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Strange Quark Matter Coupled to the String Cloud in Kaluza-Klein Theory of Gravitation

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Abstract: The solution are obtained for quark matter coupled to the string cloud in Kaluza-Klein theory of gravitation. The features of the obtained solutions are also discussed.
Keywords: Cosmology, higher dimensional space time, string cloud

I. INTRODUCTION

The idea that the world may have more than four dimensions (4D) is due to Kaluza (1921), who with a brilliant insight realized that a five dimensional (5D) manifold could be used to unify Einstein's theory of general relativity with Maxwell's theory of electromagnetism. Einstein endorsed the idea, but a major impetus was provided by Klein (1926). He made the connection to the quantum theory by assuming that the extra dimension was microscopically small, with a size infact connected via Plank's constant h to the magnitude of the electron charge e. Despite the elegance, though this version of Kaluza-Klein theory was largely eclipsed by the explosive development. First of wave mechanics and then of the quantum field theory. However, the development of particle physics led eventually to the resurgence of interest in higher dimensional field theories as a means of unifying the long-range and short range interactions of physics.

The idea of higher dimensional space time is particularly important in the field of cosmology. Since one knows that our universe was much smaller in its early stage that it is today. Indeed the present 4-dimensional stage of the universe could have been preceded by the higher dimensional stage which at later times become efectively four dimensional in the sence that the extra dimension become unobservably small due to dynamical contraction.

A good number of exact cosmological solutions of Einstein field equations with different equation of state and different symmetries, including or not a cosmological constatnt, has been found with 5D (Demarat and Hanquin (1985), Davidson and Vozmediano (1985)) and also with arbitrary number of dimension (Lorenz - Petzold (1984), (1985) and (1986)). Sahdev (1984), Emelyanov et al. (1986) and Chatterjee and Bhui (1993), have studies physics of the universe in higher dimensional space time.

Carmeli and Kuzmenko (2002) have shown that the cosmological reletivistic theory predicts the value for cosmological constant = 1:934 10 $^{-35}$ S^{-2}. This value Λ of is excellent in agreement with the measurements recently obtained by the High-z Supernovae Team and Supernovae Cosmological Project. The main conclusion of these observations is that the expansion of the universe is accelerating. Motivated by dimensional grounds with quantum cosmology, Chen and Wu (1990) have considered the variation of cosmological term as $\Lambda \propto R^{-2}$. However, a number of authors have argued in favor of the dependence $\Lambda \propto t^{-2}$. Berman (1990, 1991) has discussed the possibility of $\Lambda \propto t^{-2}$ by adding an additional term to the usual energy momentum tensor resulting in variable $\Lambda -$ term. In an attempt to modify the general theory of relativity, Al-Rawaf and Taha (1996), Al-Rwaf (1998) and Overduin and Cooperstock (1998) have proposed a model with $\Lambda = \beta\left(\frac{\ddot{R}}{R}\right)$, where β is constant. One of the motivations for introducing the $\Lambda -$term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

A gravitating object when it undergoes indefinite collapse, the end product is a singularity which is marked by the divergence of physical parameters like energy density. As the singularity is approached, the density diverges and it would there-fore be of relevance to consider the state of matter at ultra high density beyond the nuclear matter. One of such possible states could be strange quark matter which consist of u, d, and s quarks. It is the energetically most favored state of baryon matter. It could either be produced in the quark-hadron phase transition in the early Universe or at ultra high energy neutron stars converting into strange (matter) starts (Witten (1984)). In the context of gravitational collapse, which is our concern here, it is the latter process which

Volume 4 Issue X, October 2016
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would be pertinent.

The strange quark matter (SQM) fluid is characterized by the equation of state $p=(\rho-4B_c)/3$ where B_c is the bag constant indicating the difference between energy density of the perturbative and nonperturbative QCD vaccum, and ρ , p are the energy density and thermodynamic pressure of the quark matter (Witten (1984) and Weinberg (1986)). The fluid consists of zero mass particles with the QCD corrections for trace anomaly and perturbative iteractions Harko and Chang (2000). The boundary of a strange star is defined by $p\to 0$ which would imply $\rho \to B$. The typical value of the bag constant is of the order of $B_c \approx 10^{15}\, g\, cm^3$ while the energy density, $\rho \approx 5\times 10^{15}\, g\, cm^3$ (Witten, 1984)).

This shows that SQM will always satisfy the energy conditions because $\rho \geq p \geq 0$. We shall however consider the equation of state $p=(\rho-4B)/n$ as a generalization of SQM fluid, and particularly the cases $n=2, \to \infty$ corresponds to known cases of the Vaidya - de Sitter and Vaidya in constant potential both collapse respectively.

Typically strange quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark con-finement is described by an energy term proportional to the volume (Farhi and Jaffe (1984)).

In this model, quarks are though as degenerate Fermi gases, which exists only in a region of space endowed with a vacuum energy density B_c (called as the bag constant).

In the simplified version of the bag model, assuming quarks are mass less and non interacting, we then have quark pressure

$p_q = \dfrac{\rho_q}{3}$ (ρ_q is quark energy density); the total energy density is

$$\rho = \rho_q + B_c \tag{1}$$

but total pressure is

$$p = p_q - B_c \tag{2}$$

One therefore gets the equation of state for strange quark matter

$$p = \frac{1}{3}(\rho - 4B_c) \tag{3}$$

With this background in this paper we study the strange quark matter and quark matter coupled with string cloud in the presence of variable in five di-mensional Kaluza-Klein theory of gravitation. The paper is organized as follows:

In section 2 field equations and their solutions are obtained for strange quark matter for n=1. In Section 3 we obtained the solution for quark matter coupled with the string cloud. In section 4 concluding remarks are given.

II. FIELD EQUATIONS AND THEIR SOLUTIONS FOR STRANGE QUARK MATTER

We consider five-dimensional Kaluza-Klein model

$$ds^2 = -dt^2 + a^2(dx_1^2 + dx_2^2 + dx_3^2) + b^2dx_4^2; \tag{4}$$

where a and b are functions of t.

The energy momentum tensor for strange quark matter is given by

$$T_{ab} = \rho U_aU_b + p(g_{ab} + U_aU_b): \tag{5}$$

This is perfect fluid form of strange quark matter described by Eqs. (1) and (2). Here the five velocity u^a is time like vector such that $u^au_a = -1$. We use com-moving coordinate system $u^a = \delta_0^a$.

The Einstein's field equations with cosmological constant Λ can be written as

$$R_{ab} - \frac{1}{2}Rg_{ab} = -8\pi G T_{ab} - \Lambda g_{ab} \tag{6}$$

The Einstein's field equation (6) with energy momentum tensor (5) with the help of the metric (4) gives

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$$8\pi G\rho = 3\left(\frac{\dot{a}^2}{a^2} + \frac{\dot{a}\dot{b}}{ab}\right) + \Lambda \qquad (7)$$

$$8\pi G p = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - 2\frac{\dot{a}\dot{b}}{ab} - \frac{\ddot{b}}{b} - \Lambda \qquad (8)$$

$$8\pi G p = -3\left(\frac{\dot{a}^2}{a^2} + 3\frac{\ddot{a}}{a}\right) - \Lambda \qquad (9)$$

where dot (.) denotes differentiation with respects to t.

The above field Eqs. (7) - (9) have five unknowns ρ , Λ, G, a and b. Therefore, to obtain an exact solution we need one more relation. This leads to a polynomial relation between the metric coefficients

$$b = \mu a^n; \qquad (10)$$

where μ and n are constants.

By using Eq. (10) in above field Eqs. (7)-(9) we get

$$8\pi G\rho = 3(n+1)\frac{\dot{a}^2}{a^2} + \Lambda \qquad (11)$$

$$8\pi G p = -\left((n^2+n+1)\frac{\dot{a}^2}{a^2} + (n+2)\frac{\ddot{a}}{a}\right) - \Lambda \qquad (12)$$

$$8\pi G\rho = -3\left(\frac{\dot{a}^2}{a^2} + 3\frac{\ddot{a}}{a}\right) - \Lambda \qquad (13)$$

If we subtract Eq. (12) from (13) we get

$$(n-1)\left(\frac{\ddot{a}}{a} + (n+2)\frac{\dot{a}^2}{a^2}\right) = 0 \qquad (14)$$

The solution of above equation is either

$$\frac{\ddot{a}}{a} + (n+2)\frac{\dot{a}^2}{a^2} = 0 \qquad (15)$$

or

$$(n-1) = 0 \qquad (16)$$

The solution of Eq. (15) is already discussed by Yilmaz (2006). In this work we discussed only the case (n = 1) was ignored by Yilmaz (2006).

For n = 1 Eq. (10) reduced to the form

$$\frac{\dot{b}}{b} = \frac{\dot{a}}{a} \qquad (17)$$

From the above field equations (11)-(13) we get

$$\dot{\rho} + \left(\frac{3\dot{a}}{a} + \frac{\dot{b}}{b}\right)(p+\rho) = -\left(\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G}\right). \qquad (18)$$

Equation (18) can be split to give

$$\dot{\rho} + \left(\frac{3\dot{a}}{a} + \frac{\dot{b}}{b}\right)(p+\rho) = 0. \qquad (19)$$

and

Volume 4 Issue X, October 2016
ISSN: 2321-9653

$$\dot{G}+\frac{\dot{\Lambda}}{8\pi\rho}=0. \tag{20}$$

Put (n=1) in Eq. (11)-(13) we get

$$8\pi G\rho=6\frac{\dot{a}^2}{a^2}+\Lambda, \tag{21}$$

$$8\pi Gp=-3\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right)-\Lambda \tag{22}$$

From Eq. (21) and Eq. (22) we get

$$\frac{\dot{a}^2}{a^2}=\frac{8\pi G\rho-\Lambda}{6} \tag{23}$$

$$\frac{\ddot{a}}{a}=\frac{-4\pi G}{3}\left(2p+\rho+\frac{\Lambda}{8\pi G}\right) \tag{24}$$

Eq. (23) and (24) can be rewritten in terms of Hubbel parameter $H=\frac{\dot{a}}{a}$ to give, respectively

$$H^2=\frac{8\pi G\rho-\Lambda}{6} \tag{25}$$

$$H+H^2=\frac{-4\pi Ga}{3}\left(2p+\rho+\frac{\Lambda}{8\pi G}\right) \tag{26}$$

The system of Eqs. (20), (25) and (26) may be solved with the additional explicit assumption on h, G(t) and $\Lambda(t)$ in terms of t or H which itself depends on cosmic time t.

Eq. (26) for the equation of state of strange quark matter (3), gives

$$H+H^2=\frac{-20\pi G\rho}{9}-\frac{\Lambda}{6}+\frac{32\pi GB_c}{9} \tag{27}$$

Eliminating ρ between Eqs, (20) and (27), we get

$$H+H^2=\frac{-20\pi G}{9}\left(\frac{\Lambda}{8\pi G}\right)-\frac{\Lambda}{6}+\frac{32\pi GB_c}{9} \tag{28}$$

Eq. (28) can be written as

$$HH'+\frac{H^2}{a}=\frac{5GA}{18G'A}+\frac{32\pi GB_c}{9a}+\frac{\Lambda}{6a} \tag{29}$$

where prime (') denotes the derivative with respect to a.

We obtain the solution of Eq. (29) by taking the following assumption on G and Λ.

We assume that

$$G(t)=\alpha H \tag{30}$$

$$\Lambda(t)=\beta H^2 \tag{31}$$

Where α and β are dimensionless positive constants.

By using the values of $G(t)$ and $\Lambda(t)$, Eq. (29) gives

$$H'=\frac{H}{a}\left(\frac{7\beta-18}{18}\right)+\frac{32\pi\alpha B_c}{9a} \tag{32}$$

Integrating Eq. (32), we get

$$H = P_3 B_c + C_0 a^{P_1} \tag{33}$$

where $P_1 = \dfrac{7\beta - 18}{18}, P_3 = \dfrac{-64\pi\alpha}{7\beta - 18}$ and C_0 is constant of integration.

From Eq. (33) we get

$$a(t) = \left(\frac{B_c}{P_4 e^{(P_2 B_c)t} - P_5} \right)^{\frac{1}{P_1}} \tag{34}$$

From Eq. (10) and (34), we get

$$b(t) = \mu \left(\frac{B_c}{P_4 e^{(P_2 B_c)t} - P_5} \right)^{\frac{1}{P_1}} \tag{35}$$

Now substituting values of a in Eq. (21), we get

$$\rho = \left(\frac{P_6 B_c}{P_4 - P_5 e^{(-P_2 B_c)t}} \right), \tag{36}$$

From Eqs. (3) and (36) we get

$$p = \frac{B_c}{3} \left(\frac{P_6}{P_4 - P_5 e^{(-P_2 B_c)t}} - 4 \right) \tag{37}$$

By substituting values of ρ and p in Eqs. (1) and (2) we get

$$\rho_q = B_c \left(\frac{P_6}{P_4 - P_5 e^{(-P_2 B_c)t}} - 1 \right), \tag{38}$$

$$p_q = \frac{B_c}{3} \left(\frac{P_6}{P_4 - P_5 e^{(-P_2 B_c)t}} - 1 \right), \tag{39}$$

Where $P_4 = \dfrac{-C_1(7\beta - 18)}{64\pi\alpha}, P_5 = \dfrac{C_0(7\beta - 18)}{64\pi\alpha}$ *and* $P_6 = \dfrac{C_1(6+\beta)}{8\pi\alpha}$

III. FIELD EQUATIONS AND THEIR SOLUTIONS FOR THE STRING CLOUD WITH QUARK MATTER

In this section we study quark matter coupled with string cloud in the presence of $\Lambda \propto H^2$.

The energy momentum tensor for string cloud of Letelier (1979) is given by

$$T_{ab} = \rho \mu_a \mu_b - \lambda x_a x_b, \tag{40}$$

Where ρ is the rest energy for the cloud of strings with particles attached to them and λ is string tension density; they are related by

$$\rho = \rho_p + \lambda \text{ or } \rho_p = \rho - \lambda \tag{41}$$

Here we take quark instead of string cloud.

$$\rho = \rho_p + \lambda + B_c \text{ or } \rho_q + B_c = \rho - \lambda \tag{42}$$

If we put Eq. (42) into Eq. (40), then energy-momentum tensor for strange quark matter attached to the string cloud can be written as

$$T_{ab}=\left(\rho_q+\lambda+B_c\right)\mu_a\mu_b-\lambda x_a x_b \qquad (43)$$

where the five velocity μ^a is five dimensional timelike vector such that $\mu^a\mu_a=-1$, x^a is the unit spacelike vector such that $x^a x_a = 1$ in the direction $x^a=\delta_4^a$ which represent the strings directions in the cloud. i.e. the direction of anisotropy.

Using commoving coordinate system $\mu^a=\delta_0^a$.

The Einstein's field equation (6) with energy momentum tensor (40) with the help of the metric (4) gives

$$3\left(\frac{\dot a^2}{a^2}+\frac{\dot a\dot b}{ab}\right)=\rho-\Lambda, \qquad (44)$$

$$2\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}-2\frac{\dot a\dot b}{ab}-\frac{\ddot b}{b}=-\Lambda, \qquad (45)$$

$$3\left(\frac{\dot a^2}{a^2}+3\frac{\ddot a}{a}\right)=\lambda-\Lambda\,. \qquad (46)$$

Field equations (44) - (46) with $\Lambda=3\beta\dfrac{\dot a^2}{a^2}$ can be written as

$$3(1+\beta)\frac{\dot a^2}{a^2}+3\frac{\dot a\dot b}{ab}=\rho, \qquad (47)$$

$$2\frac{\ddot a}{a}+(1+3\beta)\frac{\dot a^2}{a^2}+2\frac{\dot a\dot b}{ab}+\frac{\ddot b}{b}=0, \qquad (48)$$

$$3(1+\beta)\frac{\dot a^2}{a^2}+3\frac{\ddot a}{a}=\lambda\,. \qquad (49)$$

We use geometrized units so that $8\pi G=c=1$.

The physical variables, namely the expansion and shear scalar, have the following form

$$\theta=3\frac{\dot a}{a}+\frac{\dot b}{b}, \qquad (50)$$

$$\sigma^2=\frac{3}{8}\left(\frac{\dot a}{a}-\frac{\dot b}{b}\right)^2 \qquad (51)$$

If we substitute Eq. (10) into Eq. (48), we have

$$(n+2)\frac{\ddot a}{a}+\left(n^2+n+1+3\beta\right)\frac{\dot a^2}{a^2}=0. \qquad (52)$$

Integrating Eq. (52), we obtain

$$a=\left(At+B\right)^{A_0} \qquad (53)$$

Where $A_0=\dfrac{n+2}{n^2+2n+3+3\beta}$ and A and B are the constants of integration.

From Eq. (10) and (48), we have

$$b=\mu(\mathrm{At+B})^{nA_0} \qquad (54)$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

After substituting the values of a and b in Eqs. (47), (49), (50) and (51) we get

$$\rho = \frac{A_1 (n+2)(1+n+\beta)}{(\mathrm{At}+\mathrm{B})^2}, \tag{55}$$

$$\lambda = \frac{A_1\left[1-n^2+\beta(n-1)\right]}{(\mathrm{At}+\mathrm{B})^2} \tag{56}$$

$$\rho_q + B_c = \rho_q = \rho - \lambda = \frac{A_1\left[2n^2+3n+1-3\beta\right]}{(\mathrm{At}+\mathrm{B})^2} \tag{57}$$

$$\theta = \frac{A(n+2)(n+3)}{(\mathrm{n}^2+2\,\mathrm{n}+3+3\beta)(\mathrm{At}+B)} \tag{58}$$

$$\sigma^2 = \frac{3}{8}\left[\frac{A^2 (n+2)^2 (1-n)^2}{(\mathrm{n}^2+2\,\mathrm{n}+3+3\beta^2)(\mathrm{At}+\mathrm{B})^2}\right] \tag{59}$$

Where $A_1 = \dfrac{\left[3A^2(n+2)\right]}{(n^2+2\,\mathrm{n}+3+3\beta)^2}$

IV. CONCLUSION

In this paper we have obtained the exact solution of the strange quark matter coupled with string cloud in the frame work of Kaluza-Klein theory of gravitation. In the first part of the paper we have obtained the solution for strange quark matter for (n = 1) which was ignored by Yilmaz (2006) by assuming G ⊔ H and Λ ⎕ H^2. It is observed that the scale factor a and b both are exponential functions of t and energy density is always positive. It is also observed that from Eq. (38) and (39) we get $P_q = \dfrac{\rho_q}{3}$ as [roposed by Bodmer (1971) and Witten (1984).

In the second part of the paper if $0 < n < 1$ we get $\rho_p \geq \lambda$. For (n=1) we get dust quark matter solution i.e. $\lambda = 0$ and $\rho_q + B_c = \rho_p = \rho = \dfrac{2A_1(2-\beta)}{\left(At+B\right)^2}$. For $n = \dfrac{-3 \pm \sqrt{1+24\beta}}{4}$ the matter disappears and we get geometric string solution i.e. $\lambda = \rho$ and $\rho_p = 0$.

For the five-dimensional model, we note that universe starts at an initial epoch $t = \dfrac{-B}{A}$. At $t = \dfrac{-B}{A}$ the physical parameters θ and σ^2 diverges. As t gradually increases θ and σ^2 decreases, and finally they vanish when $t \to \infty$

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Volume 4 Issue X, October 2016
ISSN: 2321-9653

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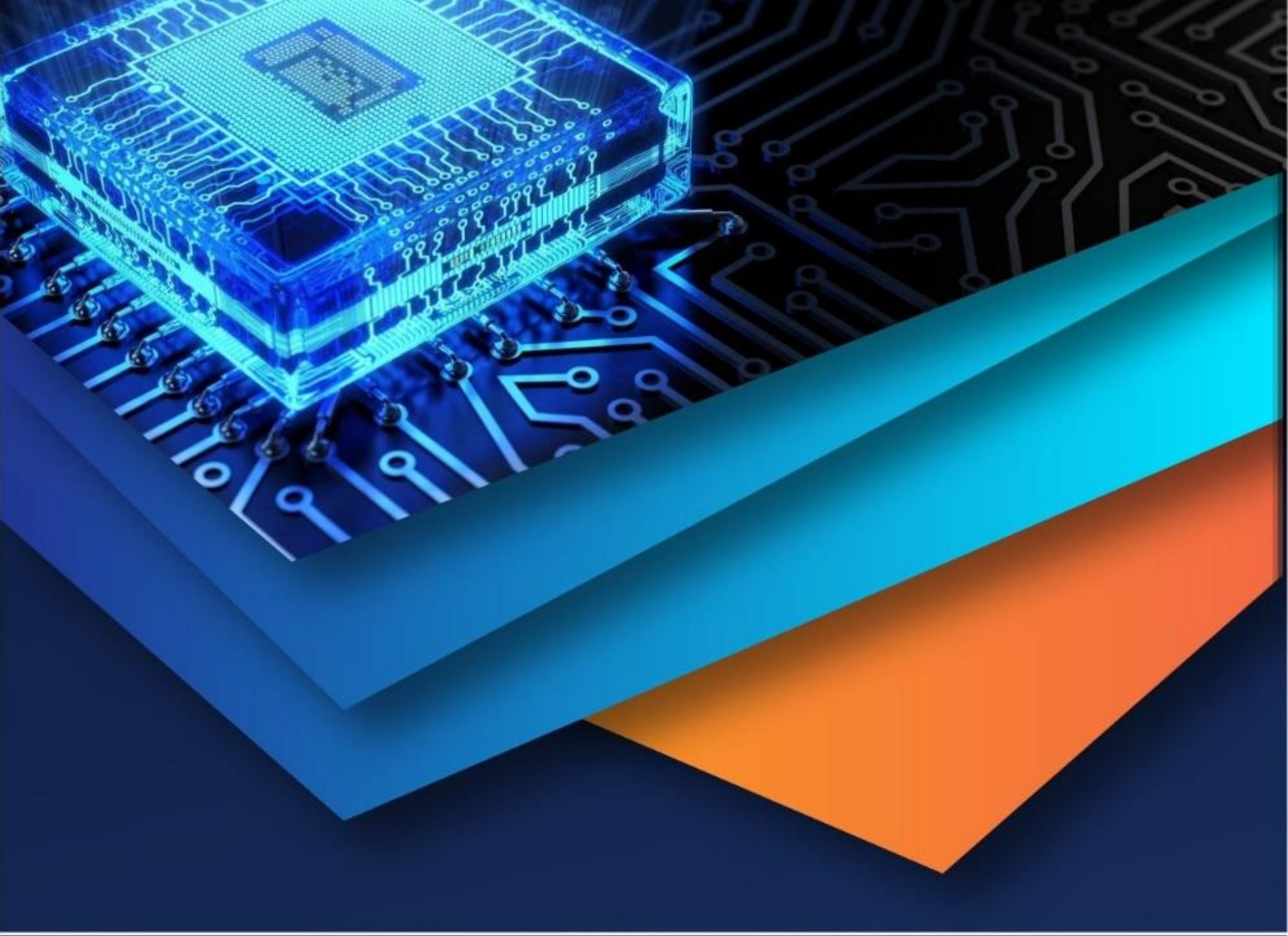

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