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# Fuzzy Inventory Model with Shortages under Fully Backlogged Using Signed Distance Method

S.K. Indrajitsingha[1], P.N. Samanta[2], U.K. Misra[3]
[1]DST INSPIRE Fellow
[1,2]P.G Department of Mathematics, Berhampur University, Bhanja Bihar-760 007, Odisha, India
[3]Department of Mathematics, NIST, Palurhills, Golanthara-761 008, Odisha, India

***Abstract- In this paper we have studied an inventory model for deteriorating items with shortages under fully backlogged condition. The analytical development is provided to obtain the fuzzy optimal solution, defuzzification by Signed Distance Method. In fuzzy environment, all related inventory parameters are assumed to be hexagonal fuzzy numbers (HFN). Suitable numerical examples are also discussed.***
***Keyword-Inventory Model, Deterioration, Hexagonal Fuzzy Number (HFN), Signed Distance Method.***
***AMS Classification No. - 03E72.***

## I. INTRODUCTION

Inventory is a physical stock of any item or resources used in organization. The stockiest run his business smoothly for keeping the physical stock or inventory. In real life inventory system, the effect of decay or deterioration is very important. Deterioration is a crucial attribute of today's esoteric economy which cannot be shrugged off. Generally, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal values of a commodity that the item cannot be used for its original purpose. Otherwise it decreases the usefulness of the product. In our daily life, most of the physical goods like medicine, alcohols, volatile liquids, blood banks, fresh products, flowers, food grains, fruits, vegetables, seafood's undergoes decay or deterioration over time. There are various types of uncertainties involved in any inventory system i.e. deterioration, shortages, holding cost, ordering cost etc. For the above uncertainties, the researchers classically modeled inventory by using probability theory. Consequently, the inventory problem of deteriorating items has been extensively studied by researchers. However, there are uncertainties that cannot be appropriately treated by using usual probabilistic model. Therefore, it becomes more convenient to deal such problems with fuzzy set theory rather than probability theory.

## II. LITERATURE SURVEY

First time in 1965, Lofti A Zadeh[7] introduced the concept of fuzzy sets. The theory of fuzzy sets attracted the attention of many researchers. In 1982, J. Kacpoyzk et. al.[4] proposed a model on long term policy-making through fuzzy decision making model. In 1983, G. Urgeletti [3] introduced the inventory control models and problems. In 1987, K.S Park [6] proposed a model on fuzzy set theoretical interpretation of economic order quantity inventory problem. In 1999, J.S Yao and H.M Lee [5] discussed on a fuzzy inventory with or without back order for for fuzzy order quantity with trapezoidal fuzzy number. In 2002, C.K Kao and W.K Hsu [2] developed a single period inventory model with fuzzy demand. In 2015, D. Stephen Dinagar [1] introduced a fuzzy inventory model with allowable shortage. In 2015, S.K. Indrajitsingha et. al [8] developed an economic production quantity model under fuzzy environment. In the proposed study, an inventory model has been developed for fixed deterioration and linear demand. All inventory parameters are fuzzified as hexagonal fuzzy numbers. The purpose of our study to defuzzify the fuzzy model by using signed distance method. The solution for minimizing the fuzzy cost function has been derived.

## III. DEFINITIONS AND PRELIMINARIES

### A. Definition

A fuzzy set $\tilde{A}$ in a universe of discourse $x$ is defined as the following set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$. Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.

### B. Definition

A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\} \subseteq X$ is called convex fuzzy set if all $\tilde{A}_x$ are convex sets i.e. for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ for every $\alpha \in [0,1]$, $\lambda x_1 + (1-\lambda)x_2 \in A_\alpha$ $\forall\, \lambda \in [0,1]$. Otherwise the fuzzy set is called non-convex fuzzy set.

 

*C. Definition*

A fuzzy set $[a_\alpha, b_\alpha]$ where $0 \le \alpha \le 1$ and $a < b$ defined on $R$, is called a fuzzy interval if its membership function is

$$\mu_{[a_\alpha, b_\alpha]} = \begin{cases} \alpha, & a \le x \le b \\ 0, & Otherwise \end{cases}$$

*D. Definition*

A fuzzy number $\tilde{A} = (a, b, c)$ where $a < b < c$ and defined on $R$, is called triangular fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ \frac{c-x}{c-b}, & b \le x \le c \\ 0, & Otherwise \end{cases}$$

When $a = b = c$, we have fuzzy points $(c, c, c) = c$. The family of all triangular fuzzy numbers on $R$ is denoted as

$$F_N\{(a, b, c) : a < b < c \ \forall \ a, b, c \in R\}$$

The $\alpha - cut$of $\tilde{A} = (a, b, c) \in F_N$, $0 \le \alpha \le 1$, is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$. Where $A_L(\alpha) = a + (b - a)\alpha$ and $A_R(\alpha) = c - (c - b)\alpha$ are the left and right endpoints of $A(\alpha)$.

*E. Definition*

A hexagonal fuzzy number $\tilde{A} = (a, b, c, d, e, f)$ is represented with membership function $\mu_{\tilde{A}}$ as

$$\mu_{\tilde{A}} = \begin{cases} L_1(x) = \frac{1}{2}\left(\frac{x-a}{b-a}\right), & a \le x \le b \\ L_2(x) = \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-d}\right), & b \le x \le c \\ 1, & c \le x \le d \\ R_1(x) = 1 - \frac{1}{2}\left(\frac{x-d}{e-d}\right), & d \le x \le e \\ R_2(x) = \frac{1}{2}\left(\frac{f-x}{f-c}\right), & e \le x \le f \\ 0, & Otherwise \end{cases}$$

The $\alpha$-cut of $\tilde{A} = (a, b, c, d, e, f)$,$0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$.

Where

$$A_{L_1}(\alpha) = a + (b - a)\alpha = L_1^{-1}(\alpha)$$

$$A_{L_2}(\alpha) = b + (c - b)\alpha = L_2^{-1}(\alpha)$$

$$A_{R_1}(\alpha) = e + (e - d)\alpha = R_1^{-1}(\alpha)$$

$$A_{R_2}(\alpha) = f + (f - e)\alpha = R_2^{-1}(\alpha)$$

Hence,

$$L^{-1}(\alpha) = \frac{L_1^{-1}(\alpha) + L_2^{-1}(\alpha)}{2} = \frac{a + b + (c - a)\alpha}{2}$$

$$R^{-1}(\alpha) = \frac{R_1^{-1}(\alpha) + R_2^{-1}(\alpha)}{2} = \frac{e + f + (d - f)\alpha}{2}$$

*F. Definition*

If $\tilde{A} = (a, b, c, d, e, f)$ is a hexagonal fuzzy number then the signed distance method of $\tilde{A}$ is defined as

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$$d(\tilde{A},\tilde{0}) = \int_0^1 d([A_L(\alpha), A_R(\alpha)],\tilde{0}) = \frac{1}{8}(a + 2b + c + d + 2e + f)$$

*G. Definition*

Suppose $\tilde{X} = (x_1, x_2, x_3, x_4, x_5, x_6)$ and $\tilde{Y} = (y_1, y_2, y_3, y_4, y_5, y_6)$ are two hexagonal fuzzy numbers and $x_1, x_2, x_3, x_4, x_5, x_6, y_1, y_2, y_3, y_4, y_5, y_6$ are all real numbers, then the arithmetical operations under function principles as follows:

i. $\tilde{X}\oplus\tilde{Y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4, x_5 + y_5, x_6 + y_6)$

ii. $\tilde{X}\otimes\tilde{Y} = (x_1y_1, x_2y_2, x_3y_3, x_4y_4, x_5y_5, x_6y_6)$

iii. $-\tilde{Y} = (-y_6, -y_5, -y_4, -y_3, -y_2, -y_1)$

iv. $\frac{1}{\tilde{Y}} = \tilde{Y}^{-1} = (\frac{1}{y_6}, \frac{1}{y_5}, \frac{1}{y_4}, \frac{1}{y_3}, \frac{1}{y_2}, \frac{1}{y_1})$

v. $\tilde{X} \oslash \tilde{Y} = (\frac{x_1}{y_6}, \frac{x_2}{y_5}, \frac{x_3}{y_4}, \frac{x_4}{y_3}, \frac{x_5}{y_2}, \frac{x_6}{y_1})$

vi. Let $\alpha \epsilon R$ , then

$$\alpha\otimes\tilde{X} = \begin{cases}(\alpha x_1, \alpha x_2, \alpha x_3, \alpha x_4, \alpha x_5, \alpha x_6), & \alpha \geq 0\\(\alpha x_6, \alpha x_5, \alpha x_4, \alpha x_3, \alpha x_2, \alpha x_1), & \alpha < 0\end{cases}$$

## IV. MATHEMATICAL MODEL

The mathematical model is developed on the basis of the following assumptions and notations

*A. Assumptions*

Single inventory will be used.

The lead time is zero.

Shortages are allowed and are fully backlogged.

Replenishment rate is infinite but size is finite.

Stock dependent demand rate is considered.

Time horizon if finite.

There is no repair of deteriorated items occurring during the cycle.

*B. Notations*

$I(t)$ -The on-hand inventory at time $t = T_1$

$a + bt$ – The demand rate at time $t$where $I(t) > 0$, $a$ and $b$ are positive constants.

$\theta$ – The constant rate of deterioration, $0 < \theta < 1$

$Q$ – The amount of inventory produced at the beginning of each period.

$T$ – The duration of a cycle.

$H_c$ – The inventory holding cost per unit.

$D$ – The amount of deteriorated units.

$D_c$ – The deterioration cost per unit time.

$S_c$ – The shortage cost per unit item.

$S$ – The initial inventory after fulfilling the back order.

$Z[A,C]$ – The total average cost.

### C. Crisp Model

It is assumed that the inventory level initially at time $t = o$ is $Q$. Due to reason of market demand and deterioration of the items, the inventory level gradually diminishes during the period $[o, T_1)$ and ultimately falls to zero at $t = T_1$. The shortage occurs during the interval $[T_1, T]$ and which are fully backlogged. The model has been shown graphically in the fig.1.

Let $I(t)$ be the on-hand inventory level at any time $t$, which is governed by the following two differential equations;

$$(4.1)\quad \frac{dI(t)}{dt} + \theta I(t) = -(a + bI(t))\,; \quad 0 \le t \le T_1$$

$$(4.2)\quad \frac{dI(t)}{dt} = -(a + bI(t))\,; \qquad T_1 \le t \le T$$

Now solving (4.1) with boundary condition $I(0) = S$,

$$(4.3)\quad I(t) = -\frac{a}{\theta + b} + e^{-(\theta+b)t}\left\{S + \frac{a}{\theta + b}\right\},$$

$0 \le t \le T_1$

The solution of (4.2) with boundary condition $I(T_1) = 0$,

$$(4.4)\quad I(t) = -\frac{a}{b} + \frac{a}{b}e^{b(T_1 - t)}\ ;\ T_1 \le t \le T$$

From (4.3), using $I(T_1) = 0$, we obtain

$$(4.5)\quad S = \frac{a}{\theta + b}[e^{-(\theta+b)T_1} - 1]$$

The total average inventory in the time interval $[0, T_1]$ is given by

$$(4.6)\quad I_1(T_1) = \frac{1}{T}\int_0^{T_1} I(t)dt = \frac{aT_1^2}{T}$$

The total number of deteriorated units during the inventory cycle is given by

$$(4.7)\quad D = \int_0^{T_1} \theta(t)I(t)dt = \frac{\theta a T_1^2}{T}$$

Now the total average inventory in the interval $[T_1, T]$ is given by

$$(4.8)\quad I_2(T_1) = \int_{T_1}^{T} I(t)dt = -\frac{a}{bT}\ [\mathrm{T} - T_1 + \frac{1}{b}e^{b(T_1 - T)} - 1]$$

Now using the equation (4.6), (4.7)and (4.8), the total average cost per unit time of the model will be

$$(4.9)\quad Z[A,C](T_1) = H_c I_1(T_1) + S_c I_2(T_1) + D_c\ \mathrm{D}$$

### D. Fuzzy Model

In the above developed crisp model, it was assumed that all the parameters were fixed or could be predicted with certainty; but in real life situations, they will be fluctuate little from the actual values. Therefore the parameters of the model could not be assumed constant. Now we consider the model in fuzzy environment. Due to fuzziness, we use the following variables:

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$\tilde{\theta}$= fuzzy rate of deterioration.

$\tilde{D}_c$=fuzzy cost of each deterioration.

$\tilde{H}_c$=fuzzy carrying cost.

$\tilde{S}_c$=fuzzy shortage cost.

Suppose $\tilde{S}_c = (b_1, b_2, b_3, b_4, b_5, b_6)$,

$\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$,

$\tilde{H}_c = (h_1, h_2, h_3, h_4, h_5, h_6)$,

$\tilde{D}_c = (d_1, d_2, d_3, d_4, d_5, d_6)$are non-negative hexagonal fuzzy numbers (HFN).

The total average cost per unit time is given by

(4.10) $\tilde{Z}[A, C](T_1) = (\eta \otimes \tilde{H}_c) \oplus (\gamma \otimes \tilde{S}_c) \oplus (\eta \otimes (\tilde{D}_c \otimes \tilde{\theta}))$

Where $\eta = \frac{aT_1^2}{T}, \gamma = -\frac{a}{bT}\,[\mathrm{T} - T_1 + \frac{1}{b} e^{b(T_1 - T)} - 1]$

(4.11)$\tilde{Z}[A, C](T_1) = (\tilde{Z}[A, C, 1](T_1), \tilde{Z}[A, C, 2](T_1), \tilde{Z}[A, C, 3](T_1),$

$\tilde{Z}[A, C, 4](T_1), \tilde{Z}[A, C, 5](T_1), \tilde{Z}[A, C, 6](T_1))$

Where

$\tilde{Z}[A, C, 1](T_1) = (\eta h_1 + \gamma b_1 + \eta d_1 \theta_1)$

$\tilde{Z}[A, C, 2](T_1) = (\eta h_2 + \gamma b_2 + \eta d_2 \theta_2)$

$\tilde{Z}[A, C, 3](T_1) = (\eta h_3 + \gamma b_3 + \eta d_3 \theta_3)$

$\tilde{Z}[A, C, 4](T_1) = (\eta h_4 + \gamma b_4 + \eta d_4 \theta_4)$

$\tilde{Z}[A, C, 5](T_1) = (\eta h_5 + \gamma b_5 + \eta d_5 \theta_5)$

$\tilde{Z}[A, C, 6](T_1) = (\eta h_6 + \gamma b_6 + \eta d_6 \theta_6)$

Now we defuzzifying the fuzzy total average cost $\tilde{Z}[A, C](T_1)$ by Signed Distance Method, we have

(4.12) $$P\left(\tilde{Z}[A, C](T_1)\right) = \frac{\eta}{8}[h_1 + 2h_2 + h_3 + h_4 + 2h_5 + h_6 + d_1\theta_1 + 2d_2\theta_2 + d_3\theta_3 + d_4\theta_4 + 2d_5\theta_5 + d_6\theta_6] + \frac{\gamma}{8}[b_1 + 2b_2 + b_3 + b_4 + 2b_5 + b_6]$$

The necessary condition for minimizing the total average cost is

(4.13) $$\frac{\partial (P\left(\tilde{Z}[A, C](T_1)\right))}{\partial T_1} = \frac{2aT_1}{12}[h_1 + 2h_2 + h_3 + h_4 + 2h_5 + h_6 + d_1\theta_1 + 2d_2\theta_2 + d_3\theta_3 + d_4\theta_4 + 2d_5\theta_5 + d_6\theta_6] + \frac{a}{bT}(1 - e^{b(T_1 - T)})(b_1 + 2b_2 + b_3 + b_4 + 2b_5 + b_6)$$

Optimal amount of the initial inventory after fulfilling backorders $\tilde{S}$ denoted by $\tilde{S}^*$ is

$$(4.14) \quad \tilde{S}^* = aT_1 + \frac{abT_1^2}{2} + \frac{a(\theta_1 + 2\theta_2 + \theta_3 + \theta_4 + 2\theta_5 + \theta_6)T_1^2}{16}$$

Optimal amount of the unit deteriorated $\widetilde{D}$ denoted by $\widetilde{D}^*$ is

$$(4.15) \quad \widetilde{D}^* = \frac{aT_1^2}{T}\left(\frac{\theta_1 + 2\theta_2 + \theta_3 + \theta_4 + 2\theta_5 + \theta_6}{8}\right)$$

Thus minimum value of the total cost denoted by $\tilde{Z}[A,C](T_1)$ is

$$(4.16) \quad \tilde{Z}[A,C](T_1) = \frac{1}{8}[\eta h_1 + \gamma b_1 + \eta d_1\theta_1] + \frac{1}{4}[\eta h_2 + \gamma b_2 + \eta d_2\theta_2] + \frac{1}{8}[\eta h_3 + \gamma b_3 + \eta d_3\theta_3] + \frac{1}{8}[\eta h_4 + \gamma b_4 + \eta d_4\theta_4] + \frac{1}{4}[\eta h_5 + \gamma b_5 + \eta d_5\theta_5] + \frac{1}{8}[\eta h_6 + \gamma b_6 + \eta d_6\theta_6]$$

## V. NUMERICAL EXAMPLE

*A. Crisp Model*

Consider $D_c = Rs.300$ per order, $H_c = Rs.15$ per unit per year, $S_c = Rs.30$ per unit per year, $\theta = 0.01$ per year, $T_1 = 0.5$ year, $T = 1, a = 100, b = 2$.

The Solution of Crisp model is:

$Z[A,C](T_1) = 924.0618$, $S^* = 86.1504$ and

$$D^* = 0.2500$$

*B. Fuzzy Model*

Suppose $\widetilde{D}_c = (100,200,300,300,400,500)$,

$\tilde{S}_c = (10,20,30,30,40,50)$,

$\tilde{\theta} = (0.006,0.008,0.01,0.01,0.012,0.014)$,

$\widetilde{H}_c = (5,10,15,15,20,25)$ are all hexagonal fuzzy numbers and $T_1 = 0.5$year, $T = 1, a = 100, b = 2$. Then the fuzzy total average cost and the fuzzy optimal backorder quantity:

$\tilde{Z}[A,C](T_1) = 931.5904, \tilde{S}^* = 75.01563,$ $\quad \widetilde{D}^* = 0.2500$

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## VII. CONCLUSION

This paper present a fuzzy inventory model for stock dependent demand rate, fixed deteriorating items with shortages under fully backlogged condition. The proposed model is developed in both crisp and fuzzy environment. In fuzzy environment, all related inventory parameters were assumed to be hexagonal fuzzy numbers. The optimal results of fuzzy model is defuzzified by Signed Distance Method. By given numerical example, it has been tested that signed distance method gives minimum cost as compare to crisp model.

## REFERENCES

[1] D. Stephen Dinagar, and J. Rajesh Kannan, On fuzzy inventory model with allowable shortage, International Journal of Pure and Applied Mathematics, vol. 99, pp. 65-76, 2015.

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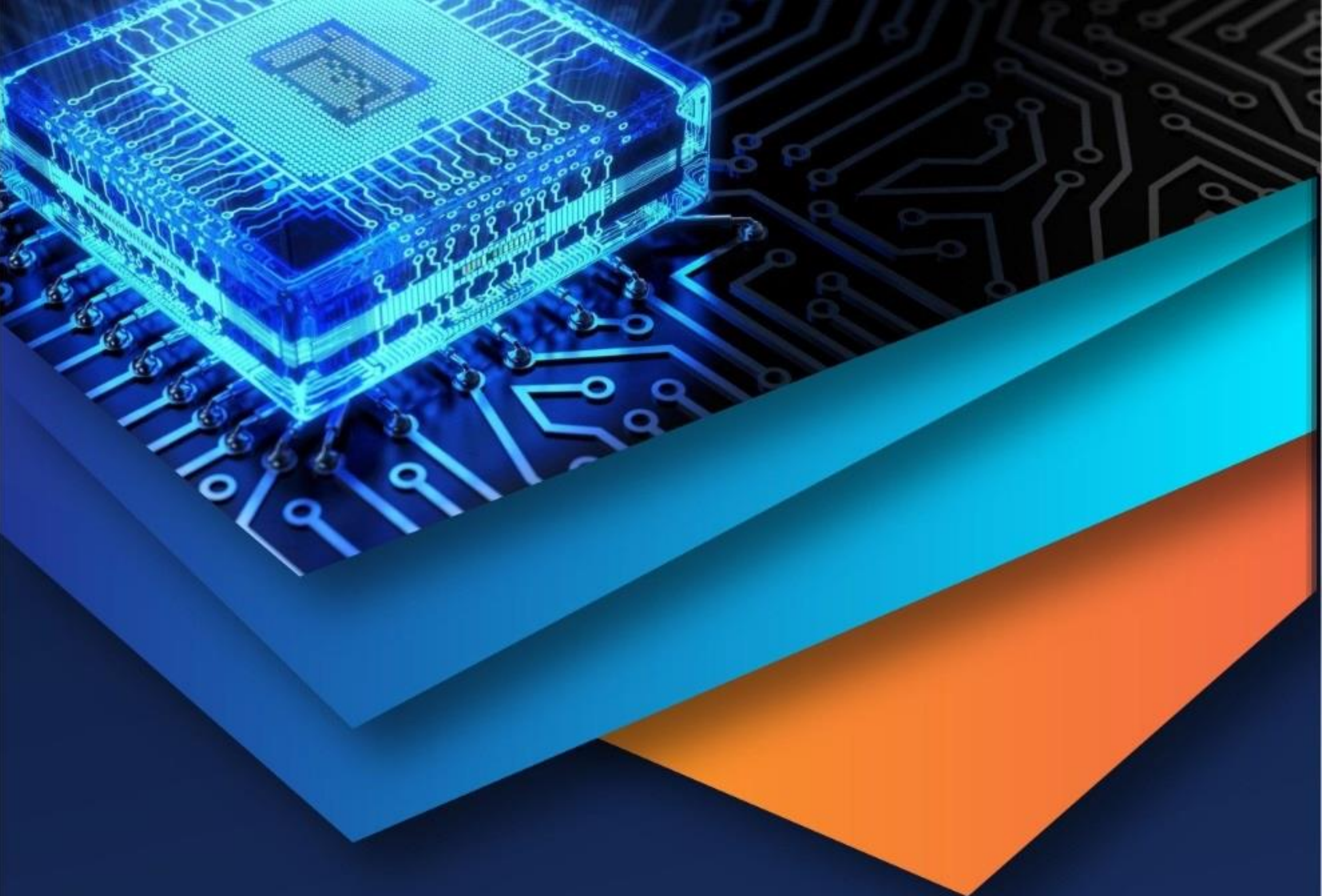
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

[2] C.K. Kao, and W.K. Hsu, A single-period inventory model with fuzzy demand, Computers and Mathematics with Applications, vol. 43, pp. 841-848, 2002.
[3] G. Urgeletti, Inventory control models and problems,European Journal of Operational Research, vol. 14, pp 1-12, 1983.
[4] J. Kacpryzk, and P. Staniewski, Long term inventory policy-making through fuzzy-decision making models, Fuzzy Sets and Systems.
[5] J.S. Yao, and H.M. lee, Economic order quantity in fuzzy sense for inventory without back-order model,Fuzzy Sets and Systems,vol. 105, pp. 13- 31, 1999.
[6] K.S. park, Fuzzy set theoretical interpretation of economic order quantity,IEEE Trans. Systems Man. Cybernet SMC,vol. 17, pp. 1082-1084, 1987.
[7] L.A. Zadeh, Fuzzy Sets,Information Control, vol. 8, pp. 338-353, 1965.
[8] S.K. Indrajitsingha, P.N. Samanta and U.K.Misra, An Economic Production Quantity Model with Time Dependent Demand under Fuzzy environment, Int. J. of Modern Sc. Eng. and Tech.,vol.2, Issue-12, pp.1-8.

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