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# Analysis of Critical Speed and Natural Frequency of Shaft with Multi – Crack and Multi Masses using different Materials

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**Abstract:** A natural frequency was analyzed and critical speed was predicted by using Campbell diagram and analysis was also performed for validation. The results represents that a solid shaft with three cracks with two masses and material like structural steel and titanium alloy the critical speed with increase in a RPM continuously found in structural steel. The natural frequency of shaft is compared by using two types of materials and is predicted that at solid shaft with multi – crack and masses of titanium alloy exhibits lower critical speed.

**Keywords:** Critical Speed, Campbell Diagram, Rotor Dynamics, Titanium alloy, Structural Steel.

## I. INTRODUCTION

In solid mechanics, inside the area of rotordynamics, the vital speed is the theoretical angular velocity that excites the natural frequency of a rotating object, which include a shaft, propeller, leadscrew, or gear. As the velocity of rotation processes the object's natural frequency, the object starts to resonate, which dramatically increases machine vibration. The resulting resonance takes place regardless of orientation. While the rotational speed is same to the numerical value of the natural vibration, then that velocity is known as critical speed [17].

Critical Speed [16]

- 1) The critical speed essentially depends on
- 2) Critical or whirling or whipping speed is the speed at which the shaft tends to vibrate violently in transverse direction.
  - a) The eccentricity of the C.G of the rotating masses from the axis of rotation of the shaft.
  - b) Diameter of the disc
  - c) Span (length) of the shaft, and
  - d) Type of supports connections at its ends.

### A. Critical Speed of Shafts

All rotating shafts, even in the absence of external load, will deflect during rotation. The unbalanced mass of the rotating object causes deflection that will create resonant vibration at certain speeds, known as the critical speeds. The magnitude of deflection depends upon the following:

- 1) Stiffness of the shaft and its support
- 2) Total mass of shaft and attached parts
- 3) Unbalance of the mass with respect to the axis of rotation
- 4) The amount of damping in the system

In general, it is necessary to calculate the critical speed of a rotating shaft, such as a fan shaft, in order to avoid issues with noise and vibration [17].

### B. Analytical model of a continuous shaft with two breathing cracks

The governing equation of lateral motion of a continuous rotating shaft with a disc located at the mid-span

$$EI \frac{\partial^4 u}{\partial x^4} - \left( \frac{EI\rho}{kG} + \rho Ar_0^2 \right) \frac{\partial^4 u}{\partial x^2 \partial t^2} + 2i\rho Ar_0^2 \Omega \frac{\partial^2 u}{\partial x \partial t} + \frac{\rho^2 Ar_0^2}{kG} \frac{\partial^4 u}{\partial t^4} - 2i \frac{\rho^2 Ar_0^2 \Omega}{kG} \frac{\partial^3 u}{\partial t^3} + \rho A \frac{\partial^2 u}{\partial t^2} = 0$$

C. Crack modeling

$$C = \begin{bmatrix} C_{yy} & C_{yz} \\ C_{zy} & C_{zz} \end{bmatrix}$$

$$f(t) = \frac{1 - \cos(\Omega t + \chi_r)}{2}$$

$$\chi_r = |\Phi_r - \Phi_1|, \quad r = 1, 2$$

$$f(t) = \frac{1}{2} - \frac{1}{4} (e^{i(\Omega t + \chi_r)} + e^{-i(\Omega t + \chi_r)})$$

II. MODELING OF PRESENT CONTINUA

The below shown figure represents the model of three crack shaft including two masses, the modal analysis is performed in present analysis by considering finite element method

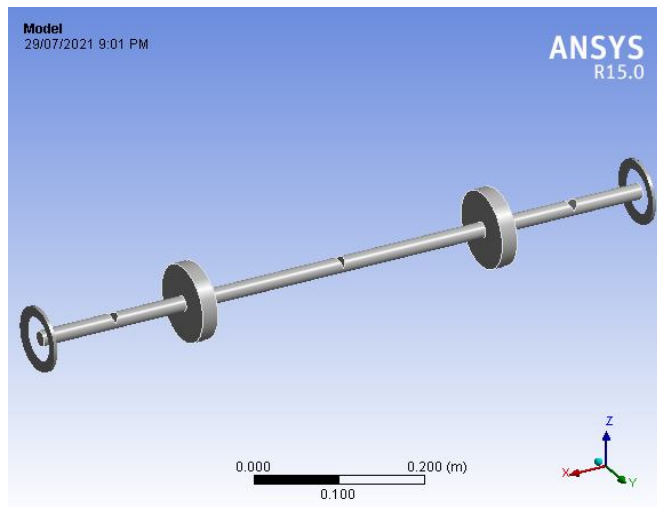


Figure 1: Model of Solid shaft with three Crack and two masses

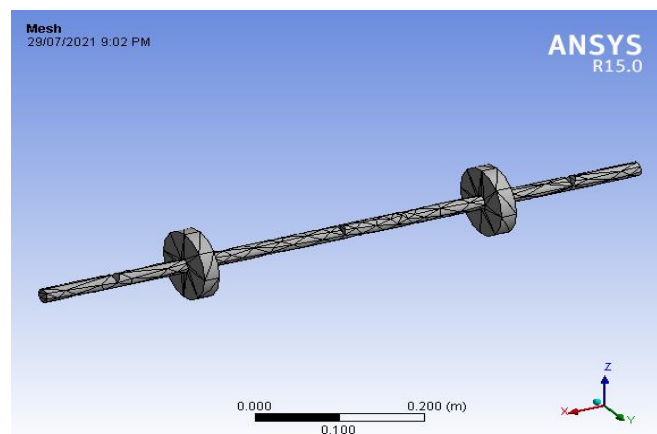


Figure 2: Meshed model of solid shaft with three crack and two masses

### III. RESULT AND DISCUSSION

#### A. Analysis of Solid Shaft with three Cracks using titanium alloy and structural steel

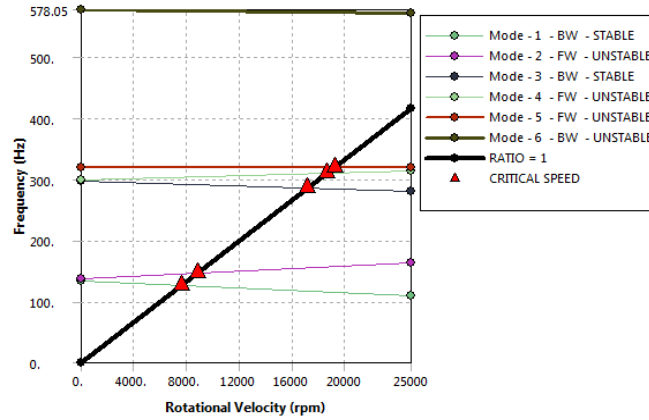


Figure No.3: Result of Campbell diagram of frequency and rotational velocity distributions along the structural steel solid shaft with Three Cracks and two masses.

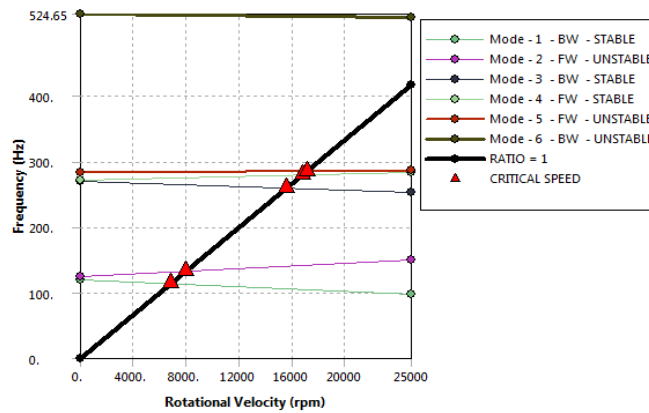


Figure No.4: Result of Campbell diagram of frequency and rotational velocity distributions along the titanium alloy solid shaft with Three Cracks and two masses.

Table No.1: Critical Speed of Solid Shaft with three Cracks and two masses (Structural Steel)

Structural Steel			
Mode	Whirl Direction	Mode Stability	Critical Speed
1	BW	STABLE	7566.8
2	FW	UNSTABLE	8795.1
3	BW	STABLE	17167
4	FW	UNSTABLE	18636
5	FW	UNSTABLE	19233
6	BW	UNSTABLE	NONE

Table No.2: Critical Speed of Solid Shaft with three Cracks and two masses (Titanium Alloy)

Titanium alloy			
Mode	Whirl Direction	Mode Stability	Critical Speed
1	BW	STABLE	6849.5
2	FW	UNSTABLE	7960.8
3	BW	STABLE	15559
4	FW	STABLE	16754
5	FW	UNSTABLE	17142
6	BW	UNSTABLE	NONE

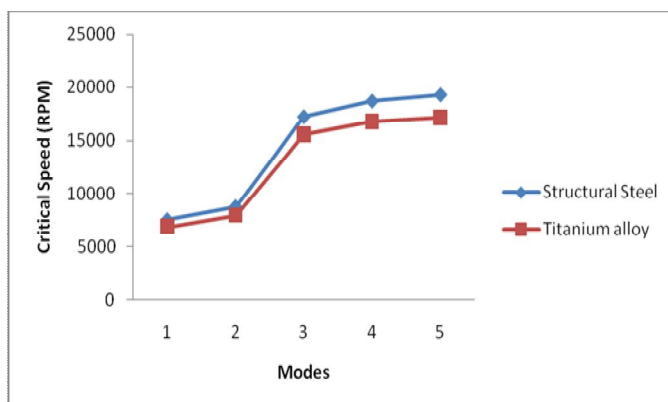


Figure No.5: Graph shows comparison of critical speed of two different materials.

Table No.3: Natural Frequency for structural steel and titanium alloy

Mode	Natural Frequency (Structural Steel)	Natural Frequency (Titanium Alloy)
1	133.09	120.48
2	137.3	124.17
3	298.13	270.29
4	298.77	270.83
5	320.54	283.41
6	578.05	524.65

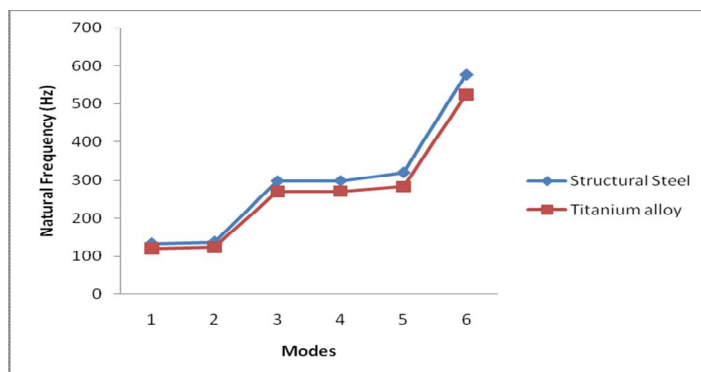


Figure No.6: Graph shows comparison of natural frequency of two different materials.



**IV. CONTOUR PLOTS OF NATURAL FREQUENCY WITH THEIR DIFFERENT MODES FOR STRUCTURAL STEEL WITH THREE CRACKS AND THREE MASS**

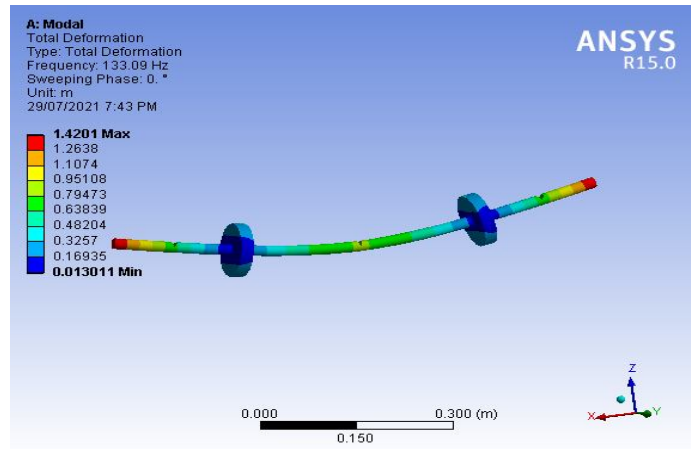


Figure No.7: First modes frequency of structural steel shaft

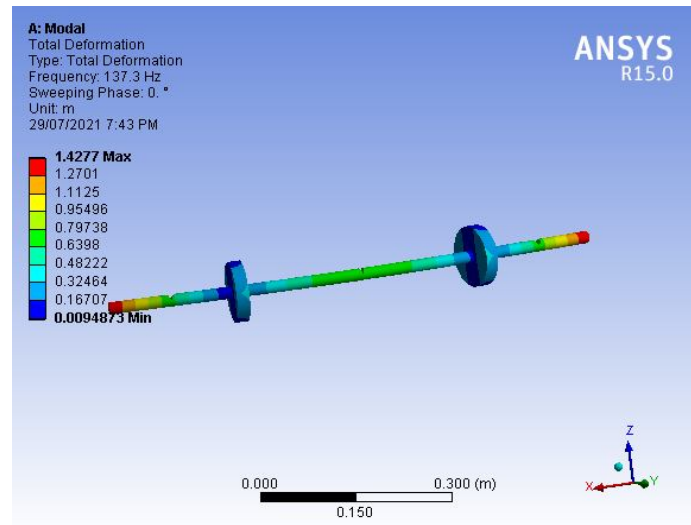


Figure No.8: Second modes frequency of structural steel shaft

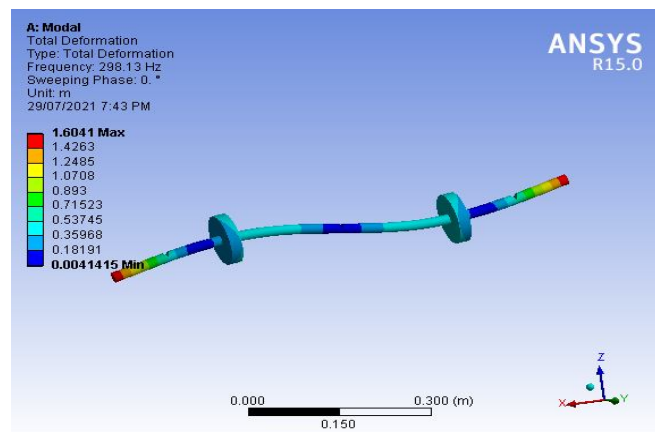


Figure No.9: third modes frequency of structural steel shaft

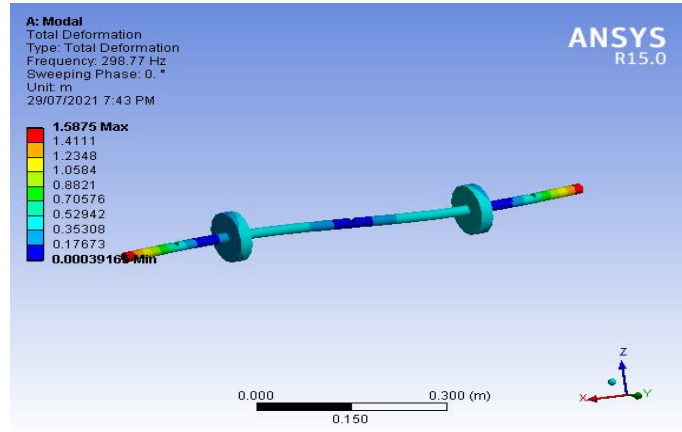


Figure No.10: fourth modes frequency of structural steel shaft

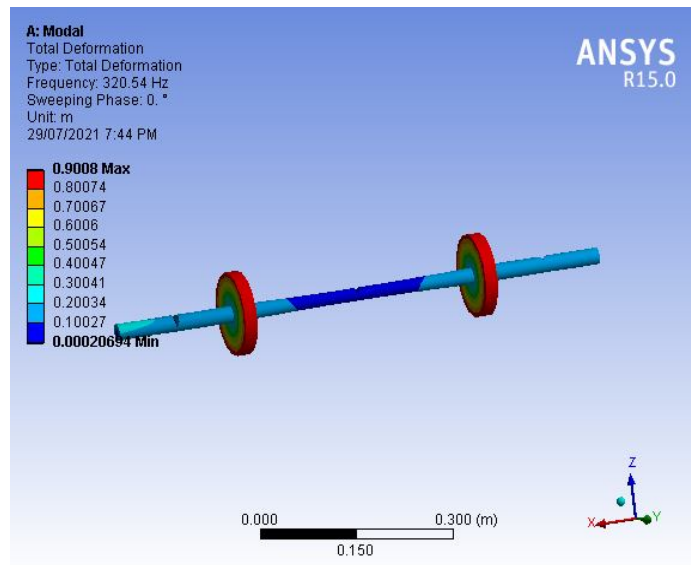


Figure No.11: fifth modes frequency of structural steel shaft

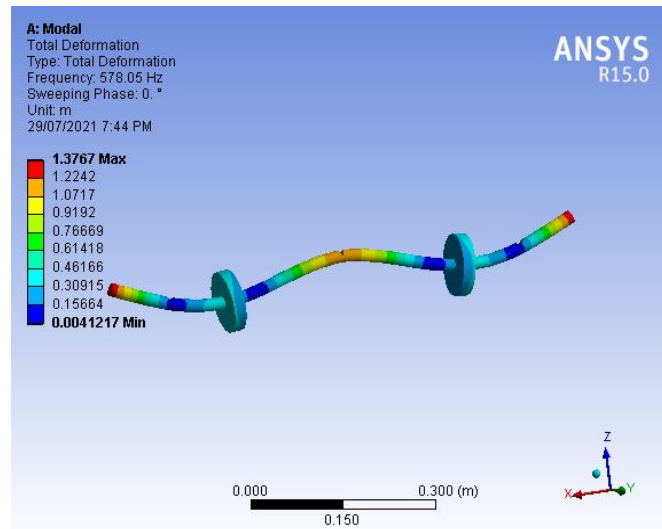


Figure No.12: sixth modes frequency of structural steel shaft

**V. CONTOUR PLOTS OF NATURAL FREQUENCY WITH THEIR DIFFERENT MODES FOR TITANIUM ALLOY WITH THREE CRACKS AND THREE MASS**

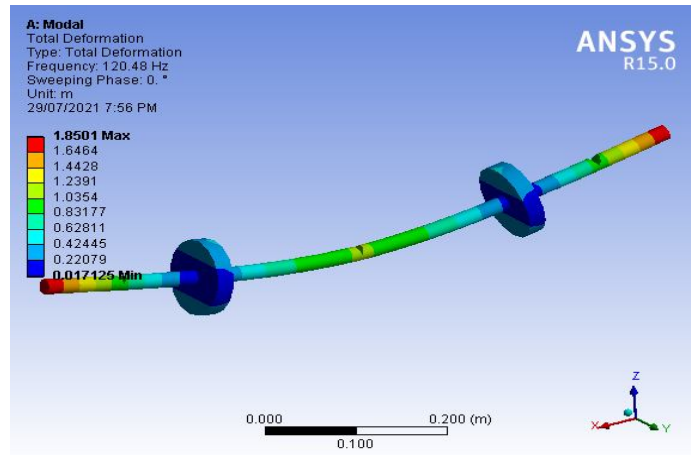


Figure No.13: first modes frequency of titanium alloy shaft

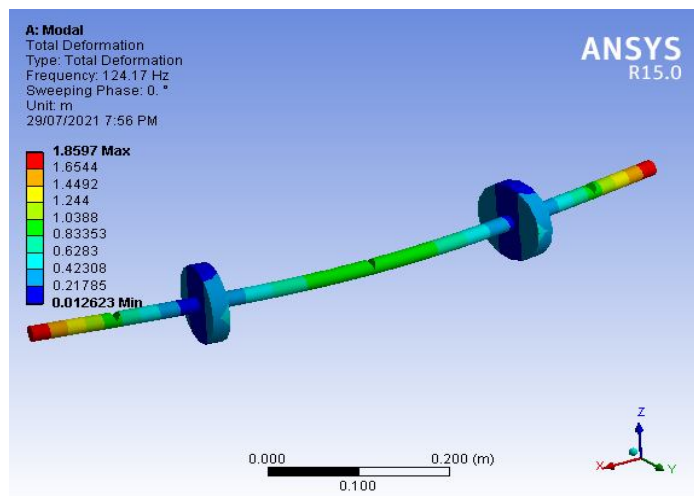


Figure No.14: second modes frequency of titanium alloy shaft

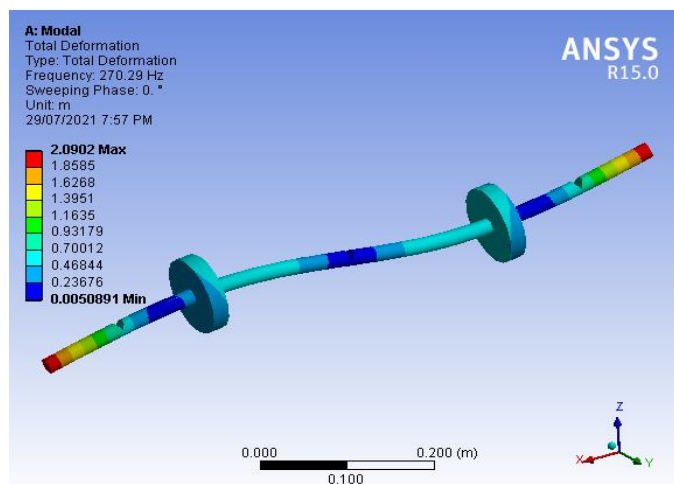


Figure No.15: third modes frequency of titanium alloy shaft



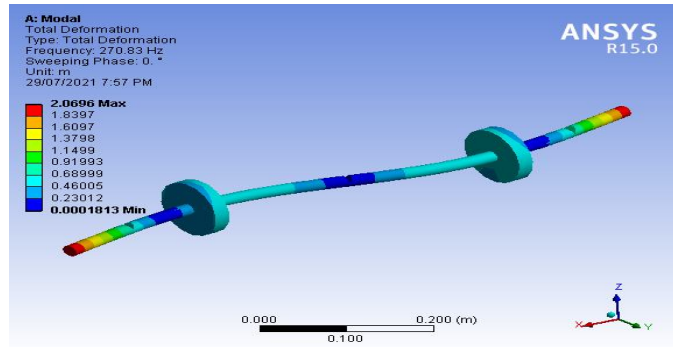


Figure No.16: forth modes frequency of titanium alloy shaft

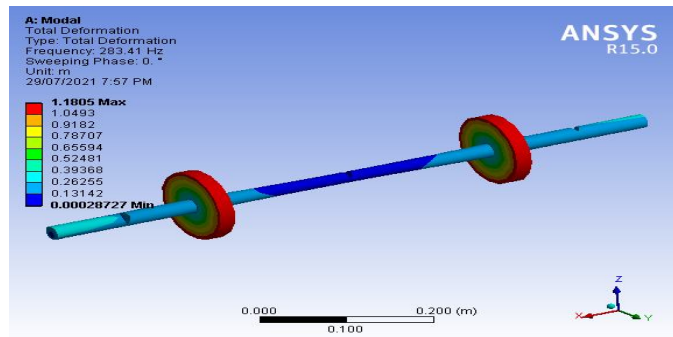


Figure No.17: fifth modes frequency of titanium alloy shaft

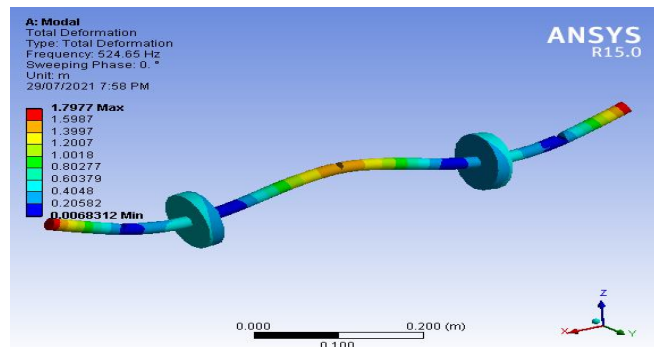


Figure No.18: sixth modes frequency of titanium alloy shaft

## VI. CONCLUSION

The effect of natural frequency along the shaft profile of two masses and three crack is observed to be minimum in titanium alloy material as compared to structural steel this is due to multi – crack effect, it is also predicted that critical speed of shaft is minimum for titanium alloy. The observed effect of critical speed is stable at low RPM as compared to structural steel shaft.

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