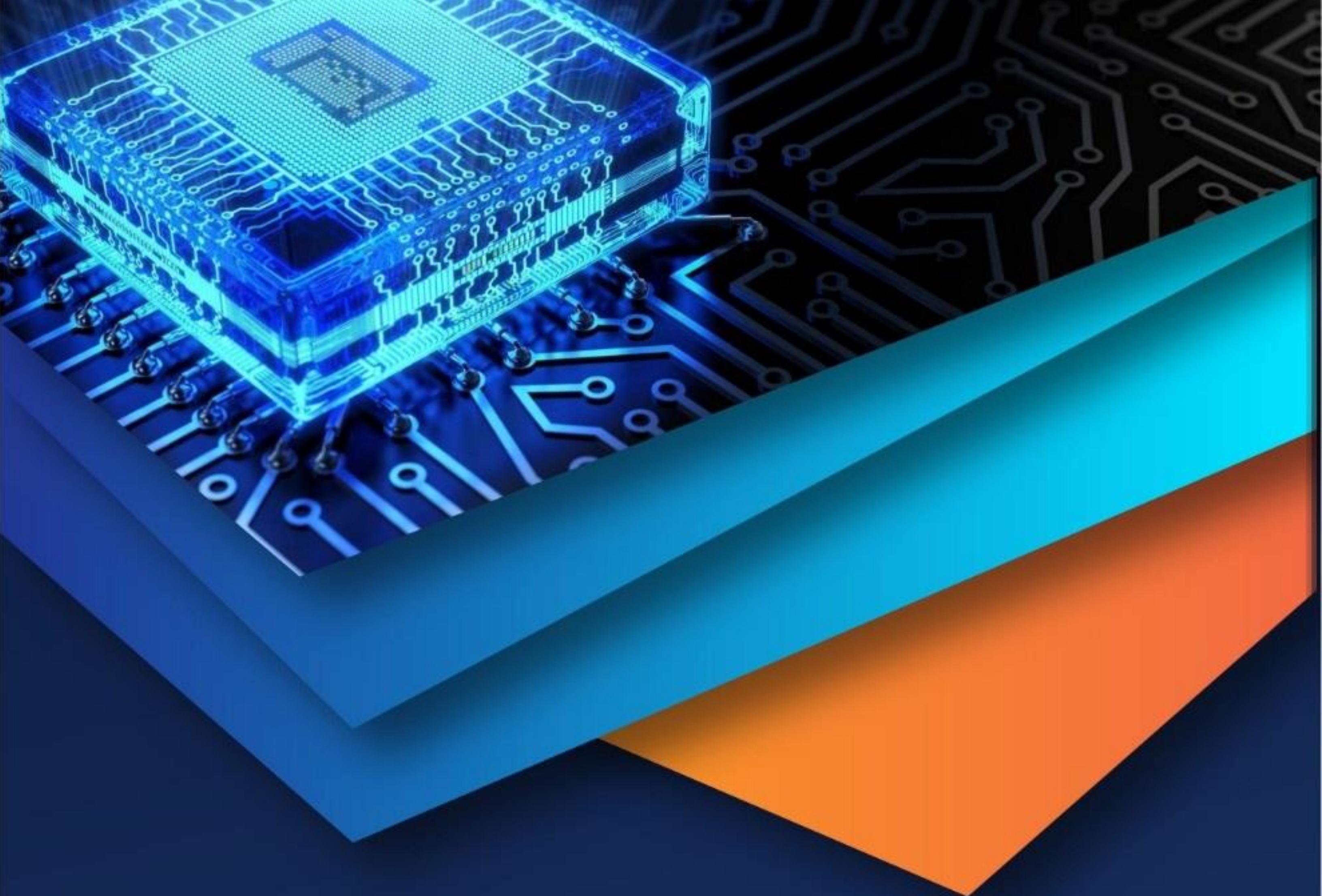

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