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Estimation of Parameters of Pert Distribution by Using Method of Moments

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Abstract: The method of moments has been widely used for estimating the parameters of a distribution. Usually lower order moments are wont to find the parameter estimates as they're known to possess less sampling variability. The method of moments may be a technique for estimating the parameters of a statistical model. It works by finding values of the parameters that end in a match between the sample moments and therefore the population moments (as implied by the model). the Method of moment Estimator is used to find out Estimates the parameters of PERT Distribution. We also compare equispaced and unequispaced Optimally Constructed Grouped data by the method of an Asymptotically Relative Efficiency. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Simulated Error (SE) and Relative Absolute Bias (RAB) for both the parameters under grouped sample supported 1000 simulations to assess the performance of the estimators.
Keywords: Method of Moments, PERT Distribution, equispaced and unequipped Optimal Grouped sample.

I. INTODUCTION

The method of moments is an alternative to the maximum likelihood method of estimating values of PERT density function parameters that describe the size distribution.

The method of moments of estimation was introduced by Karl Pearson (1894, 1895). The procedure consists of equating as many population moments to sample moments as there are parameters to estimate. Mathematical support for this procedure comes from the principle of moments as discussed intimately in Kendall and Stuart (1969). In essence, this principle says that two distributions that have a finite number of lower moments in common are going to be approximations of 1 another. Thus, the distribution of the data is approximated by equating the moments of a distributional form to the data moments. To see how this could be done with the three-parameter PERT distribution.

The method of moments considered that best estimates of the parameters of a probability distribution are those that moments of the PDF about the source are adequate to the corresponding moments of the sample data. Pearson originally considered only moments about the origin, but later it became customary to use the variance as the second central moment and the coefficient of sleekness as the standardized third central moment, to work out second and third parameters of the distribution if necessary.

In the method of moments approach, the parameters of a probability distribution model are estimated by matching the moments of the dataset thereupon of the candidate model. The number of moments required corresponds to the amount of unknown model parameters. Application of this method is simple, as closed-form expressions for the moments are often readily derived for many common distributions. However, the raw moments could also be biased thanks to the presence of outliers and/or the shortage of perfect agreement between the info and therefore the model.

The estimation of parameters of the three-parameter generalized exponential distribution introduced by Hossain and Ahsanullah [5] by using the utmost likelihood estimation and therefore the method of moments. Rameshwar D. Gupta (2000), And Debasis Kundu Studied. The Generalized Exponential Distribution: Different Method of Estimations. Recently a full derivation of the conditional moment equations was derived and numerical results show that when the utmost order of the considered moments is high, the amount of equations that have to be integrated is usually much smaller for the conditional moments approach and the resulting equations are less stiff(2013). Syed Afzal Hossain (2018) studied Estimating the Parameters of a Generalized Exponential Distribution, here discussed the maximum likelihood (ML) method and the method of moments to estimate the parameters.

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In this paper, we discuss about the estimation procedure for the unknown parameters for PERT distribution. The idea behind the Method of Moments (MoM) parameter estimation is to determine the parameters for given the sample data. We present MoM of the unknown parameters of PERT distribution using Newton-Raphson iterative procedure. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Efficiency (RE) for three parameters under sample based on 10,000 simulations to assess the performance of the estimators. A simulation study is conducted to evaluate the performance of the Method of Moment estimates. Finally, the proposed estimation method is applied on real and generalized data sets the results are given. Which illustrate Method of Moments estimation of unknown parameters for PERT distribution?

A random variable X ~ PERT (a, b, c) has probability density function and is in the form

$$f_{PERT}(x;\ a, b, c) = \frac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}};\ a < x < c \quad \ldots(1)$$

$$\alpha = \frac{4b+c-5a}{c-a}\ ;\quad \beta = \frac{5c-a-4b}{c-a}$$

(a, b, c) are parameters of PERT distribution

A random variable X ~ PERT (a, b, c) has cumulative distribution function and is in the form

$$F_{PERT}(x;\ a, b, c) = \frac{(-1)^{\alpha}\beta(\frac{z}{z-1};\alpha,\ 1-\alpha-b)}{\beta(\alpha,\ \beta)};\ a < x < c \quad \ldots(2)$$

Here, $z = \frac{x-a}{c-a}$

A random variable X ~ PERT (a, b, c) has Quantile function and is in the form

The p^{th} quantile x_p of PERT distribution is of the equation.

$$x_p = a+(c-a)\frac{\alpha+(p-\frac{5}{6})}{\alpha+\beta+\left(p-\frac{7}{6}\right)} \quad \ldots(3)$$

Let U ~ U(0,1), then equation (4.3) can be used to simulate a random sample of size 'n' from the PERT distribution as follows

$$x_i = a+(c-a)\frac{\alpha+(u_i-\frac{5}{6})}{\alpha+\beta+\left(u_i-\frac{7}{6}\right)},\ i = 1, 2, \ldots, n. \quad \ldots(4)$$

II. ESTIMATION OF PARAMETERS OF PERT DISTRIBUTION USING METHOD OF MOMENTS

If X follows the PERT distribution then K^{th} moment of PERT distribution is given by

$$E(X^k) = \int_a^c x^k f(a, b, c)dx \quad \ldots(5)$$

The moments of PERT distribution as follows

$$E(X) = \frac{a+4b+c}{6} = \mu \quad \ldots(6)$$

$$E(X^2) = \frac{a^2\beta(\beta+1)+2ac\alpha\beta+c^2\alpha(\alpha+1)}{42} \quad \ldots(7)$$

$$E(X^3) = \frac{a^3\beta(\beta+1)(\beta+2)+3a^2c\alpha\beta(\beta+1)+3c^2a\alpha\beta(\alpha+1)+c^2\alpha(\alpha+1)(\alpha+2)}{336} \quad \ldots(8)$$

$$E(X^4) = \frac{a^4\beta(\beta+1)(\beta+2)(\beta+3)+4a^3c\alpha\beta(\beta+1)(\beta+2)+6a^2c^2\alpha\beta(\alpha+1)(\beta+1)+4c^3a\alpha(\alpha+1)(\alpha+2)+c^4\alpha(\alpha+1)(\alpha+2)(\alpha+3)}{3024} \quad \ldots(9)$$

By using the (5) and (6), we get

$$\text{Mean} = \hat{\mu} = \bar{x} = \frac{a+4b+c}{6} \quad \ldots(10)$$

$$\text{Variance} = \hat{\sigma}^2 = \frac{(c-a)^2}{36} \quad \ldots(11)$$

Solving (11), we have

$$\hat{a} = \frac{(c-\hat{\sigma})}{6} \quad \ldots(12)$$

$$\hat{c} = \frac{6\bar{x}-7c+\hat{\sigma}}{24} \quad \ldots(13)$$

III. SIMULATION STUDY

In this section, we develop a simulation study. The major goal of these simulations is to calculate the efficiency of the Method of Moments estimation method for the parameters of the PERT distribution. The subsequent procedure was adopted as follows:

Step 1: Set the sample size 'n' and the vector of parameter values $\Psi = (\alpha, \beta)$.

Step 2: Using the values obtained in step (2), compute $\hat{\alpha}_{MoM}$ and $\hat{\beta}_{MoM}$ through Method of Moments.

Step3: Repeat steps (2) and (3) N times

Step4: Using $\widehat{\Psi}$ of Ψ, compute the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If $\widehat{\Psi}_{lm}$ is Method of Moments estimate method of $\widehat{\Psi}_m$, m=1, 2 where $\Psi_m$is a general notation that can be replaced by $\Psi_1 = \alpha, \Psi_2 = \beta$ based on sample l, (l =1,2,…,r), then the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

$$\text{Average Estimate } (\hat{\psi}_m) = \frac{\sum_{i=1}^{r} \hat{\psi}_{lm}}{r}$$

$$\text{Variance}(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}$$

$$\text{SD } (\hat{\psi}_m = \sqrt{\frac{\sum_{i=1}^{r} (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}}$$

$$\text{Mean Absolute Deviation}(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} Med(|\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}|)}{r}$$

$$\text{Mean Square Error } (\hat{\psi}_m) = \frac{\sum_{i=1}^{r} (\hat{\psi}_{lm} - \psi_m)^2}{r}$$

$$\text{Relative Absolute Bias}(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} |(\hat{\psi}_{lm} - \psi_m)|}{r\psi_m}$$

$$\text{Relative Error}(\hat{\psi}_m) = \frac{1}{r}\left(\frac{\sum_{i=1}^{r} MSE\sqrt{(\hat{\psi}_{lm})}}{\psi_m}\right)^2$$

The results were computed using the software R (R Core Development Team). The seed used to generate the random values. The chosen values to perform this procedure were N = 10,000, and n = (20, 40, 60,…, 200). For different population parameter values.

IV. APPLICATIONS

In this section, we considered two real data sets. First data set consists of 62 observations of the strengths of 3.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory. analyzed by Smith and Naylor (1987). The second data set is presented by Boag (1949) and is related to the ages (in months) of 18 patients who died from other causes than cancer.

In this Section, our simulation study indicated that the MoM estimators should be used for estimating the parameters of the PERT distribution. Initially, we compared the estimates obtained from the different procedures with the MoM estimator. Then, we compared the results obtained from the PERT distribution fitted by the MoM estimators with some common lifetime models, such as Uniform and Triangular distributions.

The Kolmogorov-Smirnov (KS) test is considered to check the goodness of fit. This procedure is based on the KS statistic $D_n = sup_x|F_n(x) - F_0(x)|$

Where sup_x is the supremum of the set of distances,

$F_n(x)$ is the empirical distribution function and $F_0(x)$ is cumulative distribution function. In this case, we test the null hypothesis that the data comes from $F_0(x)$ and with significance level of 5%, we will reject the null hypothesis if p value is smaller than 0.05. As discrimination criterion method, we considered the AIC (Akaike Information Criteria) computed, respectively, by

$$\text{AIC} = -2l(\widehat{\Psi}, x) + 2k$$

Where k is the number of parameters fitted and $\widehat{\Psi}$ is estimate of Ψ.

The data set consists of 62 observations of the strengths of 3.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory with (α) = 4 and

(β) = 2. The data are:

4.99, 3.97, 2.18, 3.14, 2.19, 4.96, 2.66, 4.98, 3.37, 2.85, 4.88, 3.27, 4.29,3.29, 4.10, 4.76, 4.49, 4.24, 2.85, 3.16, 2.16, 2.34, 3.84, 4.52, 2.89, 4.87, 2.87, 2.40, 4.30, 3.73, 3.45, 4.98, 4.43, 2.09, 2.30, 2.89, 2.53, 2.01, 4.94, 2.23, 4.15, 2.73, 3.59, 3.27, 4.70, 2.14, 4.84, 4.46, 4.42, 2.57, 3.64, 3.54, 3.70, 3.95, 2.98, 4.23, 3.78, 4.84, 3.54, 3.03, 2.98, 3.89. These data have also been analyzed by Smith and Naylor (1987). We obtained

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$\hat{\alpha}_{MoM}$ = 2.247 and $\hat{\beta}_{MoM}$ = 1.5874.
Results of the KS test (p value), AIC for the different probability distributions considering the above data set

Test	PERT	Uniform	Triangular
KS	0.5148	0.01254	0.1689
AIC	2015.23	2654.8	2421.13

Boag Data Set 2
The data set related to the ages (in months) of 18 patients who died from other causes than cancer extracted from Boag (1949), which considered the PERT distribution to describe such data.
0.3, 4, 7.4, 15.5, 23.4, 46, 46, 51, 65, 68, 83, 88, 96, 110, 111, 112, 132, 162.
.0.3, 4, 7.4, 15.5, 23.4, 46, 46, 51, 65, 68, 83, 88, 96, 110, 111, 112, 132, 162.
We obtained
$\hat{\alpha}_{MoM}$ = 1.1236 and $\hat{\beta}_{MoM}$ = 2.2487

Results of the KS test (p value), AIC for the different probability distributions considering the above data set

Test	PERT	Uniform	Triangular
KS	0.4965	0.0001	0.0018
AIC	1154.65	2587.25	2014.56

Comparing the empirical function with the adjusted distributions, a better fit for the PERT distribution among the chosen models can be observed. This result is confirmed from AIC, since PERT distribution has the minimum values among the chosen models. Moreover, considering a significance level of 5%, the PERT distribution was the only model in which p values returned from the KS test were greater than 0.05.
Method of Moments (MoM) for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MoMs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=5, b=6 and c=7 in Table 4.1.

TABLE-4.1
Method of Moment for Estimating the PERT Distribution (a=5, b=6 and c=7)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	3.1354	0.8885	0.9154	0.7245	1.9194	0.9989	0.7985
	b	4.0245	0.8368	0.8574	0.6123	1.9213	0.9987	0.6958
	c	5.0198	0.8924	0.9121	0.4526	1.9417	0.9754	0.6932
40	a	3.2359	0.8847	0.9032	0.6958	1.90549	0.9687	0.6985
	b	4.1287	0.7651	0.7798	0.5964	1.9172	0.9621	0.6358
	c	5.1387	0.8832	0.902	0.4625	1.8316	0.9124	0.6158
60	a	3.3165	0.8775	0.8854	0.6178	1.8244	0.8932	0.6658
	b	4.2698	0.7184	0.7265	0.4968	1.8146	0.8721	0.5987
	c	5.2364	0.8638	0.8765	0.4785	1.8099	0.8997	0.5986
80	a	3.5689	0.8065	0.8245	0.5968	1.8081	0.8365	0.5721
	b	4.5364	0.6865	0.7085	0.4352	1.7646	0.8254	0.5687
	c	5.5024	0.8435	0.8521	0.3965	1.8101	0.8387	0.5123
100	a	3.9658	0.7543	0.7956	0.4752	1.8038	0.8254	0.5236
	b	4.7698	0.6547	0.6874	0.4132	1.7313	0.8054	0.5478
	c	5.9864	0.7786	0.7865	0.2968	1.7585	0.8198	0.4587
120	a	4.1028	0.6982	0.7145	0.4325	1.7039	0.8247	0.4265
	b	5.198	0.6448	0.6654	0.3965	1.7011	0.7154	0.5368

	c	6.1586	0.7368	0.7587	0.2754	1.7033	0.7965	0.3254
140	a	4.3896	0.6462	0.7241	0.3658	1.7008	0.7154	0.5478
	b	5.2687	0.5867	0.6024	0.2965	1.6034	0.6587	0.4598
	c	6.3698	0.6458	0.6687	0.2636	1.6034	0.6687	0.3621
160	a	4.5473	0.6364	0.6879	0.2269	1.6128	0.6498	0.5147
	b	5.6935	0.4963	0.5784	0.2684	1.5035	0.5565	0.3254
	c	6.5368	0.5968	0.6178	0.2564	1.5037	0.5368	0.3124
180	a	4.7965	0.5736	0.6024	0.2154	1.6107	0.5471	0.3021
	b	5.7635	0.3987	0.4658	0.2487	1.5003	0.4921	0.2965
	c	6.7635	0.5598	0.5687	0.2015	1.5004	0.6789	0.2547
200	a	4.8657	0.5496	0.5748	0.2141	1.5151	0.5163	0.2531
	b	5.7632	0.3865	0.4187	0.1968	1.4053	0.41	0.2245
	c	6.6981	0.5269	0.5541	0.2001	1.4137	0.6352	0.2278

- *Observations*

➢ Average Estimate (AE) of PERT parameters by MLE are increased when sample size is increased.

➢ Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

Method of Moments (MoM) for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes n=20(20)200 are considered. The MoMs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=2, b=2.5 and c=3 in Table 4.2.

TABLE-4.2
Method of Moment for Estimating the PERT Distribution (a=2, b=2.5 and c=3)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	0.8954	0.7102	0.8457	0.6587	1.9987	0.8145	0.7985
	b	0.8997	0.6754	0.7854	0.5587	1.9954	0.7798	0.7625
	c	1.1354	0.5867	0.7725	0.5841	1.9854	0.6653	0.9465
40	a	0.9584	0.5768	0.6658	0.6021	1.9354	0.6458	0.6357
	b	1.1254	0.4932	0.6987	0.5487	1.9287	0.6958	0.8706
	c	1.1524	0.3987	0.6125	0.5252	1.9232	0.4236	0.7049
60	a	1.1587	0.5829	0.5921	0.5028	1.8925	0.6254	0.617
	b	1.8457	0.5487	0.5353	0.5387	0.8547	0.5869	0.5647
	c	1.8546	0.3147	0.5147	0.4587	0.8547	0.4103	0.3996
80	a	1.2987	0.4657	0.4721	0.4998	1.8657	0.4987	0.4514
	b	1.9658	0.5252	0.5224	0.5147	0.7965	0.5364	0.5291
	c	1.8965	0.2874	0.4545	0.3658	0.8547	0.2935	0.2694
100	a	1.3254	0.2699	0.5014	0.4243	1.6854	0.3698	0.3557
	b	1.9857	0.4432	0.5024	0.4848	0.6958	0.5278	0.5123
	c	2.4781	0.2568	0.404	0.2487	0.8542	0.2547	0.2483
120	a	1.4216	0.2675	0.4487	0.3998	0.5587	0.3325	0.3017
	b	1.9936	0.4325	0.4757	0.4365	0.6684	0.5187	0.4915
	c	2.4179	0.1745	0.3998	0.1587	0.8083	0.2147	0.203
140	a	1.6254	0.2287	0.4121	0.3357	0.5587	0.2854	0.2455
	b	2.0147	0.3178	0.4128	0.3024	0.6287	0.3784	0.3563
	c	2.4587	0.1254	0.3856	0.1451	0.7245	0.1957	0.1841
	a	1.7235	0.2287	0.4018	0.3354	0.5124	0.2821	0.2273

160	b	2.1021	0.1124	0.3958	0.2958	0.5247	0.2958	0.2278
	c	2.6578	0.1769	0.3364	0.1352	0.7154	0.1951	0.1922
180	a	1.7965	0.1387	0.3935	0.3232	0.4054	0.2487	0.2338
	b	2.1471	0.1287	0.3836	0.2641	0.5065	0.2547	0.2399
	c	2.7484	0.1636	0.3254	0.1547	0.6254	0.1887	0.1416
200	a	1.8045	0.1203	0.2741	0.2268	0.3698	0.2354	0.2094
	b	2.2968	0.1198	0.3134	0.1874	0.4787	0.2487	0.1802
	c	2.7689	0.1588	0.3009	0.1287	0.5247	0.1783	0.1182

- *Observations*

- Average Estimate (AE) of PERT parameters by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

Method of Moments (MoM) for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MoMs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=3, b=4 and c=5 in Table 4.3.

TABLE-4.3

Method of Moment for Estimating the PERT Distribution (a=3, b=4 and c=5)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	1.3568	0.6024	0.7635	0.9998	1.4457	0.9984	0.9978
	b	1.4758	0.3754	0.6288	0.9452	1.3982	0.9451	0.9532
	c	2.5684	0.5964	0.7621	0.9378	1.3378	0.9627	0.9598
40	a	1.3879	0.4754	0.6921	0.9753	1.3956	0.9458	0.9365
	b	1.50247	0.2351	0.4822	0.9398	1.2521	0.9358	0.9289
	c	2.6587	0.4154	0.6348	0.9287	1.2182	0.9583	0.9408
60	a	1.9015	0.4578	0.6692	0.9687	1.1578	0.9152	0.9056
	b	1.9682	0.2747	0.5127	0.9281	1.1286	0.9287	0.9158
	c	2.7548	0.2012	0.4537	0.8787	1.1175	0.9466	0.9358
80	a	2.3876	0.2966	0.5521	0.8987	1.2432	0.9118	0.9006
	b	2.4965	0.1582	0.3952	0.9077	1.1589	0.9158	0.9088
	c	2.8875	0.2147	0.4533	0.8547	1.1006	0.9337	0.9283
100	a	2.4754	0.1257	0.3662	0.8721	1.1985	0.9006	0.8998
	b	2.5164	0.1154	0.3391	0.8964	1.1268	0.9058	0.8997
	c	3.5472	0.1132	0.3295	0.8174	1.0065	0.9221	0.9118
120	a	2.5877	0.1178	0.3463	0.8521	1.1584	0.8933	0.8457
	b	2.6874	0.1025	0.3194	0.8732	1.1157	0.8965	0.8547
	c	3.8733	0.1097	0.3241	0.7985	0.9987	0.9157	0.9064
140	a	2.7015	0.1054	0.3138	0.8154	1.1068	0.8725	0.8257
	b	2.7845	0.1009	0.3126	0.8421	0.9658	0.8799	0.8331
	c	4.0157	0.0954	0.2993	0.7732	0.92543	0.9087	0.8959
160	a	2.79856	0.1124	0.3228	0.7938	0.9932	0.8598	0.8165
	b	2.8741	0.0958	0.3102	0.8365	0.9465	0.8566	0.8289
	c	4.2245	0.0942	0.3044	0.7521	0.8098	0.8354	0.8154
	a	2.8541	0.1098	0.3128	0.7784	0.9587	0.8068	0.7932

180	b	3.5413	0.0945	0.3012	0.8164	0.9087	0.8165	0.8007
	c	4.3115	0.0931	0.3011	0.7468	0.7985	0.8057	0.8009
200	a	2.8965	0.0928	0.3087	0.7158	0.8547	0.7998	0.7542
	b	3.6874	0.0929	0.2994	0.7584	0.8356	0.6875	0.6458
	c	4.5687	0.0914	0.2984	0.6952	0.7721	0.7653	0.7054

- *Observations*
- Average Estimate (AE) of PERT parameters of estimated a, b, c by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

We calculate the Method of Moments (MoM) for estimating the PERT (α, β). Newton-Raphson iterative procedure for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n = 20(20)200 are considered. The MoMs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters α =3.5 and β = 2.5 in Table 4.4.

TABLE-4.4
Method of Moments for Estimating the PERT Distributions ($\alpha = 3$, β =2.5)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	α	1.5965	0.9969	0.9972	0.9458	1.9989	0.9999	0.8998
	β	1.4469	0.9965	0.9956	0.9268	1.9968	0.9986	0.8759
40	α	1.6582	0.9921	0.9952	0.9169	1.9756	0.9965	0.8685
	β	1.5248	0.9154	0.9823	0.8965	1.9698	0.9954	0.8532
60	α	1.7156	0.9025	0.9753	0.8865	1.9154	0.9923	0.8456
	β	1.6052	0.8956	0.9523	0.8547	1.8964	0.9897	0.8532
80	α	1.8356	0.8754	0.9326	0.8469	1.8998	0.9752	0.8365
	β	1.6125	0.8721	0.9165	0.8421	1.8863	0.9568	0.8169
100	α	1.8465	0.8598	0.9026	0.7965	1.8546	0.9468	0.7986
	β	1.6358	0.8322	0.8973	0.7524	1.8769	0.9132	0.7854
120	α	1.9865	0.8169	0.8688	0.7436	1.8326	0.9056	0.7798
	β	1.6698	0.7963	0.8546	0.7165	1.8635	0.8965	0.7684
140	α	1.9966	0.7465	0.8324	0.7098	1.7965	0.8936	0.6998
	β	1.7098	0.7398	0.8147	0.6987	1.7532	0.8732	0.6854
160	α	2.1032	0.7169	0.7995	0.6856	1.7436	0.8569	0.6598
	β	1.8562	0.6958	0.7854	0.6784	1.7164	0.8469	0.6459
180	α	2.3698	0.6898	0.7098	0.6654	1.6953	0.8299	0.6398
	β	1.9965	0.6721	0.6954	0.6438	1.6623	0.8132	0.6275
200	α	2.6831	0.6588	0.6654	0.6198	1.6125	0.7986	0.6184
	β	2.1965	0.6487	0.6469	0.5968	1.5986	0.7211	0.6098

- *Observations*
- Average Estimate (AE) of PERT parameters of estimated α, β by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

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We calculate the Method of Moments (MoM) method for estimating the PERT(α , β) Newton-Raphson simulation procedure for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n = 20(20)200 are taken. The MoM s and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters α = 4.5 and β = 4 in Table 4.5.

TABLE 4.5
Method of Moments for Estimating the PERT Distribution (α = 4.5 , β =4)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	α	2.3345	0.9981	0.9977	0.8965	1.9987	0.9988	0.7845
	β	2.3243	0.9841	0.9854	0.84	1.8864	0.9854	0.8765
40	α	2.4254	0.9654	0.9735	0.8824	1.9752	0.9754	0.7564
	β	2.4032	0.9498	0.9441	0.7798	1.8765	0.9501	0.855
60	α	2.5489	0.9254	0.9365	0.8564	1.9546	0.9473	0.7211
	β	2.4583	0.9154	0.9154	0.7584	1.8547	0.9365	0.8547
80	α	2.6985	0.8904	0.8856	0.8369	1.9365	0.8954	0.6954
	β	2.5621	0.8762	0.8756	0.7432	1.8487	0.8801	0.8568
100	α	2.7698	0.8868	0.8632	0.8166	1.9147	0.8779	0.6687
	β	2.647	0.8854	0.8321	0.7285	1.8198	0.8564	0.8327
120	α	2.8647	0.8697	0.8564	0.7956	1.8989	0.8701	0.7587
	β	2.7654	0.8354	0.8187	0.7054	1.7954	0.8485	0.8147
140	α	3.1658	0.8451	0.7954	0.7564	1.8836	0.8532	0.7321
	β	2.8791	0.7264	0.7754	0.6987	1.7865	0.7765	0.7125
160	α	3.5517	0.8252	0.7548	0.7435	1.8324	0.8432	0.7121
	β	3.1478	0.7124	0.7321	0.6887	1.7654	0.7554	0.6884
180	α	3.9648	0.7927	0.7465	0.7288	1.7965	0.8177	0.7154
	β	3.4873	0.7064	0.7258	0.6754	1.7214	0.7658	0.6719
200	α	4.2956	0.7542	0.7154	0.7054	1.7436	0.7965	0.6921
	β	3.7658	0.6857	0.6987	0.6487	1.6987	0.7543	0.6154

- *Observations*
- Average Estimate (AE) of PERT parameters of estimated α, β by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

V. CONCLUSIONS

1) Method of Moments is the better one for estimating the parameters of the PERT distribution; Since Sample size increases Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both parameters are decreases.
2) The Method of Moments has the smallest Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both parameters, proving to be the efficient method.

A. Observations For The Simulation Result

1) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error (RE) of the estimators are dependent on the sample sizes.
2) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error(RE) of the estimators are independent on the population parameter values.
3) The Average Estimate (AE) of Method of Moments a, b, c estimators is increased when sample size increased.
4) The Average Estimate (AE) of Method of Moments $(\widehat{\alpha})$ and $(\hat{\beta})$ estimators is increased when sample size increased.
5) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error(RE) of Method of Moments the estimators a, b, c are decreased when sample size are increased.
6) The Variance (VAR), Standard deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), Relative Error (RE) of Method of moments $(\widehat{\alpha})$ and $(\hat{\beta})$ estimators are decreased when sample size increased.

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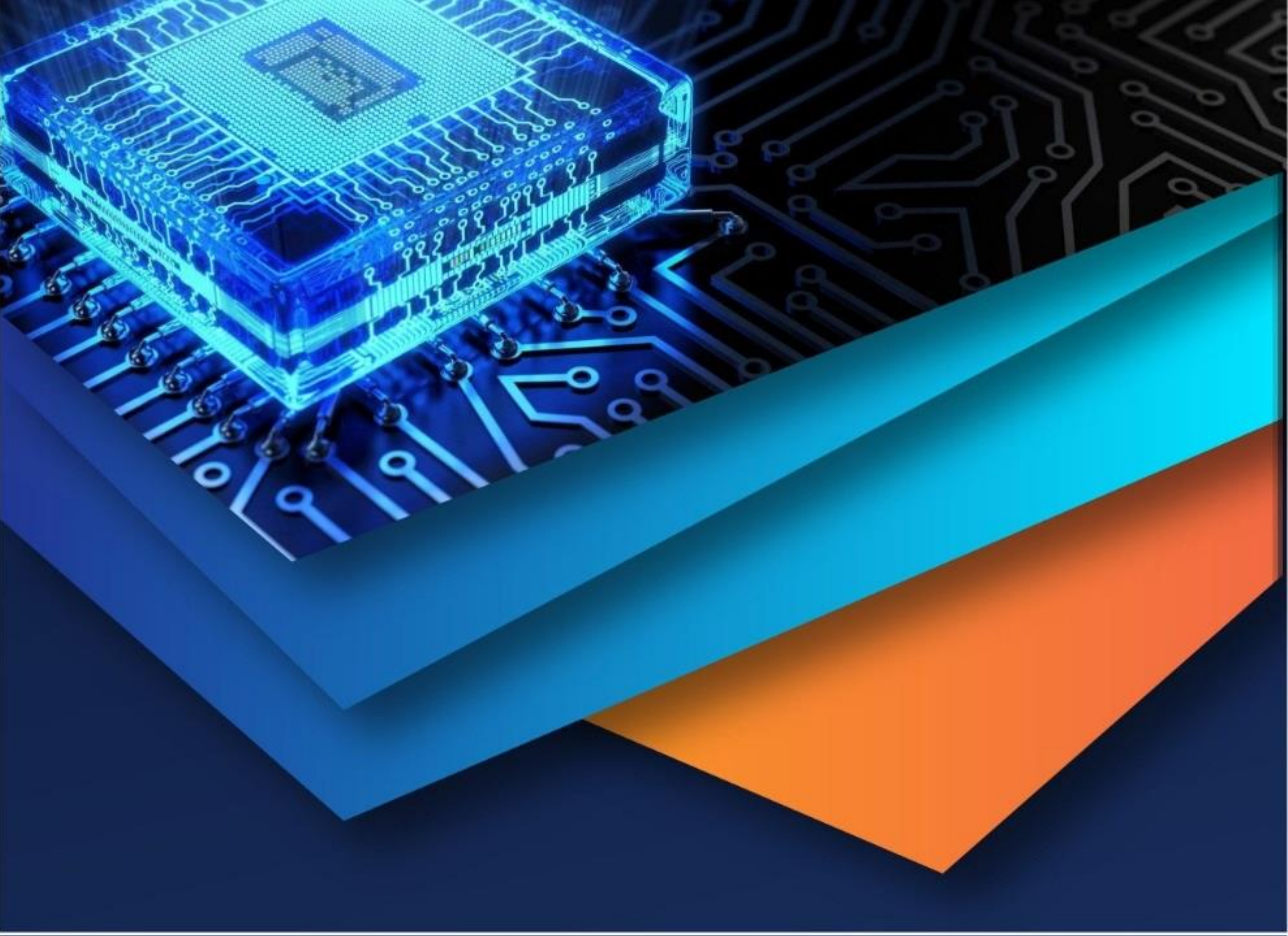

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