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# Special Rectangles and Narcissistic Numbers of Order 3 And 4

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**Abstract**— We search for infinitely many rectangles such that  $x^2 + y^2 + 3A - S^2 + k^2 + SK = \text{Narcissistic numbers of order 3 and 4}$  respectively, in which  $x, y$  represents the length and breadth of the rectangle.

Also the total number of rectangles satisfying the relation under consideration as well as primitive and non-primitive rectangles are also present.

**Keywords**—Rectangle, Narcissistic numbers of order 3 and 4, primitive, non-primitive.

## I. INTRODUCTION

The older term for number theory is arithmetic, which was superseded as number theory by early twentieth century. The first historical find of an arithmetical nature is a fragment of a table, the broken clay tablet containing a list of Pythagorean triples. Since then the finding continues.

For more ideas and interesting facts one can refer [1]. In [2] one can get ideas on pairs of rectangles dealing with non-zero integral pairs representing the length and breadth of rectangle. [3,4] has been studied for knowledge on rectangles in connection with perfect squares, Niven numbers and kepriker triples. [5-10] was referred for connections between Special rectangles and polygonal numbers, jarasandha numbers and dhuruva numbers

Recently in [11,12] special pythagorean triangles in connections with Narcissistic numbers are obtained.

In this communication, we search for infinitely many rectangles such that  $x^2 + y^2 + 3A - S^2 + k^2 + SK = \text{Narcissistic numbers of order 3 and 4}$  respectively, in which  $x, y$  represents the length and breadth of the rectangle.

Also the total number of rectangles satisfying the relation under consideration as well as primitive and non-primitive rectangles are also present.

## II. NOTATIONS

A-Area of the rectangle

S-Semi-perimeter of the rectangle

## III. BASIC DEFINITIONS

*Definition 1: Narcissistic Numbers*

An  $n$ -digit number which is the sum of  $n^{\text{th}}$  power of its digits is called an  $n$ -narcissistic number. It is also known as Armstrong number.

*Definition 2: Primitive Rectangle*

A rectangle is said to be primitive if the generators  $u, v$  are of opposite parity and  $\text{gcd}(u, v) = 1$ , where

$$x = u + v; y = u - v \quad \text{and} \quad u > v > 0$$

## IV. METHOD OF ANALYSIS

Let  $x, y$  be two non-zero distinct positive integers representing the length and breadth of a rectangle  $R$ . Let  $k \geq 0$  be any given integer.

The problem under consideration is to solve the equation

$$x^2 + y^2 + 3A - S^2 + k^2 + Sk = \text{Narcissistic Number} \quad (1)$$

To solve (1), let us introduce the linear transformation  $x = u + v$  and  $y = u - v$  ( $u > v > 0$ ) (2)

Therefore (1) reduces to

$$(u + k)^2 - v^2 = \text{Narcissistic Number} \quad (3)$$

Case 1:

Consider the 3<sup>rd</sup> order Narcissistic Number 153.

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Therefore (3) becomes,

$$(u + k)^2 - v^2 = 153$$

Applying the method of factorization, we have

u+k	13	27	77
v	4	24	76

From the above mentioned values, the following results are observed.

TABLE I

k	No.of Rectangles related to 153	Observations
0	3	2 rectangles are primitive and one is non-primitive.
1,2	2	For k = 1, both the rectangles are non-primitive. For k = 2, both the rectangles are primitive.
3-8	1	For k = 3,5,7,8, the rectangles are non- primitive. For k = 4,6, the rectangles are primitive.

Case2:

Consider the 3<sup>rd</sup> order Narcissistic Number 371.

Therefore (3) becomes,

$$(u + k)^2 - v^2 = 371$$

Applying the method of factorization, we have

u+k	30	186
v	23	185

From the above mentioned values, the following results are observed.

TABLE II

k	No.of Rectangles related to 371	Observations
0	2	Both the rectangles are primitive
1-6	1	For k = 1,3,5, the rectangles are non-primitive For k = 2,4,6, the rectangles are primitive.

Case3:

Consider the 3<sup>rd</sup> order Narcissistic Number 407.

Therefore (3) becomes,

$$(u + k)^2 - v^2 = 407$$

Applying the method of factorization, we have

u+k	24	204
v	13	203

From the above mentioned values, the following results are observed.

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TABLE III

k	No.of Rectangles related to 407	Observations
0	2	Both the rectangles are primitive
1-10	1	For k = 1,3,5,7,9, the rectangles are non-primitive For k = 2,4,6,8,10, the rectangles are primitive.

Case 4:

Consider the 4<sup>th</sup> order Narcissistic Number 8208.

Therefore (3) becomes,

$$(u + k)^2 - v^2 = 8208$$

Applying the method of factorization, we have

u+k	92	93	103	127	132	183	237	348	517	687	1028	2053
v	16	21	49	89	96	159	219	336	509	681	1024	2051

From the above mentioned values, the following results are observed.

TABLE IV

k	No.of Rectangles related to 8208	Observations
0,1	12	For k = 0 , all the rectangles are non-primitive For k = 1 , all the rectangles are primitive
2,3	11	For k = 2 , all the rectangles are non-primitive For k = 3 , there are 5 primitive and 6 non-primitive rectangles
4,5	10	For k = 4 , all the rectangles are non-primitive For k = 5 , there are 8 primitive and 2 non-primitive rectangles
6,7	9	For k = 6 , all the rectangles are non-primitive For k = 7 , all the rectangles are primitive
8-11	8	For k = 8,10 all the rectangles are non-primitive For k = 9 , there are 3 primitive and 5 non-primitive rectangles For k = 11 all the rectangles are primitive
12-17	7	For k = 12,14,16 all the rectangles are non-primitive For k = 13,17 all the rectangles are primitive For k = 15, there are 3 primitive and 4 non-primitive rectangles
18-23	6	For k = 18,20,22 all the rectangles are non-primitive For k = 19,23 there are 5 primitive and 1 non-primitive rectangles For k = 21, there are 3 primitive and 3 non-primitive rectangles
24-35	5	For k = 24,26,28,30,32,34 all the rectangles are non-primitive For k = 25,29,31,35 all the rectangles are primitive For k = 27, there are 3 primitive and 2 non-primitive rectangles For k = 33, there are 2 primitive and 3 non-primitive rectangles
36,37	4	For k = 36, all the rectangles are non-primitive For k = 37, there are 3 primitive and 1 non-primitive rectangles
38-53	3	For k = 38,40,42,44,46,48,50,52 all the rectangles are non-primitive For k = 41,43,49,53 all the rectangles are primitive

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		For $k = 39,45,47,51$ there are 2 primitive and 1 non-primitive rectangles
54-71	2	For $k = 54,56,58,60,62,64,66,68,70$ all the rectangles are non-primitive For $k = 55,59,61,67,71$ all the rectangles are primitive For $k = 57,63,65,69$ there are 1 primitive and 1 non-primitive rectangles
72-75	1	For $k = 72,74$ the rectangles is non-primitive For $k = 73,75$ the rectangles is primitive

### V. CONCLUSION

To conclude, one may search for the connections between the rectangles and Narcissistic numbers of higher order and other number patterns.

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