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Investigation of Thermal Boundary Layer of non-Newtonian Fluid Past over a Wedge

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Abstract—This paper investigates the problem of steady, laminar, two-dimensional boundary layer flow and heat transfer of an incompressible, viscous non-Newtonian fluid over a wedge. A non-Newtonian fluid represented by power law model. Similarity transformation method has been utilized to obtain the ordinary differential equations with associated boundary conditions from the partial differential equations. The transformed coupled ordinary differential equations are solved numerically. Numerical values of local Nusselt number are tabulated. The effects of various parameters on flow and the heat transfer characteristics are discussed numerically and presented graphically. Comparison of present study with the existing limiting solution is shown and examined.

Keywords— Boundary layer; non-Newtonian fluids; Power-law fluids; Wedge; Heat transfer; Nusselt number

I. Introduction

In recent years, the study of boundary layer flow of non-Newtonian fluid over a wedge has generated considerable interest for its numerous industrial and engineering applications such as the boundary layer along a liquid film, polymer processing and chemical engineering processes. Many fluids such as cosmetics and toiletries, paints, glues, multiphase mixtures, biological fluids and food items are non-Newtonian in nature (Andersson and Irgens [1], Bird et al. [2], Schowalter [3], Irvine and Karni [4], Postelnicu and Pop [5]).

Acrivos [6] examined the behavior of boundary layer flows for non-Newtonian fluids and after that; many related studies have been investigated. Astarita and Marrucci [7] and Bohme [8] proposed the mathematical models of non-Newtonian fluid for both steady and unsteady flows. The steady boundary layer flow of non-Newtonian power law fluid past a moving wedge on a flat plate was analyzed by Ishak et al. [9]. Magyari and Keller [10, 11] considered the problem of the thermal boundary layer of a moving surface. There are different non-Newtonian fluid models available in the literature. The Ostwald-de Waele model, i.e., power-law fluid model is most common type of such models (Bird et al. [2]).

Chen and Chen [12] studied similarity solutions for free convection of non-Newtonian fluids over vertical surfaces in porous media. Gorla and Kumari [13] have investigated non-similar solutions for mixed convection in non-Newtonian fluids along a wedge with variable surface temperature in porous medium. The problem of a steady stretching sheet foe power law fluid was studied by Andersson and Dandapat [14], Andersson et al. [15], Hassanien et al. [16], Chamkha [17], and Prasad et al. [18, 19].. Zhang et al. [20] analyzed the characteristics of the thermal boundary layer in Power law fluid flow on continuous moving surface. Moorthy and Senthilvadivu [21] discussed the effect of variable viscosity on free flow of non-Newtonian power law fluids along a vertical surface with thermal stratification. Surati and Timol [22] studied heat transfer in forced convection boundary layer flow of non-Newtonian fluids past a wedge.

This paper investigates the thermal boundary layer of power law fliud flow over a wedge. The numerical results of the resulting coupled ordinary differential equations are obtained using shooting technique under Matlab software. Results were given for velocity and temperature distributions, the coefficient of skin-friction and Nusselt numbers for various values of power law index, wedge parameter and different Prandlt numbers.

II. MATHEMATICAL FORMULATION

Considering a two-dimensional steady, laminar, incompressible fluid obeying the power-law model [2], flowing over a wedge with constant wall temperature, T_w in a stationary coordinate system.The governing equations of boundary layer flow are [23 and 24]

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \frac{1}{\rho}\frac{\tau_{xy}}{\partial y} \qquad (2)$$

Energy Equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

with the boundary conditions

At $y = 0$: $u = v = 0$ and $T = T_w$,

At $y \to \infty$: $u \to U(x) = Cx^m$ and $T = T_\infty$,

At $x = 0$: $u = U_\infty$ and $T = T_\infty$, (4)

where u and v are the respective velocity components in the x- and y-directions of the fluid flow, υ the kinematic viscosity of the fluid and U the reference velocity at the edge of boundary layer and is a function of x. $m = \frac{\beta}{(2\pi - \beta)}$ is the wedge parameter and β is the wedge angle.ρ is the density of fluid and α is the thermal diffusivity of the fluid, T the temperature in the vicinity of the wedge.

For power-law fluids the shear stress is defined as demonstrated by [1] and [26]

$$\tau_{xy} = K\left(\frac{\partial u}{\partial y}\right)^n \qquad (5)$$

where K is called the consistency coefficient and n is the power-law index n.

Thus, Equation (2) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n \qquad (6)$$

The following transformations are utilized to facilitate the solution of the governing equations [24]

$$\psi = \left(\frac{Kx}{\rho}\right)^{\frac{1}{n+1}} [U(x)]^{\left(\frac{2n-1}{n+1}\right)} f(\eta)] \qquad (7)$$

$$\eta = [\rho\, U(x)^{(2-n)}/Kx]^{\left(\frac{1}{n+1}\right)} y \qquad (8)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \qquad (9)$$

where η is the similarity variable and $f(\eta)$ and $\theta(\eta)$ are similarity dependent variables.

Now Equation (1) is satisfied automatically. Substituting Equations (7-9) into momentum equation (6) and energy

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equation (3) lead to the following ordinary differential equations:

$$f''' + \frac{2mn-m+1}{n(n+1)} f\,(f'')^{(2-n)} + \frac{m}{n}(1-f'^2)(f'')^{(1-n)} = 0 \qquad (10)$$

$$\theta'' + \lambda f Pr R \theta' = 0 (11)$$

where $\lambda = \frac{2mn-m+1}{n+1}$, $R = \frac{Re}{Re_{(n,x)}{}^{2/(n+1)}}$, $Re_{(n,x)} = \frac{x^n U^{2-n}}{\nu}$ is the generalized Reynolds number for non-Newtonian fluids and $Re = \frac{xU}{\nu}$ and $Pr = \frac{\rho \nu C_P}{k}$ are Reynolds number and Prandlt number, respectively.

The associated boundary conditions are:

At $\eta = 0$: $f = 0, f' = 0, \theta = 1$

At $\eta \to \infty$: $f' = 1, \theta = 0$ (12)

where primes denote differentiation with respect to η.

For Newtonian fluid, index of power law fluid *n*=1 and *m=0*, Equation (11) reduces to

$$\theta'' + \frac{1}{2} Pr(m+1) f \theta' = 0 (13)$$

The important physical quantity of interest is the Nusselt number which can be defined as

$$Nu_x = \frac{q_w x}{(T_0 - T_\infty)k} = -\theta'(0) Re_{(x,n)}{}^{\frac{1}{n+1}} (14)$$

III. Results And Discussion

All Numerical Numerical results are obtained to study the effect of the various values of the R, wedge parameter m, power law index n and Prandtl number Pr on dimensionless temperature and rate of heat transfer. Our present results have also been compared for Newtonian fluid (n = 1) with those of Bejan [27]. The comparison results are given in Table I. The numerical results for $-\theta'(0)$ are tabulated in Tables II-IV.

The temperature profiles are displayed in Figure 1-4 for prescribed values of m, n, Pr and R. We see that the temperature profiles decreases as the R, Pr and m increases but with the increase in n the temperature profiles are increases. Figure 5-7 depict the effects of the Prandtl number Pr, wedge parameter m and power law index n on Nusselt number. As Pr and m increases, the Nusselt number increase for a given n and it decreases as n increases.

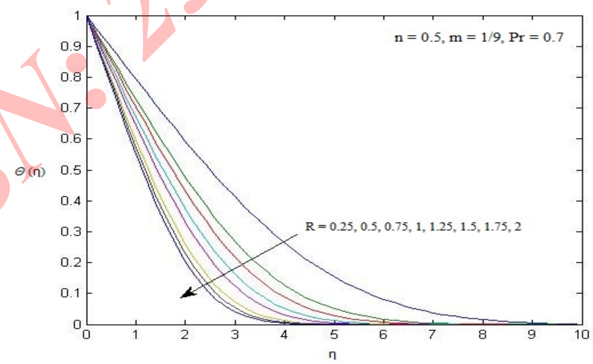

Figure 1. Temperature profiles for different values of R

when n = 0.5, m = 1/9 and Pr = 0.7

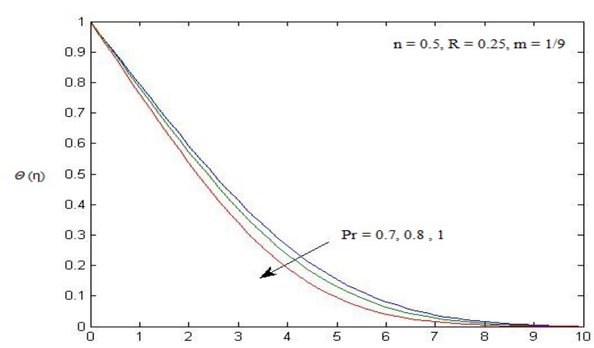

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Figure 2. Temperature profiles for different values of Pr

when n = 0.5, m = 1/9 and R = 0.25

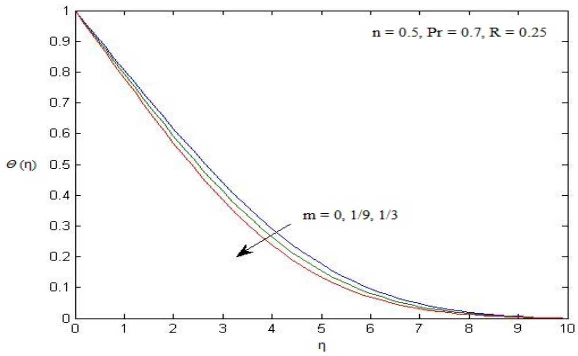

Figure 3. Temperature profiles for different values of m

when n = 0.5, Pr = 0.7 and R = 0.25

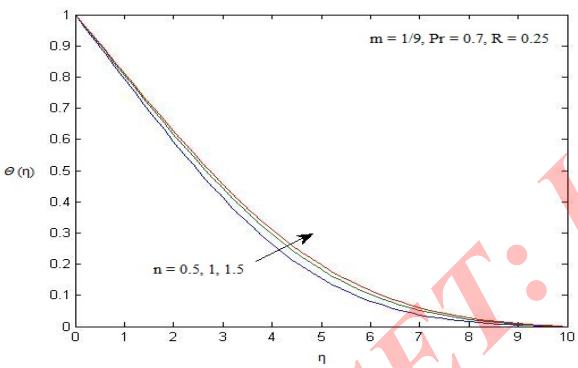

Figure 4. Temperature profiles for different values of n

when m = 1/9, Pr = 0.7 and R = 0.25

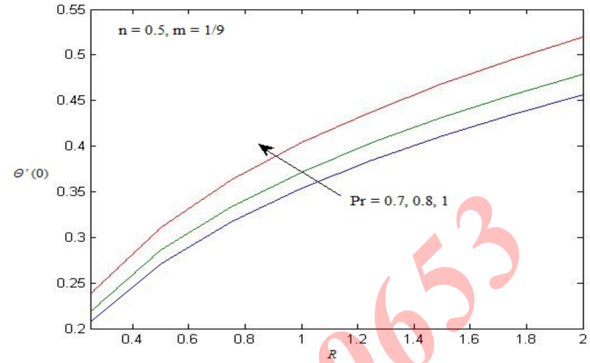

Figure 5. Variation of Local Nusselt number for different values of Pr

when n = 0.5 and m = 1/9

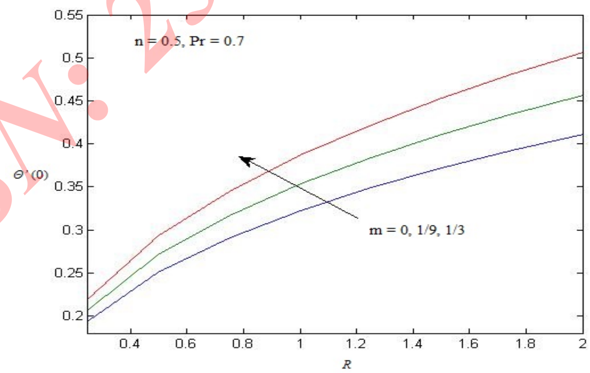

Figure 6. Variation of Local Nusselt number for different values of m

when n = 0.5 and Pr = 0.7

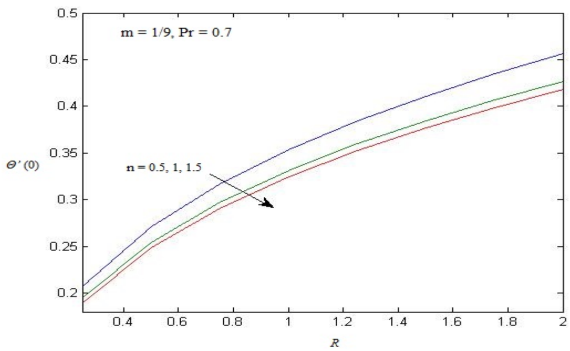

Figure 7. Variation of Local Nusselt number for different values of n

when m = 1/9 and Pr = 0.7

Table I. Comparison of the results of $-\theta'(0)$ for various values of wedge parameter m and Prandtl number Pr when $n = 1$ and $R = 1$

Table II. Values of $-\theta'(0)$ for different values of wedge parameter m and R when n = 0.5 and Pr = 0.7

R	$-\theta'(0)$		
	m = 0	**m = 1/9**	**m = 1/3**
0.25	0.1942	0.2071	0.2205
0.5	0.2511	0.2716	0.2933
0.75	0.2910	0.3173	0.3453
1.0	0.3226	0.3536	0.3870
1.25	0.3491	0.3843	0.4223
1.5	0.3722	0.4110	0.4532
1.75	0.3928	0.4349	0.4809
2.0	0.4115	0.4566	0.5061

Table III. Values of $-\theta'(0)$ for different values of Pr when n = 0.5 and m = 1/9

R	$-\theta'(0)$		
	Pr = 0.7	**Pr = 0.8**	**Pr = 1.0**
0.25	0.2071	0.2182	0.2382
0.5	0.2716	0.2860	0.3114
0.75	0.3173	0.3337	0.3628
1.0	0.3536	0.3717	0.4037
1.25	0.3843	0.4037	0.4381
1.5	0.4110	0.4316	0.4682
1.75	0.4349	0.4566	0.4950
2.0	0.4566	0.4793	0.5194

TABLE IV. VALUES OF $-\theta'(0)$ FOR $m = 1/9$ AND $Pr = 0.7$

R	$-\theta'(0)$		
	n = 0.5	**n = 1**	**n = 1.5**
0.25	0.2071	0.1946	0.1895
0.5	0.2716	0.2547	0.2486
0.75	0.3173	0.2973	0.2908
1.0	0.3536	0.3312	0.3243
1.25	0.3843	0.3597	0.3524
1.5	0.4110	0.3845	0.3769
1.75	0.4349	0.4067	0.3987
2.0	0.4566	0.4268	0.4185

m	$-\theta'(0)$					
	Pr = 0.7		**Pr = 0.8**		**Pr = 1.0**	
	Present results	**Bejan [27]**	**Present results**	**Bejan [27]**	**Present results**	**Bejan [27]**
0	0.2927	0.292	0.3069	0.307	0.3320	0.332
1/9	0.3312	0.331	0.3480	0.348	0.3778	0.378
1/3	0.3841	0.384	0.4043	0.403	0.4400	0.440

IV. CONCLUSIONS

In this paper, we have presented a thermal analysis of non-Newtonian fluids over a wedge. Numerical solutions using Runge-Kutta method with shooting technique were obtained for the temperature fields for several values of the wedge parameter *m*, Prandlt number *Pr* for temperature variation and power law index *n*. The main findings of the present analysis are as follows:

- As the wedge angle increases, the dimensionless temperature decreases and Nusselt number increases.
- The ratio of Reynolds number to the generalized Reynolds number leads to decrease the dimensionless temperature, while the rate of heat transfer at the wall increases.
- With increase in Prandtl number, the heat transfer rate increases and the dimensionless temperature decreases for both Newtonian and non-Newtonian fluids.
- Increase in power law index leads to increase of the dimensionless temperature and decrease of the heat transfer rate.

NOMENCLATURE

u fluid velocity component along x- direction within boundary layer

v fluid velocity component along y-direction within boundary layer

$U(x)$ free stream velocity

m wedge parameter

n index of power law fluid

dp/dx pressure gradient in x-direction

$Re_{(x,n)}$ generalized Reynolds number

C_{fx} generalizedlocal skin friction coefficient

$C(n)$ generalized shear stress coefficient

L characteristic length

f dimensionless stream function

ψ stream function

η streching function

τ_{xy} shear stress

K consistency coefficient

ρ density of the fluid

β wedge angle

τ_w wall shear stress

Subscripts

w wall conditions

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