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Static Analysis of Laminated Composite Plate Using First Order Shear Deformation Theory

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Abstract: Composites are used comprehensively in constructional company. Instead of conventional materials engineers are experimenting new materials every day in which composites are providing solution to many structural applications. MATLAB software is used in the investigation of static performance of laminated plates. Designing of composite laminate is very complex and requires lot of computational effort. It is very difficult to solve composite numerical problems manually. Developing MATLAB code for the analysis of laminated composite plate using finite element analysis. To determine the deflection and transverse shear stress of square laminated plate which is simply supported and subjected to uniform pressure. To explore the influence of modular ratio on deflection and transverse shear stress for different materials. Development of MATLAB code for the interpretation of first order shear deformation theory.
Keywords: Laminated plate, composites, MATLAB, shear deformation theories, Finite element analysis

I. INTRODUCTION

Composites are combination of two or more materials that are merged together to give distinct combination of properties. These materials have different physical, chemical properties and are insoluble. So obtained materials have high strength and high specific modulus. They are much lighter in weight as compared to conventional materials.

In history we have seen use of composite in form of mud walls, houses, boats, bamboo laminated boards, etc. Egyptian civilization in 1600 B.C. used straw and mud for house settlements. Fibrous plants, bone, wood etc., were used as binders in those times. Then there was origin of synthetic fibers like polyesters, vinyl etc. which were sturdier than those acquired from animals and plants. In 1900's we have seen much advancement in composites with introduction of carbon, glass fibers. These materials have higher modulus of elasticity and had very high strength. They were used in making aircrafts, jets, aeronautical equipment and gears., etc. Government of India had launched project Advance Composite Mission for developing motivation for the use of composite materials for building and construction.

The constituents in composites are of two types. One is reinforcing phase and other is matrix. The reinforcement is predominantly type of fibers which are poor in compression but good in tension. The fibers can be of type fiber- short or long (carbon, aramid, boron, glass, etc.) or yarn or fibrous composites (cotton, jute, sisal, hemp, coir's etc.) These fibers are entranced or held over in materials called matrix. Matrix is mainly continuous. Matrix can be polymers (epoxides, polyester, nylon) or ceramics (Sic, glass ceramics etc.) or metal (magnesium alloys, aluminum alloys, titanium etc.) Matrix is kind of binder which joints fibers.

A. Classification Of Various Plate Theories

To examine the static and dynamic performance of plates we have various plate theories present. But taking into considerations the geometrical properties and plate material, we must get familiar with different theories present as same theories cannot be applied to every condition.

1) Thin plate

a) Classical plate theory and Kirchhoff's plate theory can be used for thin plates.
b) It is valid when length to thickness ratio is greater than 20 (a/h=s>20).
c) It is two-dimensional theory.
d) This theory is based on plane stress assumption.
e) This theory can be applied for both isotropic and anisotropic (for small and large deformation).
f) The assumptions taken is that there is no elongation through thickness and there is no rotation of planes.
g) This theory is just accurate.

2) *Moderately Thick Plate:* Few shear deformation theories are accessible for analysis of moderately thick plates. When aspect ratio i.e., the width thickness ratio is greater than 6 and less than 20 (a/h=s,20<s>6) this theory is valid. First order shear deformation theory (FSDT) and higher order shear deformation theory (HSDT) are used for moderately thick plates. Both these theories can be applied to isotropic and anisotropic materials. FSDT requires correction factor and therefore gives slightly less accurate results. Early's 70 FSDT theories were used for analysis but now at present we are introduced with materials like composite laminates, functionally graded materials, piezometric or advance ceramic therefore much refined theories are required. HSDT doesn't require shear correction factors and gives much accurate results. These theories are two-dimensional theories. They can resist shear and transverse deformation. They are much accurate than the theories suitable for thin plates.
3) *Thick Plates:* Theories of elasticity are used for thick plates. These are three- dimensional theories and are more accurate than other theories. If the material is isotropic then FSTD or HSDT can anticipate marginally accurate behavior but if the material is anisotropic then ESL theories cannot be much accurate. As for the anisotropic number of layers have different properties so the displacement value will also be different and therefore number of variables will increase. This increases the computation efforts. Layer wise plate theory are much refined theories.
4) *Classical Laminated Plate Theory:* CLPT is the simplest of all the theories present for analysis of thin plates. It is an expansion of Kirchhoff 's plate theory. CLPT is appropriate for analysis of thin plate. Plane stress assumptions are taken in classical laminate plate theory and deformation due to shear is neglected. The results obtained from CLPT are adequate and just accurate. Since this theory does not deal with shear deformation, the value of buckling load is more than actual and similarly deflection is lesser than actual. For different modes this theory cannot predict accurate behavior of sandwich plates.

The displacement field of CLPT is given in form of:

$$u(x,y,z) = u_o(x,y) - z\frac{\partial w_o}{\partial x} \quad (1)$$

$$v(x,y,z) = v_o(x,y) - z\frac{\partial w_o}{\partial y} \quad (2)$$

$$w(x,y,z) = w_o(w,y) \quad (3)$$

u_o = displacement component along x direction
v_o = displacement component along y direction
w_o = displacement component along z direction

5) *First Order Shear Deformation Theory:* FSDT is relevant to thin and moderately thick plates. Unlike classical lamination plate theory, it considers shear deformation. It is an extension of Mindlin –Reissner theory. FSDT cannot predict the interlaminar stresses therefore a shear correction factor is used. It is much simpler to use and could be solve in various commercial software. It gives more accurate result than classical lamination plate theory.

The displacement field is given as:

$$u(x,y,z) = u_o(x,y) + z\theta_x(x,y) \quad (4)$$

$$v(x,y,z) = v_o(x,y) + z\theta_y(x,y) \quad (5)$$

$$w(x,y,z) = w_o(x,y) \quad (6)$$

6) *Higher Shear Deformation Theory:* It is suitable for applying to thick plates. The impact of deformation due to normal transverse and transverse shear is taken into consideration. Shear correction requirement is not there in this theory. It gives much accurate results. They represent kinematics in a decent way, but computational effort is more therefore it is used whenever required.

The displacement field is given as:

$$u(w,y,z) = u_o - z\frac{\partial w_o(x,y)}{\partial x} + \varphi(z)\varphi_x(x,y) \quad (7)$$

$$v(x,y,z) = v_o - z\frac{\partial w_o(x,y)}{\partial y} + \varphi(z)\varphi(x,y) \quad (8)$$

$$w(x,y,z) = w_o(x,y) \quad (9)$$

7) *Refined Plate Theory:* Zig zag plate theory and higher order zig zag theory were much accurate than other shear deformation theories. These theories can examine interlaminar stresses. Since number of layers increments, number of unknowns likewise increments. This increases the computational cost. If the material is isotropic then theories like CLPT, FSDT, HSDT can predict the behavior of plate almost accurately. But if the materials are anisotropic then single equivalent theory could not determine finer results, for such cases refined plate theories proves to be valuable. In this a MATLAB program is developed for the finite element analysis of laminated plates using First Order Shear Deformation Theory.**[1]**

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II. LITERATURE REVIEW AND INFERENCES

Srinivas et al. (1970) developed exact analysis of three-dimensional theory of elasticity for bending, static and free vibration of rectangular laminated composite plates. Taking different modular ratios, deflection and stresses for thin plate for simply supported boundary condition has been found and error had been presented. For static and dynamic conditions results are been validated with Reissner's theory and thin plate theory. Similarly, for buckling analysis , the results are being compared with Mindlin's theory.**[2]**S. Srinivas (1973) had previously postulated exact analysis and studied the behavior of laminated plate under bending and buckling. In this paper he simplified the previous theory of exact analysis and reduced the three dimensional to two-dimensional theory. He carried out the analysis with same boundary conditions, modular ratios and thickness and found that it had good agreement with solutions obtained from exact analysis of three-dimensional theory of elasticity. This refined theory could be applicable for anisotropic plates and could be used for common support conditions.**[3]** J. N. Reddy et al. (1985) presented analysis of laminated plates using higher shear deformation theory of simply supported plates. The results were matched with FSDT and CLPT. There was no requisite to use shear correction factors and the results attained were nearer to those attained from three dimensional exact solutions. Natural frequencies and buckling loads were calculated for few numerical examples using HSDT. They examined the stability of isotropic and orthotropic plates**.[4]** B. N. Pandya et al. (1987) composed a displacement model expressed from higher order shear deformation theory and analyzed thick laminated plate for bending. This theory unlike FSDT did not use shear correction factor. Few examples with different material and conditions were taken. Deflection of the plate and interlaminar stresses were calculated by developing a computer program. The results obtained were compared with three- dimensional model closed form solutions. The response was precise with former theory.**[5]** J. N. Reddy et al. (1991) evaluated progress of different types of finite element model in field of finite element analysis of laminated plates. In this paper two such finite where models were taken and was observed that they provide with better accuracy. It was much refined theory which could possibly be used for coding in commercial finite element software.**[6]** P. C. Dumir et al. (2001) presented a paper on evaluation of transverse shear deformation of laminated plate using orthogonal point collocation method and assessed with first order shear deformation theory and classical plate theory of laminates. The response of the plate for both simply supported and clamped conditions were considered by changing the modular ratios. The consequences of using CLPT and FSDT was investigated.**[7]** A. J. M. Ferreira et al. (2005) had evaluated free vibration of symmetrical laminated plates. They used displacement model of first order shear deformation theory and for transverse stress, eigen value he used multiquadric radial basis function. He showed that convergence of transverse deflection, natural frequency was very reasonable. This method could be used as substitute of other finite element models.**[8]** Jun-Sik Kim et al. (2006) formulated an augmented version of first order shear deformation theory. This theory was used to analyze the static performance of laminated and sandwich plates. Transverse deflection and stresses were calculated and validated with his previous research papers. The enhanced FSDT was also compared to three dimensional exact solutions. The proficiency of this theory was confirmed by showing of few examples and compared with higher shear deformation theories. It was seen that it gave outstanding results. **[9]** Oh et al. (2008) presented an enhanced FSDT theory which used the displacement field of (HOZT) higher zig zag theory thereby developed enhanced first order shear deformation theory. So developed theory can be used to any commercial software without cogent modifications as such. He used ANSYS to compare numerical results and measure up with FSDT and HOZT. Several stacking sequences were studied for plates and simple sandwich plates. The finite model gave close and proximate results for HOZT and EFSDT but were in less agreement with FSDT. **[10]** Y. X. Zhang et al. (2008) studied various research papers based on analysis of laminated plates using finite element model. The paper was ardent to lately developed finite model for analysis of laminated plates. Effect of bending, buckling, static and free vibration was reviewed in this paper. The development in shear deformation theories with the advance in finite element model and how these models helping in reducing computational efforts.**[11]** Bhar et al. (2010) conducted a comparative study of laminated plates using first order shear deformation theory and higher order shear deformation theory. Different shear deformation model was taken with different boundary conditions. Plates stiffener dimensions were changed, and it hence was seen that by changing rather increasing the thickness of plates FSTD could not hold the same result for transverse stresses. It was indicated that for better and accurate result, whenever there is variation of thickness , how HSDT is significant than FSDT.**[12]** SS Aliedin et al. (2011) proposed three alteration method for the analysis of laminated plates and functionally graded plate. They used first order shear deformation theory model and reformed the theory to develop mode for elastostatic analysis. Comparisons were done with multilayer composite plates and their counterpart functionally graded plate. Many methodologies were given to convert composite plate into comparable functionally graded plate.**[13]** Lx Peng et al. (2011) had analyzed flexure behavior of laminated composite plate using FSDT. In this paper no mesh was required to obtain stiffness equation of laminated plates. The stiffness equation was modified, and the proposed theory resulted in much adjustable value than finite element model.

He presented his theory by comparing with few numerical examples and found results harmonious with examples.**[14]** M. Shahbazi et al. (2012) sourced two problems of static analysis of laminated plates solved using mesh free method. They found few inconsistencies in the response of bending when width to height ratio was changed. They presented a cohesive method of boundary conditions which helped in getting over such divergence in results. They also offered few examples of angle ply and cross ply laminate analysis by the proposed theory.**[15]** Huu Tai- Thai et al. (2013) composed first order shear deformation theory with only four unknowns. He analyzed laminated plates using modified theory for free vibration and bending analyses. They analytically solve the governing equation of Hamilton and modified to a simpler version of FSDT which had only four unknowns. They verified the results with few numerical models, and it was seen that less computational effort was needed for solving laminates.**[16]** CMC Roque (2013) developed MATLAB coding using symbolic math toolbox for analysis of flexure of laminated plates. He used radial basis functions using third order shear deformation theory. A MATLAB program was developed. He demonstrated how such coding can be helpful for other shear deformation theory by simply writing the governing equations and equilibrium equations in terms of engineering constants.**[17]** J. L. Mantari et al. (2015) modified the conventional first order shear deformation theory by adding few indeterminate terms and reducing the number of unknowns in the equilibrium conditions. This reduces the number of unknown equations from five to four which makes this theory less demanding for calculations. This theory was verified by Navier's solution. The studies showed that results were in good agreement with each other.**[18]** Tiantang Yu et al. (2015) presented iso geometrical analysis of buckling and vibration of laminate plates. Several cut outs were made by level set method and then analysis was done to find in plane stress using FSDT displacement model. The solution from IGA analysis for various lay outs and different fiber orientation.[1**9]** J. Eisentrager et al. (2015) presented the paper on application of first order shear deformation theory on laminated plate and photovoltaic panel. This paper examined how FSDT could be applied to symmetric laminate plate with unsymmetrical photovoltaic panels having anisotropic properties. Transverse shear stresses and deflection were calculated with finite element model and were matched with closed form solution.[**20]** J Belinha et al. (2016) used different meshless method combined with first order shear deformation theory for analysis of composite laminated plates. The results were compared with exact solutions. Coding for analysis of laminates was developed in MATLAB. These methods were compared with published literature and the values were adjacent to those obtained from program.**[21]** Marjanovic et al. (2016) formulated multilayer finite plate element rectangular in shape from the postulate of Reddy's higher shear deformation theory. The HSDT model was reduced to FSDT model. The model was carried out in MATLAB code and was further verified from Abaqus commercial software. The HSDT model and FSDT model were corelated and was seen that HSDT exhibit finer results.[**22]** Osama Mohammed El mardi (2016) presented that when dynamic relaxation method was used with finite difference method, the response of bending in rectangular plates were compatible with published analytical and numerical methods. He used Fortran language for analysis of moderately thick plates finding the deflection. He changed the layup configuration, boundary condition, mess sizes, and observed the convergence of the results. He concluded that factor like load application and layup configuration affects the deflection of the plates. The dynamic relaxation program was based on first order shear deformation theory.[**23]** Hossein Zamanifar et al. (2018) developed a MATLAB program for analysis of static, free, and forced vibration of corrugated core sandwich plate using FSDT. He validated his results from printed literature trying several boundary conditions, mechanical and geometrical properties. **[24]** Dongyan Shi et al. (2018) used simple first shear order shear deformation theory (SFSDT) to analyze free vibration of plates of different shapes like that of trapezoidal, rectangular or skew plates. This shortened theory was much easier than first order shear deformation theory. Mainly simply supported laminated plates were studies for laminate behavior**.[25]**

III. PLATE THEORIES

For many engineering uses and functions plates are widely used around the world. They are valuable in making decks, bridges, giving footings in foundation, etc. Thickness of the plates plays an important role as behavior of plate is largely affected by lateral loads which bring about bending in plates. Therefore, flexure analysis is important when we study laminates behavior.

A. Classical Laminated Plate Theory

CLPT therory is adjunct to classical plate theory. It is derived and based on Kirchoffs theory of plate which is further extension of Euler Bernoulli beam theory. It is suitable for applying to thin plates. It neither contemplate normal deformation nor transverse shear.

The assumption taken in hypothesis of Kirchoffs states that

1) After deformation, straight line normal to mid surface remains straight.
2) The transverse normal does not undergo any enlongation.
3) After deformation, thickness of the plate remains the same ie the tranverse shear strain is zero.

4) Plane stress assumptions are used.

Classical laminated plate theory gives satisfactory and moderate results for thin laminates. This theory overlooks shear deformation effects. This theory undervalue deflection and overvalue buckling load.

Assumptions made in classical lamination plate theory is equivalent to those made by Naiver and Bernoulli theory for thin plates. Therefore, solution given by Navier for simply supported rectangular plate is described below. Defection of the plate was given by means of double infinite series in expression of trigonometric series.

$$w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}\ f_m(x) g_n(y) \tag{10}$$

After applying boundary conditions for simply supported edges, the deflection could be written as

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}\ \sin\alpha_m x \sin\beta_n y \tag{11}$$

Where $\alpha_m = \frac{m\pi}{a}$, $\beta_n = \frac{n\pi}{b}$, and A_{mn} is constant

For defining the load, we have used double Fourier series

$$q(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn}\ \sin\alpha_m x \sin\beta_n y \tag{12}$$

Coefficient of the series is given as:

$$q_{mn} = \frac{4}{ab} \int_0^a \int_0^b q(x,y) \sin\alpha_m x \sin\beta_n y\, dx\, dy \tag{13}$$

Thus, the deflection attained is

$$w = \frac{a}{D\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{q_{mn}}{(m^2 + \frac{n^2 a^2}{b^2})} \sin\alpha_m x \sin\beta_n y \tag{14}$$

For uniformly distributed load, deflection can be written as

$$q_{mn} = w = \frac{16qa^4}{D\pi^6} \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \frac{\sin\alpha_m x \sin\beta_n y}{mn\left(m^2 + \frac{n^2a^2}{b^2}\right)^2} \tag{15}$$

$$w_{max} = \frac{16qa^4}{\pi^6 D} \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \frac{\sin\frac{m\pi}{2}\sin\frac{n\pi}{2}}{mn\ (m^2 + n^2)^2} \tag{16}$$

When only first and second terms are taken, the summation of the series is obtained as

$$w_{max} = 0.00416\frac{qa^4}{D} \tag{17}$$

After the convergence of the series maximum deflection resulted as

$$w_{max} = 0.00406\frac{qa^4}{D} \tag{18}$$

B. First Order Shear Deformation Theory

FSDT is based on Mindlin – Reissner theory of plates which is an adjunct of Kirchhoff's theory of plate. It is suitable for applying to moderately thick plates. On top and bottom surface of the plates this theory contravenes the equilibrium equations, therefore a shear correction factor is used. The variation between actual stress and assumed constant stress is balanced by this shear correction factor. Apparently FSDT seemed uncomplicated and effective for analyzing structural problems.

Indeed, FSDT can be used for moderate approach to response of laminates whereas commercial codes based on finite element analysis necessitate requirement of input parameters of shear correction factors. FSDT can evaluate value of in-plane and out of plane stresses and once it is predicted the zone of maximum stresses, zig zag theories can be used for further failure and delamination. Boundary and loading conditions and geometric properties affect shear correction factor.

The displacement field is given as

$$u(x,y,z) = z\Psi_x(x,y) \quad (19)$$

$$v(x,y,z) = z\Psi_y(x,y) \quad (20)$$

$$\omega(x,y,z) = \omega_0(x,y) \quad (21)$$

Where ψ_x = rotation of element about x axis

ψ_y= rotation of element about y axis

ω_o= displacement in z direction

Strains are given as:

$$\epsilon_x = z\frac{\partial\Psi_x}{\partial x} \quad (22)$$

$$\epsilon = z\frac{\partial\Psi_y}{\partial y}, \epsilon = 0 \quad (23)$$

$$\gamma_{xy} = \left(\frac{\partial\Psi_x}{\partial y} + \frac{\partial\Psi_y}{\partial x}\right)z \quad (24)$$

$$\gamma_{yz} = \Psi_y + \frac{\partial\omega_0}{\partial y} \quad (25)$$

$$\gamma_{zx} = \Psi_x + \frac{\partial\omega_0}{\partial x} \quad (26)$$

In classical plate theory the deformation due to bending, i.e., transverse shear stress is zero but in FSDT transverse shear stress is not zero.

$$\Psi_x = -\frac{\partial\omega_0}{\partial x}, \quad \Psi_y = -\frac{\partial\omega_0}{\partial y} \quad (27)$$

Using the expression of displacement, the governing differential equation of first order shear deformation theory is given by

$$\frac{\partial^2\psi_x}{\partial x^2} + \left(\frac{1-v}{2}\right)\frac{\partial^2\Psi_x}{\partial y^2} + \left(\frac{1+v}{2}\right)\frac{\partial^2\Psi_y}{\partial x\partial y} - \frac{6(1-v)k^2}{h^2}\left(\Psi_x + \frac{\partial\omega_0}{\partial x}\right) = 0 \quad (28)$$

$$\frac{\partial^2\Psi_y}{\partial y^2} + \left(\frac{1-v}{2}\right)\frac{\partial^2\Psi_y}{\partial x^2} + \left(\frac{1+v}{2}\right)\frac{\partial^2\Psi_x}{\partial x\partial y} - \frac{6(1-v)k^2}{h^2}\left(\Psi_x + \frac{\partial\omega_0}{\partial y}\right) = 0 \quad (29)$$

$$Ghk^2\left[\nabla^2\omega_0 + \left(\frac{\partial\Psi_x}{\partial x} + \frac{\partial\Psi_y}{\partial y}\right)\right] + q = 0 \quad (30)$$

$$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \quad (31)$$

For obtaining transverse deflection of a plate, which is simply supported, the load is articulated as

$$q(x,y) = \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty} q_{mn} \sin\alpha_m x \sin\beta_n y \quad (32)$$

After applying boundary conditions

$$\omega_0 = \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty} A_{mn} \sin\alpha_m x \sin\beta_n y \quad (33)$$

$$\Psi_x = \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty} B_{mn} \cos\alpha_m x \sin\beta_n y \quad (34)$$

$$\Psi_y = \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty} C_{mn} \sin\alpha_m x \cos\beta_n y \quad (35)$$

The deflection is given as

$$\omega_o = \sum_{m=1,2}^{\infty} \sum_{n=1,2}^{\infty} \frac{\left\{1 + \frac{D}{k^2 Gh}(\alpha_m^2 + \beta_n^2)\right\}}{D(\alpha_m^2 + \beta_n^2)} q_{mn} \sin \alpha_m x \sin \beta_n y \tag{36}$$

IV. FINITE ELEMENT ANALYSIS OF LAMINATED PLATES

Displacement field is given by:

$$u(x, y, z) = u_o(x, y) + z\theta_x(x, y) \tag{37}$$

$$v(x, y, z) = v_o(x, y) + z\theta_y(x, y) \tag{38}$$

$$w(x, y, z) = w_o(x, y) \tag{39}$$

Strains are derived from displacement

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{Bmatrix} \tag{40}$$

Constituents of deformation are termed as

$$\begin{Bmatrix} \epsilon_{xx}^m \\ \epsilon_{yy}^m \\ \epsilon_{xy}^m \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_o}{\partial x} \\ \frac{\partial v_o}{\partial y} \\ \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \end{Bmatrix}; \quad \begin{Bmatrix} \epsilon_{xx}^f \\ \epsilon_{yy}^f \\ \gamma_{xy}^f \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \end{Bmatrix} \tag{41}$$

$$\begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w_o}{\partial x} + \theta_x \\ \frac{\partial w_o}{\partial y} + \theta_y \end{Bmatrix} \tag{42}$$

Matrices of strain displacement are

A. Membrane Segment

$$B_{m=}^s \begin{bmatrix} \frac{\partial N}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} & 0 & 0 & 0 \end{bmatrix} \tag{43}$$

B. Bending Segment

$$B^s_{m=}\begin{bmatrix} 0 & 0 & 0 & \frac{\partial N}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N}{\partial y} \\ 0 & 0 & 0 & \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} \end{bmatrix} \quad (44)$$

C. Shear Segment

$$B^s_{m=}\begin{bmatrix} 0 & 0 & \frac{\partial N}{\partial x} & N & 0 \\ 0 & 0 & \frac{\partial N}{\partial y} & 0 & N \end{bmatrix} \quad (45)$$

Stresses- strain relations:

$$\sigma = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{E}{1-\upsilon^2}\upsilon & \frac{E}{1-\upsilon^2} & 0 \\ \upsilon\frac{E}{1-\upsilon^2} & \frac{E}{1-\upsilon^2} & 0 \\ 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = D\epsilon \quad (46)$$

$$\Gamma = \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} K_1 G & 0 \\ 0 & K_2 G \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = D_c\gamma \quad (47)$$

The stiffness matrix is a summation of five components:

$$K^s = K^s_{nn} + K^s_{nf} + K^s_{fn} + K^s_{ff} + K^s_{cc} \quad (48)$$

where,

$$K^s_{nn} = \sum_{k=1}^{nc}\int B_n^T D_k B_n (z_{k+1} - z_k)dA \quad (49)$$

$$K^s_{nf} = \sum_{k=1}^{nc}\int B_n^T D_k B_f \frac{1}{2}(z^2_{k+1} - z^2_k)dA \quad (50)$$

$$K^s_{fn} = \sum_{k=1}^{nc}\int B_f^T D_k B_n \frac{1}{2}(z^2_{k+1} - z^2_k)dA \quad (51)$$

$$K^s_{ff} = \sum_{k=1}^{nc}\int B_f^T D_k B_f \frac{1}{3}(z^3_{k+1} - z^3_k)dA \quad (52)$$

$$K^s_{cc} = \sum_{k=1}^{nc}\int B_c^T D_k B_c (z_{k+1} - z_k)dA \quad (53)$$

K^e_{nn} = segment stiffness
K^e_{mf} = segment bending coupling stiffness
K^e_{fm} = segment bending coupling stiffness
K^e_{ff} = segment stiffness
K^e_{cc} = segment stiffness

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V. RESULTS AND DISSCUSSIONS

In this research we have taken material properties of four different composite plates.

The shape of the plate is square and is simply supported. The thickness of the plate is 0.01m. To reduce the stiffness matrix of orthotropic material from (6×6) to (5×5) we have eradicated ε_z by assuming $\sigma_z = 0$. This is plane stress assumption.

Analysis of laminated plate having three layers and subjected to uniform pressure, simply supported on all edges is examined. For several values of R, the convergence study of transverse deflection and resultant stress is studied for all four materials. These four materials are taken from references. These materials are being repetitively used by researches to prove, analyze their theory. Hence these materials were chosen as core materials. Therefore, the convergence of transverse shear stress with mesh refinement is calculated. For finding the stiffness of core materials, MATLAB code was used.

A. *MATLAB Validation of The Model*

For the validation of MATLAB code, the program is used to determine the transverse deflection and transverse shear stress from a published literature **[26]** in which First Order Shear Deformation Theory is used for analysis of laminated plate.

Effect of varying modular ratio on transverse shear stress and deflection for a simply supported square laminated plate under uniform pressure

R=5						
Source	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
A.J.M Ferreira[27]	252.0836	58.8628	45.4232	9.8846	3.8311	2.5319
FOST[26]	236.10	61.87	49.50	9.899	3.313	2.444
Present	236.9041	57.3371	45.8697	9.1739	2.6736	13.679

Table 1: Validation table for R=5

B. *Convergence Study*

Effect of varying modular ratio on transverse shear stress for a simply supported square laminated plate under uniform pressure(t=0.01m) for Material 1.

1) *Material 1 [28]*

$E_1 = 40MPa, E_2 = 1MPa, E_3 = 1MPa, G_{12} = 0.6MPa, G_{23} = 0.5MPa, G_{13} = 0.6MPa, \nu_{12} = 0.25, \nu_{23} = 0.25, \nu_{13} = 0.25$

Stiffness matrix:

$$\overline{Q} = \begin{bmatrix} 40.1674 & 0.3347 & 0.3347 & 0 & 0 \\ 0.3347 & 1.0695 & 0.2695 & 0 & 0 \\ 0.3347 & 0.2695 & 1.0695 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.6 \end{bmatrix}$$

R=5						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6×6	1436.6	116.9215	93.5372	18.7074	1.9843	9.9215
12×12	1404.3	120.8270	96.6616	19.3323	2.1686	10.8430
24×24	1397.5	121.7739	97.419	19.4838	2.2623	11.3115
30×30	1396.7	121.8870	97.5096	19.5019	2.2813	11.4063

Table 2: Convergence study of square laminated plate under uniform pressure, R=5 (Material 1)

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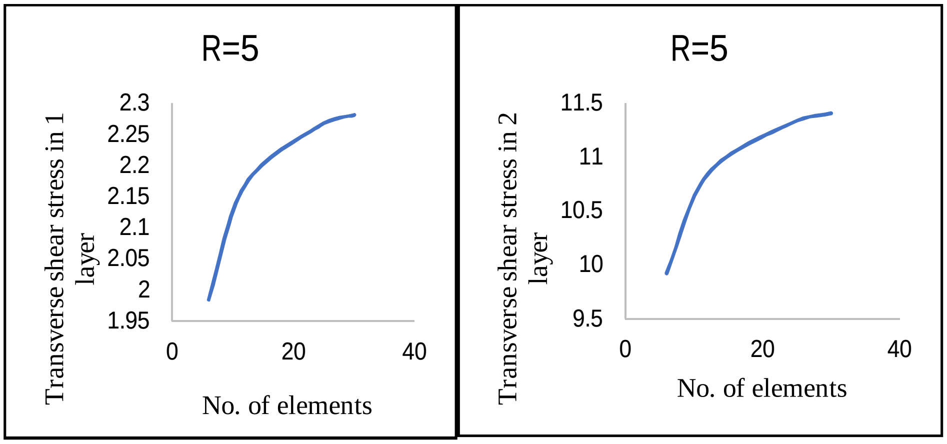


Figure 1: Variation of transverse shear stress when R=5 (Material 1)

R=10						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	1136.9	120.2890	96.2312	9.6231	1.2337	12.3371
12 × 12	1107.7	124.8957	99.9166	9.9917	1.3568	13.5676
24 × 24	1104.4	126.0105	100.8084	10.0808	1.4180	14.1798
30 × 30	1100.6	126.1435	100.9148	10.0915	1.4303	14.3028

Table 3: Convergence study of square laminated plate under uniform pressure, R=10 (Material 1)

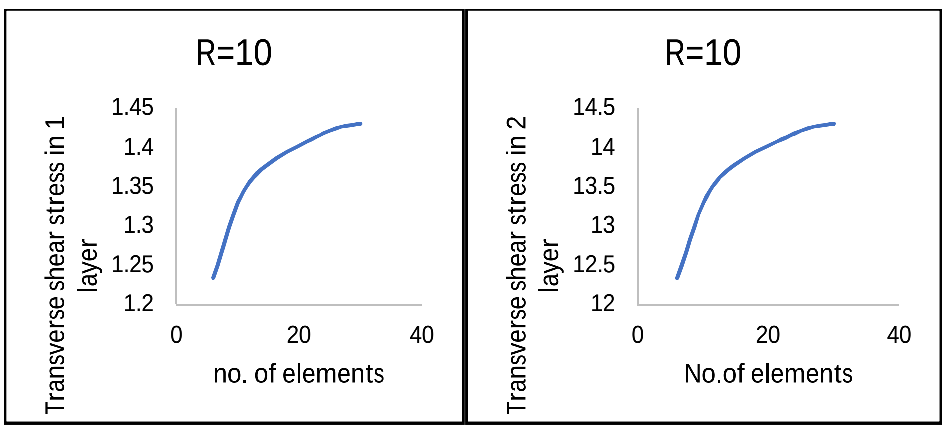


Figure 2: Variation of transverse shear stress when R=10 (Material 1)

R=15						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	1010.9	117.5031	94.0025	6.2668	0.8823	13.2344
12 × 12	983.7958	122.3375	97.8700	6.5247	0.9745	14.6172
24 × 24	977.8508	123.5137	98.8110	6.5874	1.0200	15.2994
30 × 30	977.1567	123.6541	98.9233	6.5949	1.0291	15.4360

Table 4: Convergence study of square laminated plate under uniform pressure, R=15 (Material 1)

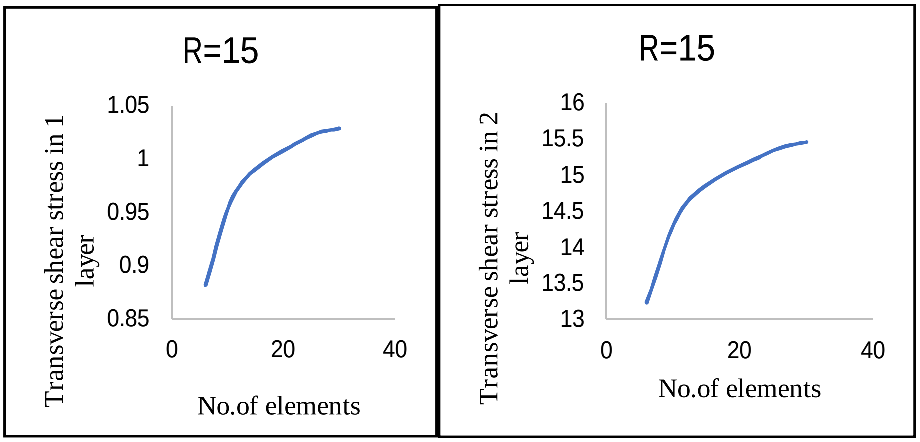
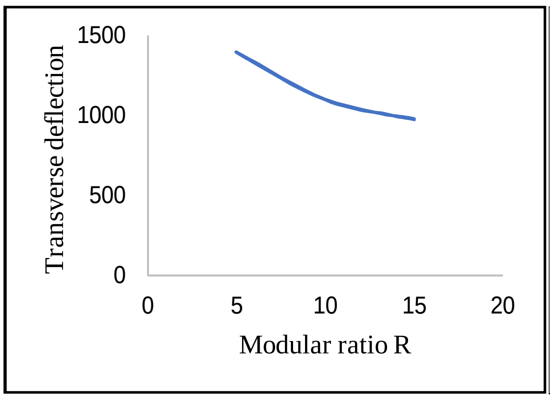


Figure 3: Variation of transverse shear stress when R=15 (Material 1)

Effect of varying modular ratio on transverse deflection for a simply supported square laminated plate under uniform pressure(t=0.01m) for Material 1



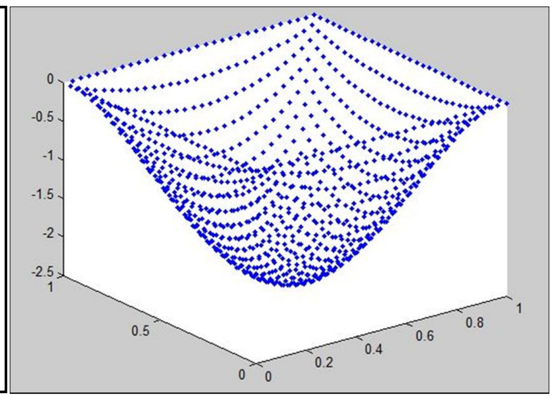


Figure 4: Variation of transverse deflection (R=5,10,15) (Material 1) Figure 5: Deformed shape of square laminated plate (Material 1)

Effect of varying modular ratio on transverse shear stress for a simply supported square laminated plate under uniform pressure (t=0.01m) for Material 2.

2) Material 2 [29]

$E_1 = 50MPa, E_2 = 50MPa, E_3 = 50MPa, G_{12} = 21.7MPa, G_{23} = 21.7MPa, G_{13} = 21.7MPa, \nu_{12} = 0.15, \nu_{23} = 0.15, \nu_{13} = 0.15$

Stiffness matrix

$$\overline{Q} = \begin{bmatrix} 57.7950 & 9.3168 & 9.3168 & 0 & 0 \\ 9.3168 & 57.7950 & 9.3168 & 0 & 0 \\ 9.3168 & 9.3168 & 57.7950 & 0 & 0 \\ 0 & 0 & 0 & 21.700 & 0 \\ 0 & 0 & 0 & 0 & 21.700 \end{bmatrix}$$

R=5						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	124.3595	25.9943	20.7954	4.1591	1.4298	7.1488
12 × 12	124.1049	26.2885	21.0308	4.2062	1.6110	8.0552
24 × 24	124.0735	26.3803	21.1043	4.2209	1.7132	8.5662
30 × 30	124.0704	26.3917	21.1134	4.2227	1.7347	8.6736

Table 5: Convergence study of square laminated plate under uniform pressure, R=5 (Material 2)

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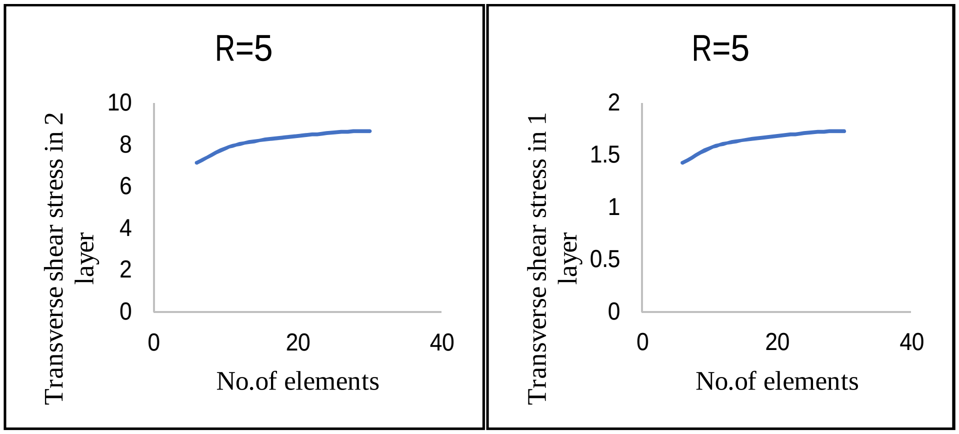

Figure 6: Variation of shear stress when R=5 (Material 2)

R=10						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	77.1791	28.4511	22.7609	2.2761	0.9235	9.2347
12 × 12	76.6921	28.7340	22.9872	2.2987	1.0416	10.4157
24 × 24	76.5990	28.8236	23.0589	2.3059	1.1075	11.0754
30 × 30	76.5884	28.8348	23.0679	2.3068	1.1214	11.2139

Table 6: Convergence study of square laminated plate under uniform pressure R=10 (Material 2)

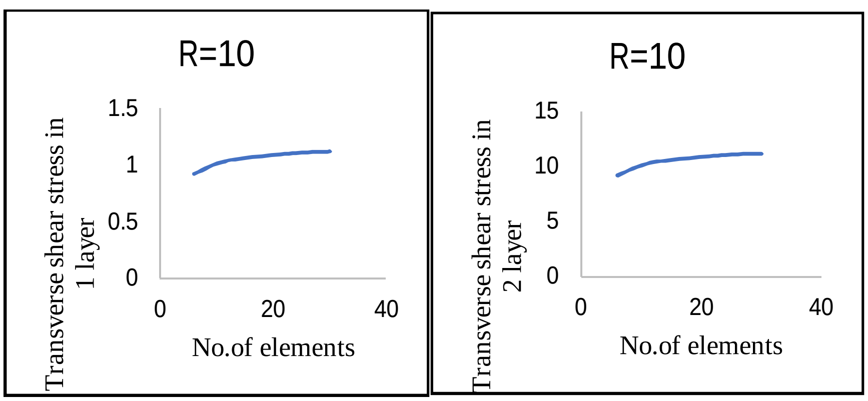

Figure 7: Variation of shear stress when R=10 (Material 2)

R=15						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	59.4118	29.3742	23.4994	1.5666	0.6823	10.2345
12 × 12	58.8384	29.6393	23.7114	1.5808	0.7704	11.5558
24 × 24	58.7221	29.7231	23.7785	1.5852	0.8192	12.2881
30 × 30	58.7087	29.7336	23.7869	1.5858	0.8294	12.4415

Table 7: Convergence study of square laminated plate under uniform pressure R=15 (Material 3)

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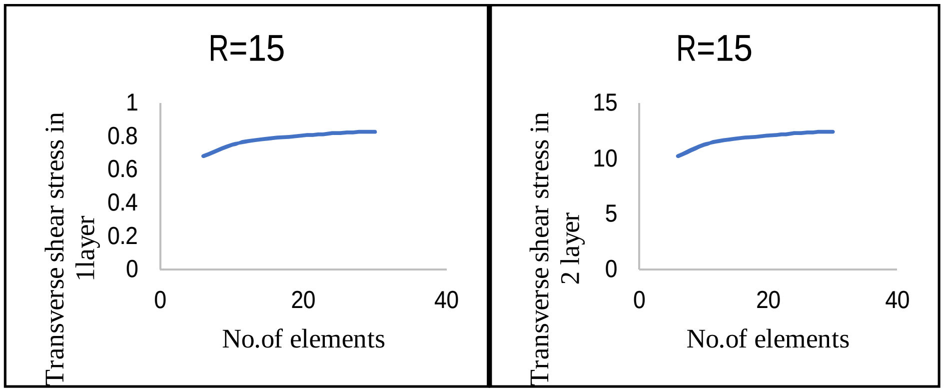
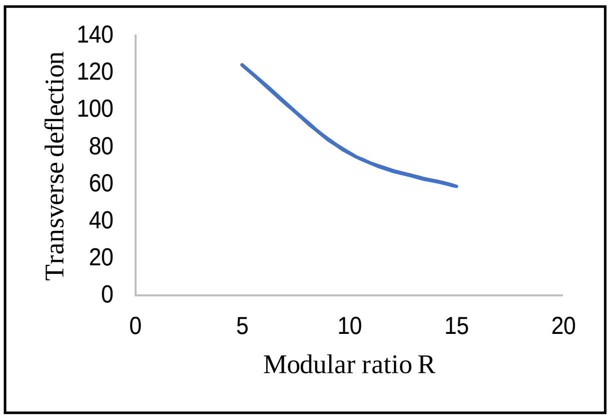


Figure 8: Variation of shear stress when R=15 (Material 2)

Effect of varying modular ratio on transverse deflection for a simply supported square laminated plate under uniform pressure (t=0.01m) for Material 2.



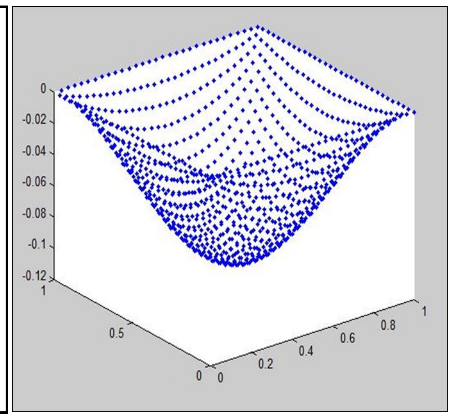


Figure: 9 Variation of transverse deflection (R=5,10,15) (Material 2) Figure: 10 Deformed shape of square laminated plate (Material 2)

Effect of varying modular ratio on transverse shear stress for a simply supported square laminated plate under uniform pressure (t=0.01m) for Material 3.

3) Material 3 [30]

$E_1 = 25MPa, E_2 = 1MPa, E_3 = 1MPa, G_{12} = 0.5MPa, G_{23} = 0.2MPa, G_{13} = 0.5MPa, \nu_{12} = 0.25, \nu_{23} = 0.01, \nu_{13} = 0.25$

Stiffness matrix:

$$\overline{Q} = \begin{bmatrix} 25.1269 & 0.2538 & 0.2538 & 0 & 0 \\ 0.2538 & 1.0027 & 0.0126 & 0 & 0 \\ 0.2538 & 0.0126 & 1.0027 & 0 & 0 \\ 0 & 0 & 0 & 0.200 & 0 \\ 0 & 0 & 0 & 0 & 0.500 \end{bmatrix}$$

R=5						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	1158.2	112.1469	89.7175	17.9435	0.9267	4.6337
12 × 12	1139.7	115.8582	92.6866	18.5373	1.0158	5.0789
24 × 24	1135.9	116.7835	93.4268	18.6854	1.0608	5.3041
30 × 30	1135.4	116.8944	93.5155	18.7031	1.0699	5.3497

Table 8: Convergence study of square laminated plate under uniform pressure R=5 (Material 3)

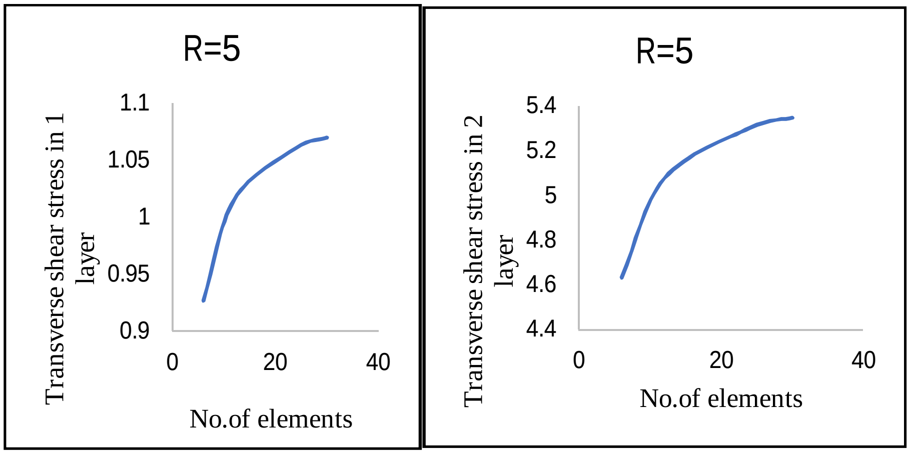


Figure: 11 Variation of shear stress when R=5 (Material 3)

R=10						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	895.5581	117.7822	94.2258	9.4226	0.5824	5.8236
12 × 12	877.2109	121.8587	97.4869	9.7487	0.6403	6.4025
24 × 24	873.2413	122.8783	98.3027	9.8303	0.6634	6.6937
30 × 30	872.7784	123.0005	98.4004	9.8400	0.6752	6.7524

Table 9: Convergence study of square laminated plate under uniform pressure R=10 (Material 3)

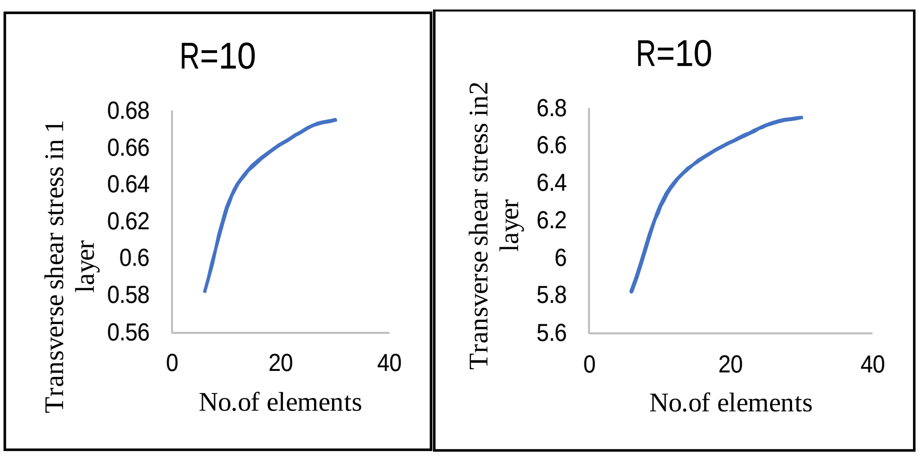


Figure 12: Variation of shear stress when R=10 (Material 3)

R=15						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	792.9925	117.7807	94.2246	6.2816	0.4217	6.3248
12 × 12	774.8679	121.9163	97.5330	6.5022	0.4644	6.9667
24 × 24	770.8950	122.9550	98.3640	6.5576	0.4859	7.2891
30 × 30	770.4305	123.0795	98.4636	6.5576	0.4903	7.3541

Table 10: Convergence study of square laminated plate under uniform pressure R=15 (Material 3)

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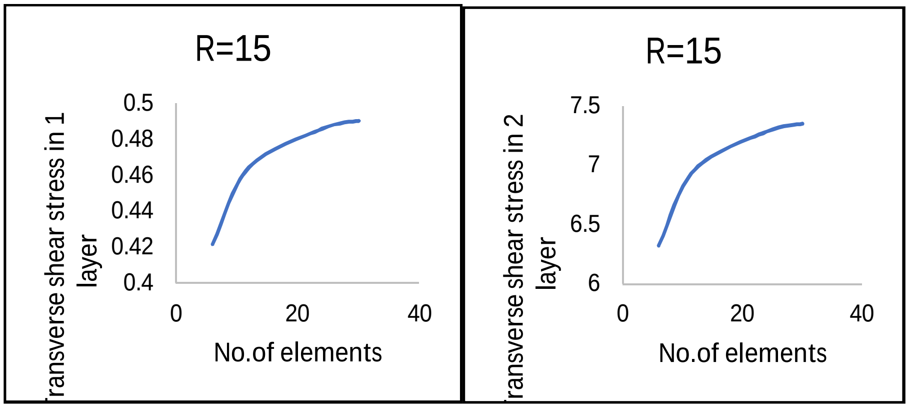
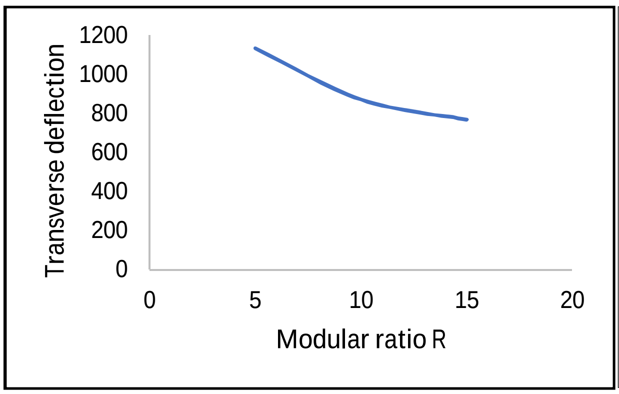


Figure: 13 Variation of shear stress when R=15 (Material 3)

Effect of varying modular ratio on transverse shear deflection for a simply supported square laminated plate under uniform pressure (t=0.01m) for Material 3.



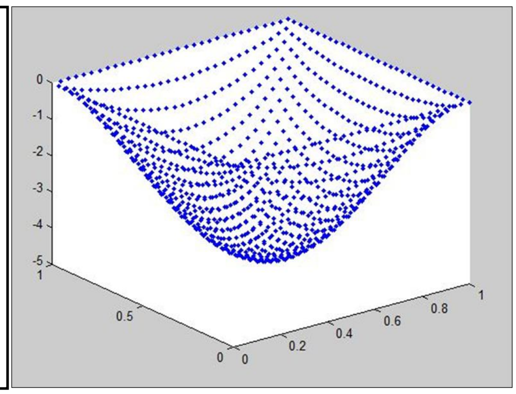


Figure: 14 Variation of transverse deflection (R=5,10,15) (Material 3)   Figure: 15 Deformed shape of square laminated plate (Material 3)

Effect of varying modular ratio on transverse shear stress for a simply supported square laminated plate under uniform pressure (t=0.01m) for Material 4.

4) *Material 4 [29]*

$$E_1 = 10MPa, E_2 = 10MPa, E_3 = 10MPa, G_{12} = 0.5MPa, G_{23} = 0.5MPa, G_{13} = 0.5MPa, \nu_{12} = 0.00001, \nu_{23} = 0.00001, \nu_{13} = 0.00001$$

Stiffness matrix:

$$\overline{Q} = \begin{bmatrix} 10 & 0.0001 & 0.0001 & 0 & 0 \\ 0.0001 & 10 & 0.0001 & 0 & 0 \\ 0.0001 & 0.0001 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

R=5						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	269.8422	23.6530	18.9224	3.4845	1.4498	7.2488
12 × 12	264.4463	23.8849	19.1079	3.8216	1.6441	8.2204
24 × 24	263.2888	23.9496	19.1597	3.8319	1.7493	8.7465
30 × 30	263.1543	23.9575	19.1660	3.8332	1.7711	8.8557

Table 11: Convergence study of square laminated plate under uniform pressure R=5 (Material 4)

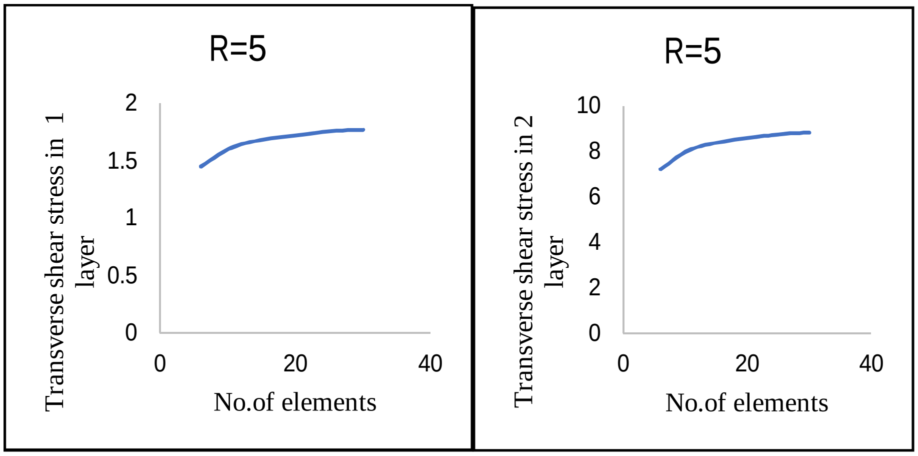


Figure 16: Variation of shear stress when R=5 (Material 4)

R=10						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	216.1197	25.8969	20.7175	2.0717	0.9334	9.3336
12 × 12	210.6182	26.1365	20.9092	2.0909	1.0598	10.5982
24 × 24	209.4264	26.2014	20.9611	2.0961	1.1279	11.2790
30 × 30	209.2876	26.2093	20.9674	2.0967	1.1420	11.4201

Table 12: Convergence study of square laminated plate under uniform pressure R=10 (Material 4)

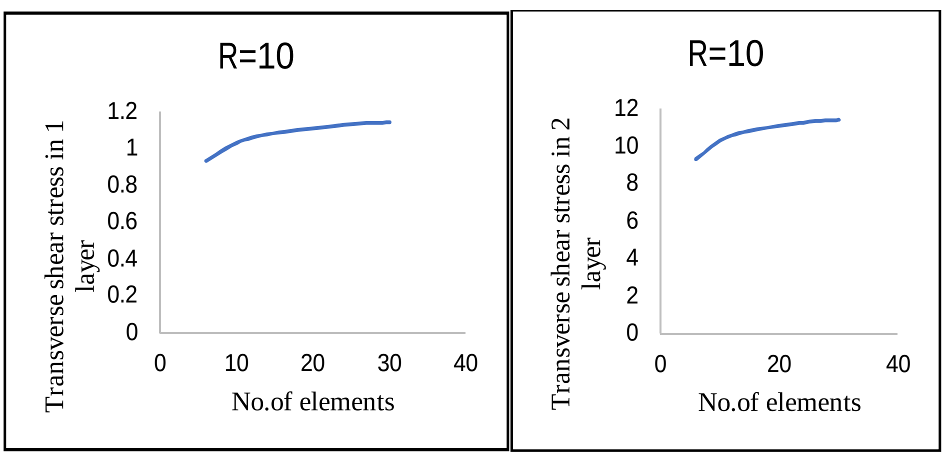


Figure :17 Variation of shear stress when R=10 (Material 4)

R=15						
Mesh size	$\overline{\omega}$	$\overline{\sigma_x^1}$	$\overline{\sigma_x^2}$	$\overline{\sigma_x^3}$	$\overline{\tau_{xz}^1}$	$\overline{\tau_{xz}^2}$
6 × 6	195.9632	26.7424	21.3939	1.4263	0.6882	10.3225
12 × 12	190.4205	26.9833	21.5867	1.4391	0.7819	11.7281
24 × 24	189.2158	27.0475	21.6380	1.4425	0.8322	12.4830
30 × 30	189.0751	27.0553	21.6442	1.4429	0.8426	12.6392

Table 13: Convergence study of square laminated plate under uniform pressure R=15 (Material 4)

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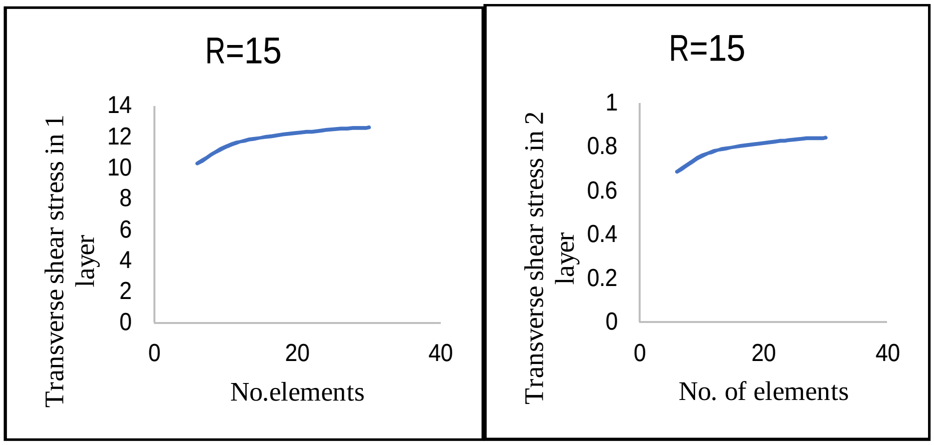
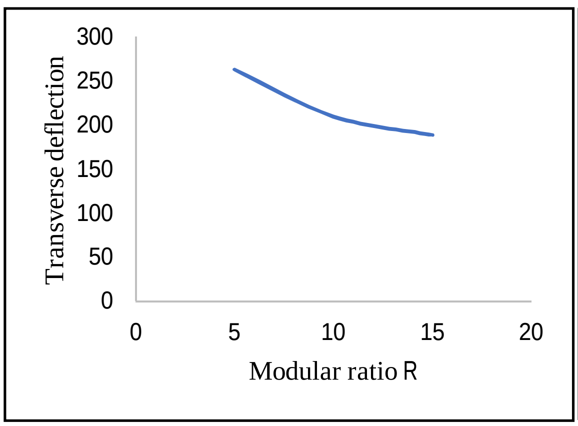


Figure 18: Variation of shear stress when R=15 (Material 4)

Effect of varying modular ratio on transverse deflection for a simply supported square laminated plate under uniform pressure (t=0.01m) for Material 4.



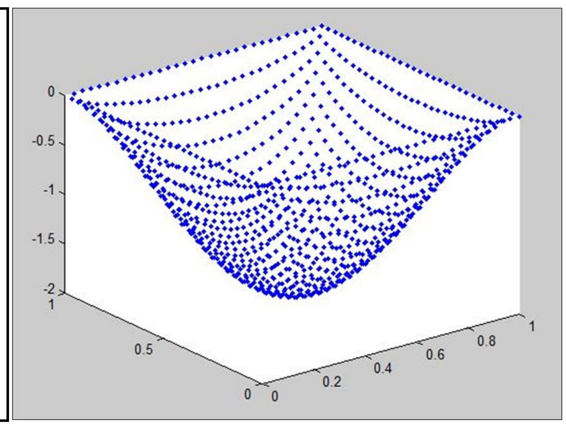


Figure:19 Variation of transverse deflection (R=5,10,15) (Material 4) Figure: 20 Deformed shape of square laminated plate (Material 4)

VI. CONCLUSION AND FUTURE SCOPE

A. Conclusions

A simply supported laminated plate under uniform pressure with three layers was examined using First Order Shear Deformation Theory. MATLAB code was generated for finite element analysis of laminated plate. For several values of modular ratios(R=5,10,15) variation in transverse deflection and transverse shear stress is investigated. By increasing the number of element size of the plate, convergence of values of deflection and shear stress could be seen. Stress and shear stress were calculated for each layer. This was repeated for all four materials which were taken for study. By increasing the modular ratio of the plate, shear stress in three layers was seen to be decreasing. The value for transverse shear stress varied along the thickness of each layer. The current work holds good agreement when evaluated from literatures for static analysis. Graphical variation of transverse shear stresses is continuous at the layer interface which is the true behavior of composites. It delivered almost accurate result when compared to the theory presented in base paper. This code could be helpful in analyzing laminated plate using FSDT by varying the thickness of plate, load condition, boundary condition, number of layers etc.

B. Future Scope

We can further simplify the FSDT theory by reducing the number of unknowns. This makes easier for researcher to solve problems and implement in any programming. To determine the interlaminar stress using FSDT, it is not be possible therefore shear correction factor is being used. Working should be done in area of simplification of these theories as such theories are analytically very difficult to solve. Very little research work is available on the effects of moving loads on sandwich structures. Therefore, the study of moving load must be carried out.

The effects of dampers can also be studied. As the viscos-elastic materials are available as construction materials, their use in sandwich structures may give better results. Effects of rotatory motion and torsion also need to be studied. Due to application of sandwich structures in aeronautical, avionics and satellite structures, the change in stresses due to air drag can be carried out.

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