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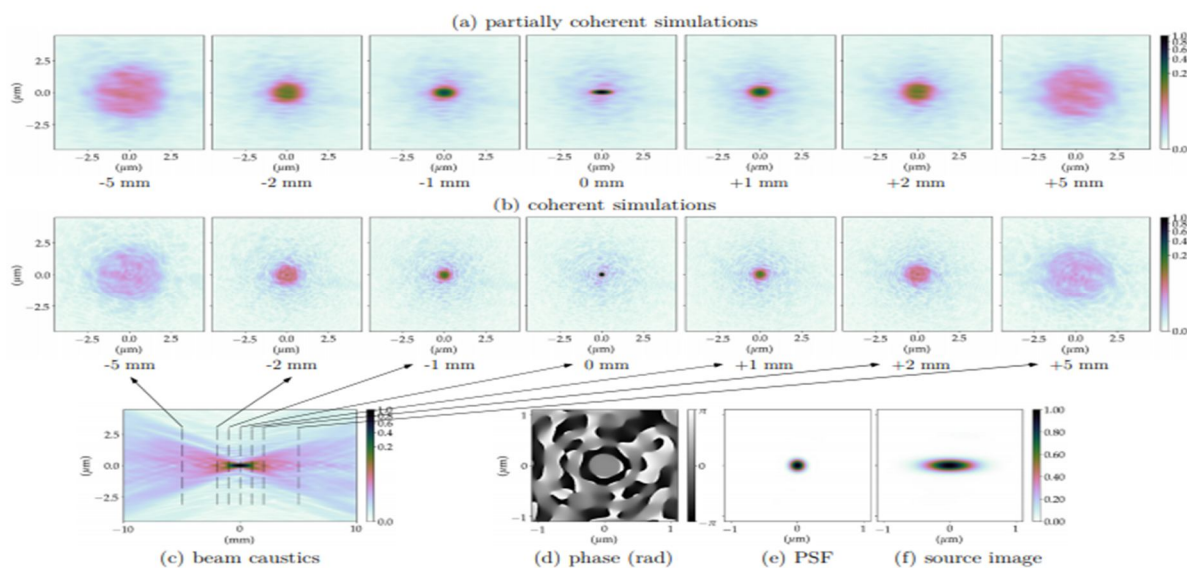
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Applications of Mathematics and Nuclear Physics in Medicine

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(Celestre et al., *Modelling phase imperfections in compound refractive lenses* 2019)

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Abstract: *This research paper examines the applications of mathematics and physics in medicine. Mathematical equations governing the physics behind nuclear medicine are deconstructed; the equations include the radioactive decay equation, nuclide uptake formula, formula for the total electric field, quaternions and their role in the geometry of CT and x-ray scans, the use of complex numbers and quaternions in CAT scans, and the utilization of partial differential equations in image processing. On the other hand, physics supports medicines in different areas, including medical imaging through popular MRI and ultrasound techniques. It contributes to therapy through radiation, ultrasonic technologies, laser physics, and vibrational medicine. Nuclear medicine relies on radioactive tracers, while non-ionizing radiation technologies support laser surgery; ultrasound imaging, and UV light treatments are used for diagnosing and treating chronic illnesses. Radiation oncology medical physicists apply medical physics for the assessment and monitoring of the safety of staff and patients involved in radiation therapy, with electromagnetism offering support in neural engineering, signal analysis, quantum electronics, and in studying the nervous system*

I. INTRODUCTION

The application of mathematics and nuclear physics in medicine is gaining momentum in the contemporary world. Mathematics is being applied in several medical fields: cardiovascular diseases, clinical schedules and tests, biofluids, genetics, and epidemiology. Nuclear physics in medicine involves using radiation to diagnose, treat and prevent diseases (Mohammadian-Behbahani & Saramad, 2018). Most of the modern procedures in hospitals today involve some form of radiation or radiology. The two methods are among the best life-saving techniques since they are safe, painless, and do not require anesthesia. The two methods are also applicable in a span of medical specialties: cardiology, pediatrics, and psychiatry.

This academic research paper aims to examine the applications of mathematics and nuclear physics in medicine. It will do so by deconstructing mathematical formulae that govern the physics behind nuclear medicine and explaining the physics behind specific medical procedures. In addition, the paper will provide the specific applications of mathematics and physics in supporting the advancement of medical practices. Finally, the discussion establishes the contributions that advancement in mathematics and physics offers in medicine, especially in diagnosing, monitoring, managing, and treating terminal illnesses.

II. MATHEMATICS AND EQUATIONS THAT GOVERN PHYSICS IN NUCLEAR MEDICINE

A. Radioactive Decay and Nuclide Uptake

Nuclear medicine is rapidly developing, with the optimization of therapy and diagnostics requiring numerous calculations. The use of radioactive resources in cancer treatment and management, especially in shrinking tumors, killing cancerous cells and tissues, and reducing pain, requires accurate calculation and measurement of radioactive decay, isotopes, and other processes. The rate of radioactive decay (A) is directly proportional to the number of atoms of the nuclide and its decay constant, λ (Świątaszczyk & Pilecki, 2013). Therefore, the calculation of the radioactive decay is indicated by formula $A = \lambda N$ where A refers to total activity. An equation for determining the number of atoms in nuclide one (1) after the time (t) when the number of atoms at the beginning is $t=0$ is shown below, representing a simple radioactive decay equation;

$$N_t = N_0 e^{-\lambda_1 t}$$

Successive radioactive decay describes a decay of nuclide 1 that produces nuclide two and many others, as shown in the sequence below;



The equation for the n -th nuclide is:

$$N_n(t) = N_1 0 \cdot \lambda_i^{-1} \cdot \sum_{i=1}^n \lambda_i \alpha_i e^{-\lambda_i t}, \text{ where } \alpha_i = \prod_{\substack{j=1 \\ j \neq i}}^n \frac{\lambda_j}{(\lambda_j - \lambda_i)}$$

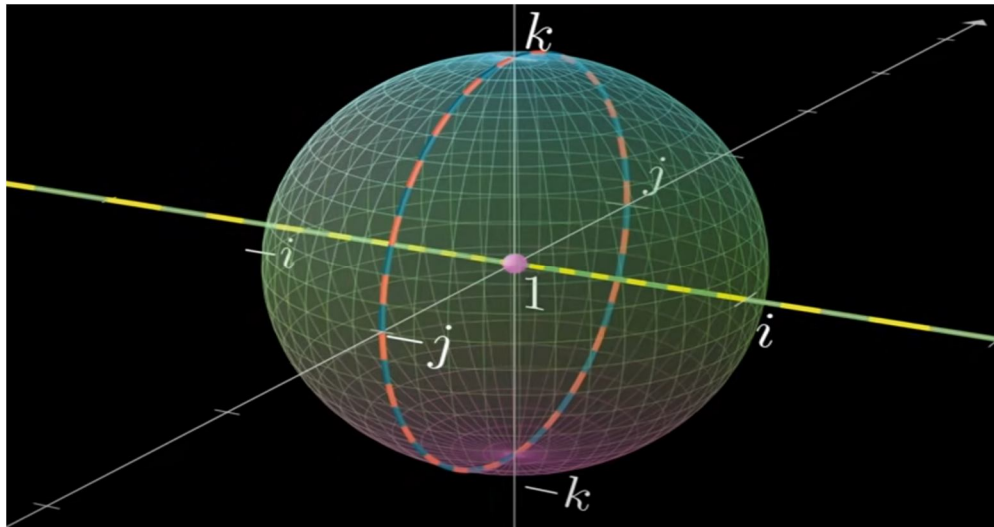
Similar equations can be derived for the activity of the n -th nuclide, the number of atoms in the second nuclide, and other quantities under certain assumptions, such as cases where decay constants for all nuclides in a given chain are assumed to be the same. Calculating radionuclide uptake is also an essential component of applying such a concept as a medical technique. First, background radiation is estimated by creating the first region of interest (ROI) over a target organ and the second in the vicinity of the organ. The formula below allows for the calculation of nuclide uptake;

$$\text{Uptake} = \frac{\text{patient} - \text{background}}{\text{nuclide} - \text{background}} (100\%)$$

B. Complex Numbers and Quaternions in Medicine Imaging

Complex numbers and quaternions in medicine are utilized in performing CAT scans using the X-ray principle. Conventional 2-dimensional complex numbers are expressed in the form $a + bi$, where a and b are real numbers, and i represents the fundamental unit of an imaginary number ($\sqrt{-1}$). Quaternions are a 4-dimensional extension of traditional 2-dimensional complex numbers and are expressed in the general form;

$$Z = bi + cj + dk, \text{ where } a, b, c, d \text{ are real numbers and } i, j, k, \text{ are distinct square roots of } -1$$



Stereographically Projected Model of Quaternion Multiplication

The geometry of CT scans and X-ray imaging utilize complex numbers and quaternions. When X-rays pass through the body, they are attenuated at different levels; MRI and CT scans perform rigid 3-dimensional transformations, so is it feasible to represent that using quaternions. Each quaternion is iterated as an interpolated change that is computed for position and orientation. In addition, the interpolated rotation is a small-angle approximation of a rotation quaternion that is linear in three parameters (Carter, 2003). It is essential to parametrize $ax + by = c$ in a radon transform representing trajectories of different beams entering the body when conducting CT scans and X-ray imaging. The parametrization of lines $ax + by = c$ uses a , b and c as constants as shown below:

$$\frac{a}{\sqrt{a^2 + b^2}}x + \frac{b}{\sqrt{a^2 + b^2}}y = \frac{c}{\sqrt{a^2 + b^2}}$$

whereby the two coefficients, $\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}$ Define a point on a unit circle. Thus, the angle θ corresponding to that point on the unit circle is calculated through;

$$\theta = \cos^{-1} \frac{a}{\sqrt{a^2 + b^2}}$$

C. Calculations in Medical Imaging, Radiation, and Electromagnetics

In medical imaging, radiation, and electromagnetics, mathematical approaches are used through partial differential equations (PDEs). Methods based on PDE are popular for image processing have been popular in the previous years. For instance, a person can think of an image as a map $I: D \rightarrow C$, i.e., to any point x in the domain D , I associate a “color” $I(x)$ in a color space C . The algorithms can be used to deform the image into different shapes by introducing time (t) as shown in the formula

$$\frac{\partial I}{\partial t} = \mathcal{F}[I]$$

Whereby $I(x, t): D \times [0, T] \rightarrow C$ is the evolving image, \mathcal{F} is an operator which characterizes the given algorithm, and the initial condition is the input image I_0 .

According to Šagátová et al. (2020), the acceleration of the beams in radiation therapy is also calculated through mathematical calculations to determine their intensity.

For instance, the average beam current (I_{AV}) can be calculated from the peak current using the formula $I_{AV} = \frac{1}{T} I_{PEAK} t_p$ Where T is the inverse of the pulse frequency, and t_p is the pulse duration. Maxwells equations are used in the electromagnetism process in medicine. For instance, when illuminating the head by electromagnetic fields, the general governing equation for the total electric field can be calculated using the formula;

$$\nabla \times \left(\frac{\pi}{2} \nabla \times E \right) - \omega^2 \epsilon E = 0$$

Whereby $E = E^{inc} + E^{sct}$ is the summation of the incident and scattered fields.

III. APPLICATIONS OF NUCLEAR PHYSICS IN MEDICINE

A. Medical Imaging Physics

Medical imaging physics deals with different areas such as testing, optimization, and quality assurance of diagnostic radiology. The imaging physics is also applied in quality assurance of diagnostic radiology in computed tomography, radiographic X-rays, fluoroscopy, and non-ionizing radiation modalities such as Magnetic Resonance Imaging (MRI) and ultrasound. Diagnostic radiology is a process that involves undertaking a range of imaging procedures to obtain images inside the body (Mohammadian-Behbahani & Saramad, 2018). The diagnostic radiologist then interprets the generated images to diagnose the illness or injuries inflicted on those examined. In modern medicine, diagnostic radiology is at the center of clinical decision-making. X-rays are a form of radiation that is similar to microwaves and radio waves. X-radiation has a high energy level that allows the X-rays to penetrate the body and create images.

Fluoroscopy medical imaging shows continuous X-ray images on a monitor, which helps in a detailed examination of a body (Fischer et al., 2017). Fluoroscopy is used in different examinations and procedures to diagnose and treat patients. Fluoroscopy includes catheter insertion and manipulation, Barium X-rays and enemas, Orthopedic surgery, and Angiograms. Just like the other X-ray procedures, fluoroscopy has some risks, such as relatively high radiation doses, especially when devices are placed in the body. MRI is a scanning procedure that uses radiofrequency pulses and strong magnets to generate signals from the body. The signals are perceived by a radio antenna and processed through the computer to produce images that show inside the body (Fischer et al., 2017). MRI is among the best ways of showing problems such as the spine, knee, and brain injuries. The technique is also used in providing additional information on ultrasound and X-rays.

Ultrasound is used for the high-frequency soundwaves to produce an image or picture on the screen that shows inside of the body. The process is conducted by different health professionals such as sonologist, radiologists, and sonographers using a transducer, a hand-held device moved around the body. The transducer transmits the high-frequency sound waves into the body. Different sound waves and reflected from soft tissues and body parts in different ways. The sound waves are converted to electrical impulses that produce the images observed on the screen. There are several advantages associated with ultrasound, such as its painless and does not involve radiation indicating that it is safe. In addition, the high-frequency sound waves ensure that the images produced are highly detailed and can show the fine parts of the body. Ultrasound is critical for the imaging of babies and children since it can picture while in movement.

B. Radiation Therapy

Radiation therapy is a form of cancer treatment that utilizes beams of high energy to kill cancer cells. The method often uses X-rays, but also photons and other forms of energy are used. The radiation damages cells by destroying the genetic material that determines how cells develop and divide. Both healthy and affected cells are damaged; however, the aim is to kill few normal cells (Lustberg et al., 2017). During the process, gamma rays and high-energy rays constating photons attack the tumor and kill the damaged cells. In addition, the subatomic particles, photons, are used to remove electrons out of the tumor cells atoms and destroy the larger cells by destroying the chemical compounds in the cells (Chandarana et al., 2018). For many years, radiotherapy has remained indispensable in the treatment of different types of cancer. The two types of radiation used in cancer treatment are internal radiation therapy and external beam radiation. Through internal radiation therapy, the implant containing the radiation is placed close to the cancer cells. The implant comes in different forms, such as wire, tube, pellets, and seeds. Systematic radiation is a form of internal radiation therapy whereby a patient swallows a radioactive substance that moves around the body, killing the cancerous cells. Moreover, the radioactive substance can also be injected through the veins. On the other hand, external beam radiation is commonly used in cancer treatment, whereby the energy beams used to treat cancer cells come from outside the body. The doctors aim the beams professionally in the body until they reach the cancer cells.

C. Electromagnetics

Electromagnetism deals with an electromagnetic force that occurs between the electronically charged particles. In healthcare, electromagnetism is applied in neural engineering, signal analysis in medicine, quantum electronics, and the nervous system (Sebens, 2019). Neural engineering is a biomedicine discipline in which mathematical, computational, and engineering technology methods combine with biology and neuroscience techniques. Neural engineering enhances the understanding of the nervous system and improves the body's performance after diseases or injuries (Li et al., 2020). The applications include neuroimaging, computer interfaces, and neuroinformatics. Quantum physics and technology are highly applied in healthcare in terms of sensors and computing. The healthcare processes in quantum are applied in artificial intelligence, nanotechnology, and quantum computing. Several healthcare companies use nanotechnology to develop quantum sensors that help detect diseases faster and accurately (Thew et al., 2019).

D. Physics of Therapy

Physics of therapy incorporates several activities that include ultrasonic therapy, laser physics, and vibrational medicine. Physical therapists use ultrasound therapy to promote tissue healing and relieve pain (De Lucas et al., 2020). The technique is essential when the patients have problems such as osteoarthritis, bursitis, and carpal tunnel syndrome. The main types of ultrasound therapy are mechanical and thermal, whereby they both utilize sound waves produced by a transducer to penetrate the soft tissues. De Lucas et al. (2020) indicate that thermal ultrasound therapy uses a more continuous transmission of sound waves when compared to mechanical ultrasound therapy that uses pulses of sound waves.

Vibrational medicine is crucial for several conditions such as anxiety, depression, headaches, and sleep disorders. Vibration healing is an alternative therapy that utilizes mechanical vibration used to treat particular health problems and injuries. The technique is often used in various medical settings, such as rehabilitation, massage therapy, and physical therapy (Callery & Rowbottom, 2021). The vibrations used in healing provide mechanical signals to the bones and muscles that help stimulate several growth factors. The alternative vibration medicine or healing is used to treat different health conditions such as multiple sclerosis, metabolic syndrome, pain, and sports injuries (Abramavičius et al., 2020).

Lasers are often used in healthcare to cut, coagulate and ablate tissues. The laser systems produce different wavelengths of varying energy levels and pulse duration. The laser beams' computer systems allow the procedures to be performed precisely, quickly, and better. The lasers are used in widespread applications such as bronchoscopy, laparoscopy, and endoscopy. The appropriate use of lasers requires trained clinicians who understand the laser delivery system and the laser-tissue interactions. Medical lasers produce photons of electromagnetic energy that are used to ablate, cut or coagulate the tissues.

E. Physics of Materials and Mechanics

Nuclear medicine uses radioactive tracers to assess, diagnose, and treat diseases. Special designed cameras are used in the process whereby the doctors track the path of these radioactive tracers. Oh (2020) states that the modalities used in the process are the Single Photon Emission Computed Tomography (SPECT) and Positron Emission Tomography (PET). SPECT shows how the blood flows in the tissues and organs. According to Lee and Park (2020), SPECT helps diagnose seizures, infections, stress fractures, and tumors. SPECT is a nuclear imaging scan that uses a radioactive tracer and computed tomography (CT) to produce images (Oh, 2020). The tracer allows the doctors to observe how the blood flows inside the blood. Before the SPECT scan is done, a tracer is injected into the patient's bloodstream. The tracer is radiolabeled; hence it emits gamma rays that are detected by the CT scanner. The computer collects the information from the gamma rays and displays it on the CT cross-sections. Sometimes the sections are combined to form 3-D images.

The radioisotopes used in the labeling of tracers are iodine-123, thallium-201, fluorine-18, and technetium-99m. Such radioactive elements penetrate the body and are easily detected by the scanner. The type of tracer used depends on what the doctor wants to measure from the patient. For instance, when the doctor wants to locate a tumor in the body, radiolabeled glucose determines its metabolism in the tumor. SPECT is different from a PET scan in that the tracer stays in the bloodstream rather than being absorbed in the bloodstream. The technique limits the images taken to where the blood flows. PET scan shows how the tissues and organs function through a radioactive tracer (Smye & Frangi, 2021). PET can detect diseases before they are shown in the other imaging tests. Depending on the tissue being tested, the tracer can be swallowed, inhaled, or injected. PET is useful in revealing several conditions such as brain disorders, heart disease, and cancers.

Moreover, physics of materials and mechanics is applied in nanoscience and nanotechnology, rotary molecular motors, physics of bio fabrication, and regenerative medicine.

Nanotechnology is the study and application of small things applied and used across the science fields such as biology and physics. Nanotechnology has significant impacts in the healthcare field, whereby doctors use nanoparticles to improve medical imaging, promote drug delivery, and target tumors (Anjum et al., 2021). Some of the nanoparticle-based treatments can find tumors and deliver the drug for treatment. Leso et al. (2019) demonstrate that nanotechnology is also crucial in increasing the speed of DNA sequencing and promoting wound treatment.

F. Non-Ionizing Medical Radiation Physics

Non-ionizing radiation (NIR) refers to enough radioactive energy for excitation rather than producing charged ions when passing through a matter. NIR is extensively used in healthcare applications such as laser surgery, ultrasound imaging, and UV light treatments (Hansson-Mild et al., 2019). Within the electromagnetic spectrum, NIR is located below the ionizing radiation band, such as the X-rays. Therefore, NIR has less energy when compared to ionizing radiation; hence cannot remove electrons from an atom. In addition, NIR is grouped according to different wavelengths and frequencies, which have different effects on the body. According to Hansson-Mild, et al. (2019), when the exposure limits are exceeded, the patients will likely endure severe effects of non-ionizing radiation. Therefore, safety measures through evidence-based information are needed to prevent the side effects when using NIR.

G. Radiation Oncology and Medical Imaging

Radiation Oncology physics involves the interaction of radiation with the human body when used in identifying the location and treatment of cancer (Jarrett et al., 2019). Radiation Oncology Medical Physicists are used in evaluating and monitoring the safety of staff and patients involved in radiation therapy. Radiation oncology uses high-energy X-rays that are delivered through a linear accelerator. According to (Jarrett et al., 2019), X-rays are painless and invisible, and critical in treating different cancers. Medical imaging physics refers to creating different images in the body for diagnostic and treatment purposes. Medical imaging physics incorporates processes such as MRI, X-ray radiography, Elastography, and tactile imaging. During the process, the X-rays are produced in a continuous spectrum with energy that enables them to perform the intended purpose (Suzuki, 2017). Sometimes, the X-rays are filtered with aluminum to remove low-energy photons that have a low contribution to the image being produced. Although medical radiation has expanded worldwide, there is a need to protect patients from exposure to high radiation that might compromise their health. Integrating radiation protection into good medical practice provides a platform for different stakeholders to collaborate and improve radiation safety standards (Suzuki, 2017). Improving risk assessment and communication remains critical in ensuring doctors and patients are protected from the effects of radiation.

IV. CONCLUSION

There is no doubt that Mathematics and Nuclear Physics are widely applied the medical field. The aspects of this paper that analyse the implications of complex Mathematics in medicine are particularly important as they allow complex analysis and pure Mathematics to be viewed in a different light—a more pragmatic light. The applications of Mathematics and Physics in medicine are important and continue to grow. In this paper, research into existing and developing applications of Mathematics and Physics in the realm of practical medicine has been presented.

REFERENCES

- [1] Abramavičius, S., Venslauskas, M., Vaitkus, A., Gudžiūnas, V., Laucius, O., & Stankevičius, E. (2020). Local vibrational therapy for essential tremor reduction: a clinical study. *Medicina*, 56(10), 552.
- [2] Anjum, S., Ishaque, S., Fatima, H., Farooq, W., Hano, C., Abbasi, B. H., & Anjum, I. (2021). Emerging Applications of Nanotechnology in Healthcare Systems: Grand Challenges and Perspectives. *Pharmaceuticals*, 14(8), 707.
- [3] Callery, E. L., & Rowbottom, A. W. (2021). Vibrational spectroscopy and multivariate analysis techniques in the clinical immunology laboratory: a review of current applications and requirements for diagnostic use. *Applied Spectroscopy Reviews*, 1 (1), 1-30.
- [4] Carter, M. (2003, August 15). Applications of Quaternions in Medical Imaging. Quaternions – Application Center. <https://www.maplesoft.com/applications/view.aspx?SID=4886&view=html>.
- [5] Celestre, R., Berujon, S., Roth, T., Sanchez del Rio, M., & Barrett, R. (2019). Modelling phase imperfections in compound refractive lenses. <https://arxiv.org/pdf/1911.06712.pdf>.
- [6] Chandarana, H., Wang, H., Tijssen, R. H. N., & Das, I. J. (2018). The emerging role of MRI in radiation therapy. *Journal of Magnetic Resonance Imaging*, 48(6), 1468-1478.
- [7] De Lucas, B., Pérez, L. M., Bernal, A., & Gálvez, B. G. (2020). Ultrasound therapy: Experiences and perspectives for regenerative medicine. *Genes*, 11(9), 1086.
- [8] Fischer, P., Faranesh, A., Pohl, T., Maier, A., Rogers, T., Ratnayaka, K., & Hornegger, J. (2017). An MR-based model for cardio-respiratory motion compensation of overlays in X-ray fluoroscopy. *IEEE transactions on medical imaging*, 37(1), 47-60.



- [9] Hansson-Mild, K., Lundström, R., & Wilén, J. (2019). Non-Ionizing radiation in Swedish health care—exposure and safety aspects. *International journal of environmental research and public health*, 16(7), 1186.
- [10] Jarrett, D., Stride, E., Vallis, K., & Gooding, M. J. (2019). Applications and limitations of machine learning in radiation oncology. *The British journal of radiology*, 92(1100), 20190001.
- [11] Lee, S. J., & Park, H. J. (2020). Single-photon emission computed tomography (SPECT) or positron emission tomography (PET) imaging for radiotherapy planning in patients with lung cancer: a meta-analysis. *Scientific reports*, 10(1), 1-11.
- [12] Leso, V., Fontana, L., & Iavicoli, I. (2019). Biomedical nanotechnology: Occupational views. *Nano Today*, 24, 10-14.
- [13] Li, S., Ma, Z., Cao, Z., Pan, L., & Shi, Y. (2020). Advanced wearable microfluidic sensors for healthcare monitoring. *Small*, 16(9), 1903822.
- [14] Lustberg, T., van Soest, J., Jochems, A., Deist, T., van Wijk, Y., Walsh, S., & Dekker, A. (2017). Big data in radiation therapy: Challenges and opportunities. *The British journal of radiology*, 90(1069), 20160689.
- [15] Mohammadian-Behbahani, M. R., & Saramad, S. (2018). Integral-equation based methods for parameter estimation in output pulses of radiation detectors: Application in nuclear medicine and spectroscopy. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors, and Associated Equipment*, 887, 7-12.
- [16] Oh, J. S. (2020). Nuclear Medicine Physics: Review of Advanced Technology. *Progress in Medical Physics*, 31(3), 81-89.
- [17] Šagátová, A., Fülöp, M., Pavlovič, M., Sedlačková, K., & Nečas, V. (2020). Electron-beam accelerator with conversion to X-rays: Optimal radiation type according to application. *Radiation Physics and Chemistry*, 172 (4), 108789.
- [18] Sebens, C. T. (2019). Electromagnetism as quantum physics. *Foundations of Physics*, 49(4), 365-389.
- [19] Smye, S. W., & Frangi, A. F. (2021). Interdisciplinary research: shaping the healthcare of the future. *Future Healthcare Journal*, 8(2), 218.
- [20] Suzuki, K. (2017). Overview of deep learning in medical imaging. *Radiological physics and technology*, 10(3), 257-273.
- [21] Świątaszczyk, C., & Pilecki, S. E. (2013). Calculations in nuclear medicine—application of free online software. *Nuclear Medicine Review*, 16(2), 103-108.
- [22] Thew, R., Jennewein, T., & Sasaki, M. (2019). Focus on quantum science and technology initiatives around the world. *Quantum Science and Technology* 5(1), 010201.
- [23] 3Blue1Brown. (2018). Visualizing quaternions (4d numbers) with stereographic projection. Video.



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