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Vibration Analysis of Cantilever Beam: An Experimental Study

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Abstract-- The purpose of this study is to investigate the natural frequencies of a cantilever beam experimentally and validate the result with finite element analysis of the beam. The finite element analysis is performed in MATLAB programming and the output from the program is validated with the analysis of the beam in ANSYS. The free vibration response of the beam with lumped end mass subjected to an initial disturbance is also studied in this paper. Different end masses are attached at the free end of the beam to study the influences of these masses on the natural frequencies of the beam. The values of material parameters i.e. density and modulus of elasticity etc considered during experiment are measured experimentally in the testing lab. This experimental setup can be used to measure the natural frequencies of any cantilever beam whose material properties are unknown.
Keywords- Natural frequency, Cantilever beam, mass, vibration.

I. INTRODUCTION

Cantilever beam is considered as one of the most fundamental structure in the field of structural mechanics. Cantilevers are widely found in construction mainly in bridges and balconies. Detection of the natural frequencies of the structure is considered as the most important observation in the vibration analysis as it is needed to avoid resonant condition. Therefore so many studies have been carried out previously to detect the natural frequencies of the cantilever beam. All the parameters which are the main agent to affect the natural frequencies of cantilever beam are needed to be considered. One of such parameter for small and slender structure of cantilever beam is mass of an accelerometer itself that affects the exact measured natural frequency values. It is found that the mass of an accelerometer affects the natural frequencies according to its location and the ratio of its mass to the mass of beam.

Hamdan and Jubran [1] have closely studied the modal analysis of cantilever beam with translational restraint due to spring at the tip end and an intermediate position of additional mass. The free and forced response has been studied by Galerkin's method.

Kotambkar [2] have studied the effect of mass attachment on natural frequency of free-free beam using analytical, numerical and experimental investigation. In many situations, the mass of accelerometer is ignored, however when lighter structures are investigated this effect must be count.

Liu and Haung [3] studied free vibration of beam. The beam was hinged by a rotational spring at one end and carried a concentrated mass at arbitrary location. For vibration analysis of beam Laplace Transformation method was used.

Wang [4] proposed the vibration analysis of uniform beam with lumped mass in both translational and rotary inertias condition. Finite element analysis is used to form expression of frequency for different attached location of lumped mass.

Kaunadis [5] analyzed the vibration analysis of cantilever beam in free and forced condition. The cantilever beam was loaded at different locations.

In this paper the natural frequencies of two different cantilever beams made of Aluminum and Iron are measured experimentally with and without the presence of end masses. The finite element analysis of those beams is done in ANSYS, as well as in MATLAB programming. The material properties i.e. density and modulus of elasticity considered during modal analysis are tested experimentally at the testing lab.

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II. ANALYSIS

A. SDOF model analysis

The governing equation for free undamped vibration of single degree of freedom (SDOF) system is given by,

$$m\ddot{y} + ky = 0 \qquad \text{….. (1)}$$

$$\ddot{y} + \frac{k}{m}y = 0$$

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{….. (2)}$$

Where

ω= Natural frequency of system in rad/s

m =Mass of beam in kg

k =Stiffness of the system in N/m.

The stiffness of a cantilever beam at the free end of the beam is $k = \frac{3EI}{l^3}$ …… (3)

Where

E =Young's modulus of the beam in N/m^2

l =Total length of beam in m.

I =Moment of inertia of the beam cross section given by

$$I = \frac{bd^3}{12}\ m^4$$

Where

b = breadth of the beam cross section

d = width of the beam cross section

Case 1: Calculation of natural frequency of Cantilever beams without end mass

From equation 1

$$\omega = \sqrt{\frac{k}{m}}\ \text{rad/s} = 1/2\Pi\sqrt{\frac{k}{m}}\ \text{Hz} \quad \text{…… (4)}$$

$$m = mac + mb$$

Where

mac =Mass of accelerometer

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=4.69 gm=.00469Kg (measured)

mb= $\frac{33}{140}M$ is the equivalent lumped mass of the beam at the free end of the beam, where M=mass of total beam

Case 2: Calculation of natural frequency of Cantilever beams with end tip mass

With end tip mass m_e the value of mass for the system is equal to

m =mac +mb+ me

Now the value of natural frequency can be found with the help of eq. (4)

B. M.D.O.F Analysis of the cantilever beam

In a multi-degree of freedom (MDOF) model of the beam let the $n \times 1$ vector $\{X\}$ contains the displacements of the beam at its degrees of freedom. If the mass, stiffness and damping matrices of sizes $n \times n$ are represented by the symbols $[M]$, $[K]$ and $[C]$ respectively, the governing equation of free vibration of motion for the system can be written as follows:-

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{0\} \quad \text{……… (1a)}$$

in state space notations,

$$\begin{Bmatrix}\{\ddot{X}\}\\ \{\dot{X}\}\end{Bmatrix} = -\begin{bmatrix}[M]^{-1}[C] & [M]^{-1}[K]\\ -[I] & [0]\end{bmatrix}\begin{Bmatrix}\{\dot{X}\}\\ \{X\}\end{Bmatrix} \quad \text{..(2a)}$$

$$\text{Or, } \{\dot{Z}\}=[A]\{Z\} \quad \text{…………….. (2b)}$$

The mass, stiffness matrices of the beam elements are given below which are assembled to form the overall mass and stiffness matrices.

[K]=Element stiffness matrix

$$= \frac{EI}{l^3}\begin{bmatrix}12 & 6l & -12 & 6l\\ 6l & 4l^2 & -6l & 2l^2\\ -12 & -6l & 12 & -6l\\ 6l & 2l^2 & -6l & 4l^2\end{bmatrix}$$

[m]=Element mass matrix

$$= \frac{\rho Al}{420}\begin{bmatrix}156 & 22l & 54 & -13l\\ 22l & 4l^2 & 13l & -3l^2\\ 54 & 13l & 156 & -22l\\ -13l & -3l^2 & -22l & 4l^2\end{bmatrix}$$

The Eigen values of [A] matrix will give the natural frequencies of the beam.

C. Analysis of beams through ansys

The finite element analysis of the beam is done using the popular software package ANSYS. The ANSYS results are used to validate the results obtained from the finite element programming in MATLAB. The beam3 element is considered to mess the beam whereas mass21 element is considered to model the tip mass of the beam. The analysis is done for the two different beams made up of iron and aluminium materials. The different mode shapes obtained for aluminium in ANSYS is shown below .The same analysis

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is also done for iron beam.

1) Analysis of aluminum without mass

The three mode shapes obtained of aluminium beam from ANSYS is shown below.

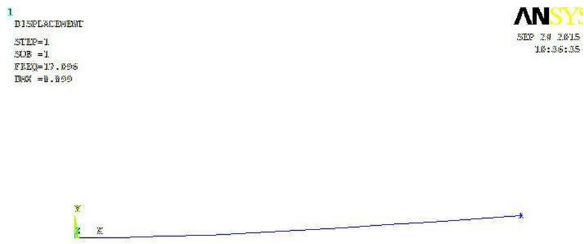

Fig.2.3.1.1 First mode shape of beam

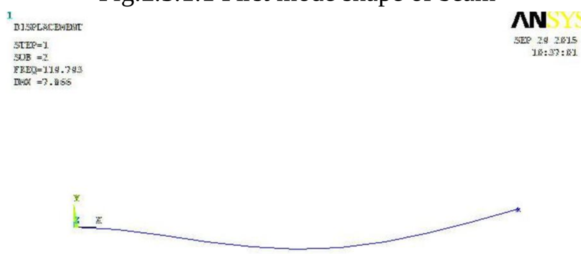

Fig. 2.3.1.2 Second mode shape of beam

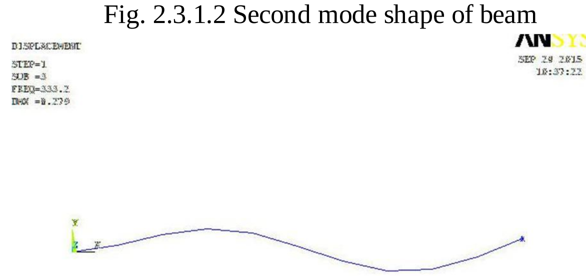

Fig. 2.3.1.3 Third mode shape of beam

2) Analysis of aluminium with first sample of end tip mass weighs .0426kg

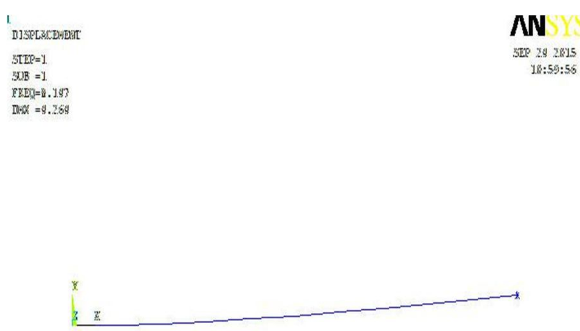

Fig. 2.3.2.1 First mode shape with first sample of end tip mass

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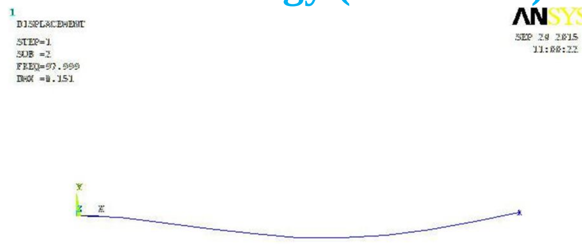

Fig.2.3.2.2Second mode shape with first sample of end tip mass

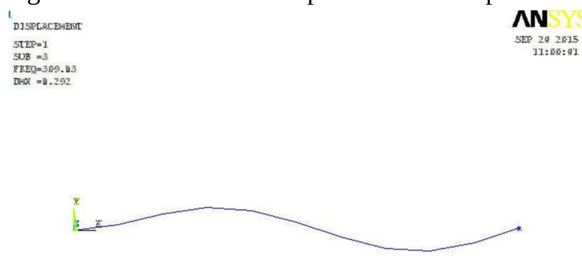

Fig. 2.3.2.3 Third mode shape with first sample of end tip mass

3) *Analysis of aluminium beam with second sample of end tip mass weighs .100kg*

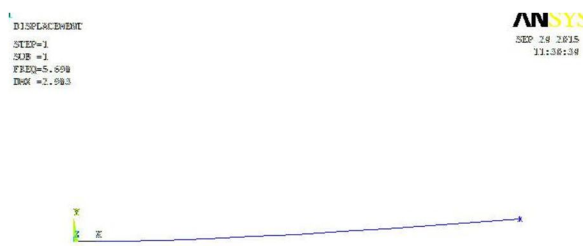

Fig.2.3.3.1First mode shape with second sample of end tip mass

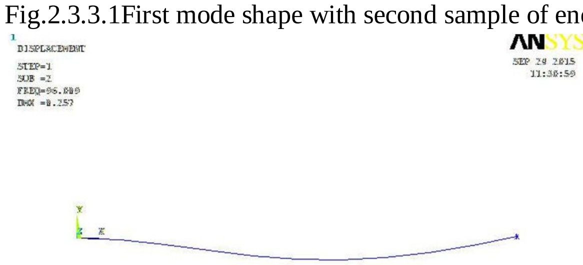

Fig.2.3.3.2Second mode shape with second sample of end tip mass

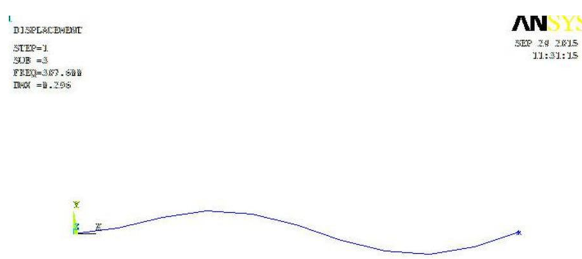

Fig. 2.3.3.3 Third mode shape with second sample of end tip mass

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D. Analysis of beams through experiment

All the material properties of the beam and the dimensions are measured in the lab. Fig 2.4.1 shows the experimental set up of a complete UTM machine in which the young modulus of the material is measured experimentally. Two slender, elastic beams of aluminum and iron are considered for frequency analysis. The lengths of beams are measured by ruler scale; whereas width and height are measured by electronic gauge. The masses of beams and mass of accelerometer are measured by the weighing machine

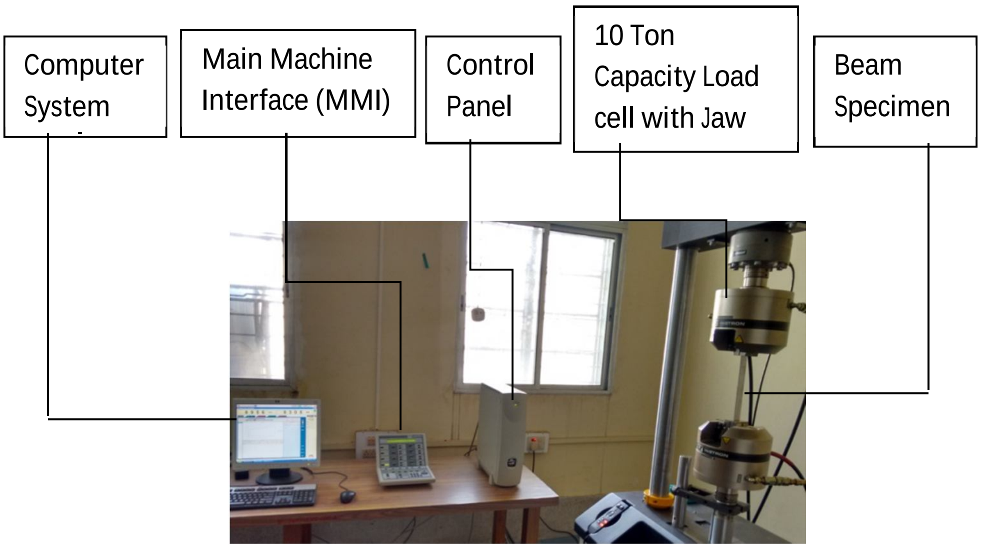

Fig. 2.4.1 UTM Machine with all accessories

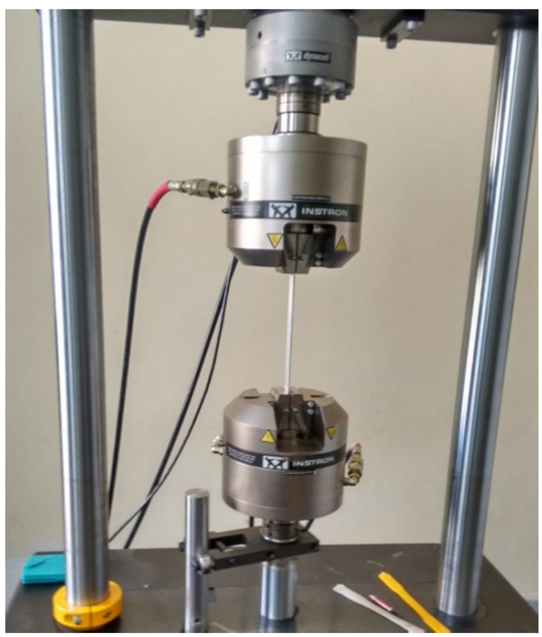

Fig. 2.4.2 Close view of specimen testing.

The main parts of UTM machine is shown in Fig 2.4.1. The beam specimen is fixed in between two jaw faces with the help of

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remote guide (Fig 2.4.2). The UTM machine applies force on specimen by two of its jaw tangentially. The power application of jaw is guided hydraulically by power pack and chiller oil pulling (not shown in fig.). The whole setup is controlled by control panel which give instruction to MMI which further connected with computer system to provide required value on its screen.

The fig.2.4.3 shows the experimental setup of for the analysis of natural frequency of beam with accelerometer sensor. The setup also consists of FFT analyzer with computer and its accessories. For further analysis of the effect of different masses on the natural frequency of cantilever beam, free end of cantilever is loaded with different masses along with accelerometer and the result of this experiment is observed on the computer screen for the aluminium and iron beams. The setup can be used to analyze any cantilever beams with different masses to measure their natural frequencies.

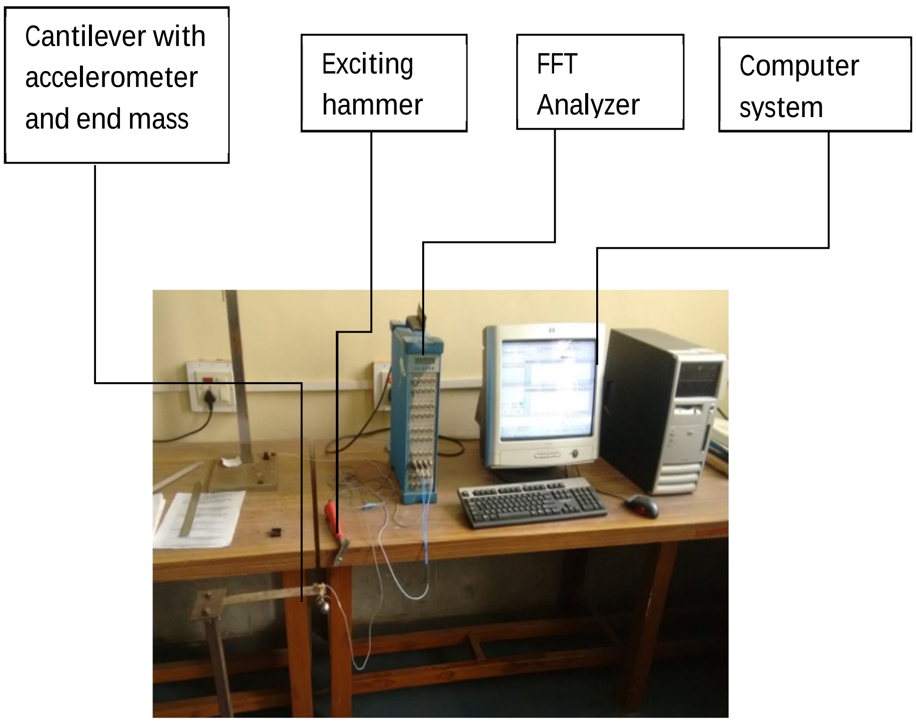

Figure2.4.3: Experimental set up

The output shown on computer screen after analysis with and without end tip masses for aluminium beam is shown below.

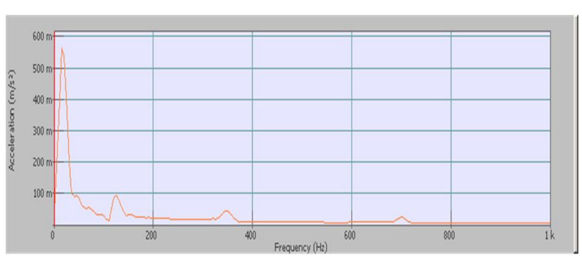

Fig.2.4.4: Output of FFT analyzer for aluminium beam with no tip mass attached at free end of cantilever beam

The same analysis is also done for iron beam with same experimental setup.

III. NUMERICAL STUDY

In this work iron and aluminium cantilever beams are considered for the analysis. The length of the iron cantilever beam is 240 mm

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and the cross section is 30X 1.16 mm^2. The length of the aluminium cantilever beam is 252 mm and the cross section is 24X 2.04 mm^2. All the dimensions and material properties of the beams are listed below in Table 1. The first three natural frequencies are calculated for the above mentioned beams using finite element analysis with and without the presences of the end mass. The finite element analysis is done using the developed codes in MATLAB and also by ANSYS software. Using the above mentioned experimental setup the natural frequencies of the beams are measured experimentally with and without the presence of the end mass. The analytical and the experimental results of the natural frequencies for the above mentioned beams are tabulated below. Table 2 to 7 shows the value of natural frequencies of Al and Fe beam obtained analytically and experimentally with and without end tip masses (mass of accelerometer is also considered).

TABLE I: DIFFERENT PARAMETERS OF BEAMS.

Parameters	Aluminum	Iron
Length (m)	0.252	0.240
Width (m)	0.024	0.030
Height (m)	0.00204	0.00116
Mass (kg)	0.0324	0.0645
Density (kg/m^3)	2632	7729
M.I.(m^4)	$1.67x10^{-11}$	$3.9x10^{-12}$
Modulus of Elasticity(N/m^2)	$45.25x10^9$	$103.5x10^9$

TABLE 2: NATURAL FREQUENCY OF AL WITHOUT END MASS.

Natural Frequency(Hz)	SDOF Analysis	MDOF		Experimental Analysis
		In House FEM code	ANSYS	
1st	17.193	16.7	17.096	17.5
2nd		113.8	114.743	115.7
3rd		331.1	333.2	332.7

TABLE 3: NATURAL FREQUENCY OF AL WITH FIRST SAMPLE OF END MASS

Natural Frequency (Hz)	SDOF Analysis	MDOF		Experimental Analysis
		In House FEM code	ANSYS	
1st	8.148	7.98	8.14	8.3
2nd		96.89	97.99	96.8
3rd		306.8	309.9	309.4

TABLE 4: NATURAL FREQUENCY OF AL WITH SECOND SAMPLE OF END MASS

Natural Frequency (Hz)	SDOF Analysis	MDOF		Experimental Analysis
		In House FEM code	ANSYS	
1st	5.7	5.43	5.698	5.59
2nd		94.8	96.089	95.8
3rd		304.8	307.68	306.9

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TABLE 5: NATURAL FREQUENCY OF FE WITHOUT END MASS.

Natural Frequency (Hz)	SDOF Analysis	MDOF		Experimental Analysis
		In House FEM code	ANSYS	
1st	10.6	9.96	10.47	10.5
2nd		66.3	67.216	66.9
3rd		190.2	191.257	192.5

TABLE 6: NATURAL FREQUENCY OF FE WITH FIRST SAMPLE OF END MASS

Natural Frequency (Hz)	SDOF Analysis	MDOF		Experimental Analysis
		In House FEM code	ANSYS	
1st	5.958	5.678	5.955	6.2
2nd		54.67	55.887	56.5
3rd		172.9	173.394	173.6

TABLE 7: NATURAL FREQUENCY OF FE WITH SECOND SAMPLE OF END MASS

Natural Frequency (Hz)	SDOF Analysis	MDOF		Experimental Analysis
		In House FEM code	ANSYS	
1st	4.32	4.09	4.302	4.45
2nd		53.4	54.027	53.8
3rd		169.8	171.2	170.9

IV. CONCLUSION

In this study the natural frequencies of the beams are measured experimentally. The finite element analysis of the beam is done using the developed codes in MATLAB and in ANSYS software. It is very much clear that there is a very close agreement between the experimental and analytical values. The accelerometer is used here to measure the vibration parameters. It is placed at the free end of the beam. The mass of the accelerometer is considered in the analysis and the effect of end tip mass on natural frequencies is also studied in this paper. It is observed experimentally as well as analytically that the higher values of end mass will lower the natural frequency of the beam.

V. ACKNOWLEDGMENTS

All the experiments of this study were conducted in the Central Instrumentation Facility (CIF) Lab of BIT, Mesra, Ranchi and the results obtained from those experimental set up were compared with the finite element analysis. The photographs of the experimental setup (fig.no.2.4.1, 2.4.2, 2.4.3) are taken in CIF Lab.

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