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A New Bio Mathematical Model to Find the Age Specific Coronary Heart Disease Mortality Rates Using Takacs Formula

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Abstract: The gender difference (gender gap) in mortality due to coronary heart disease (CHD) decreases with age. This relationship has not been well characterized in diverse populations. To examine the gender gap in coronary heart disease (CHD) mortality across age groups, we examine a doubly controlled process of servicing machines. The classical system treated by Takacs is equipped with $m + 1$ unreliable machines served by one repairman. In the present modification of this model, the failure rates and the repair time may be controlled with respect to the state of the system. The process describing the number of intact machines is considered. To derive its steady state distribution in the form of a simple explicit formula, the author introduces an auxiliary model with m unreliable machines and a single repairman who keeps working even when all machines are intact.

Key Words: Coronary Heart Disease, Takacs Formula, Duality Principle & Gamma Distribution

I. INTRODUCTION

Although Coronary Heart Disease (CHD) is the leading cause of death in men and women, age-specific CHD mortality rates are strikingly higher in men compared with women.1 In general, both CHD incidence and mortality rates in women lag 10 years behind those of men [2] & [12]. It is well established that the gender difference is more pronounced at younger ages, such that 1 in 17 women has had a coronary event before age 60, in contrast with 1 in 5 men. The gender difference has been reported to decrease with age, and after age 60, CHD accounts for 1 in 4 deaths in both sexes [4-9].

A multi-channel loss queueing system with control of input stream and service. The servicing facility of the system contained m parallel channels processing a stream of singly arriving customers. No customer was accepted when the servicing facility was occupied. The input stream and service were subject to a comprehensive control. This model can also be interpreted as a system of m unreliable machines served by a single repairman with corresponding service and failure rates control. More specifically, each of the working machines can break down with a rate dependent on the total number of intact machines. The repair time also depends upon the number of intact machines.

The repairman is not idle even when all machines become intact. At these times the repairman leaves the system and comes back later with a new machine that immediately replaces an available defective machine. If no machine breaks down during the repairman's absence, a substitution takes place at the repairman's own choice, but the number of working machines does not change and this action is supposed not to affect the future status of the system. Since the repairman leaves the system to get new equipment, we can restrict his absence to a definite time. So the control can particularly be applied to this situation [3].

A. *Model Description*

1) *Model 1:* The system consists of $m + 1$ unreliable machines served by a single repairman. Denote by Z_t^1 the number of intact machines at time $t \geq 0$. When the repair of a last defective machine is completed and the total number of intact machines becomes $m + 1$, the repairman is idle until the next breakdown. Let $\tau_1, \tau_2, \dots$ be the successive instants of the completion of machine repairs. The repair time of the nth machine is distributed in accordance with $A\,\xi_n(x) \in \{A_k(x); k = 0,1, \dots, m\}$ (a tuple of arbitrary $d.f.$'s), where $\xi_n := Z_{\tau_n}^1, n = 1,2, \dots$. That is, the repair time depends upon the number of intact machines at the moment immediately before the completion of the preceding service. The working machines perform certain jobs. Within the interval $[\tau_n, \tau_{n+1})$ the continuous durations of each job are conditionally independent given ξ_n and exponentially distributed

with parameter $\mu_{\xi_n} \in \{\mu_k, k = 0,1,\dots,m+w\} \subset \mathbb{R}_+ \setminus \{0\}$.

2) *Model 2:* There are a maximum of m working unreliable machines and one repairman. The total number of intact machines at time $t \geq 0$ is denoted by Z_t. Unlike Model 1, there are no idle periods. At certain epochs of time the repairman temporarily abandons the system to acquire a new machine. Let T_n be an epoch when a machine is completely repaired. If at this time the total number of intact machines is m, the repairman leaves the system and returns to the system at time T_{n+1} with a new machine to replace any machine that has failed during the repairman's absence. In other words, if the repairman returns to fewer than m working machines, then the number of working machines increases by one. If no machine has failed to this time, a replacement still occurs but at the repairman's own choice (in this case, without any effect on the system). In both cases used machines arc removed from the system. Consequently, $T_1, T_2, \dots$ are the moments when the repairman completes a job (repair of a machine or acquisition of new equipment). At time T_n he begins with the repair of a current machine or leaves the system if no defective machine is available. At the time T_{n+1} he completes the repair or returns to the system with a new machine. The length of the interval $[T_n, T_{n+1})$ is distributed according to the $d.f.$

$$A_{X_n}(x) \in \{A_k(x); k = 0,1,\dots,m\} \tag{1}$$

Where

$$X_n := Z_{T_n}, n = 0,1,\dots \tag{2}$$

For instance, control can be applied to the period of the repairman's absence distribute differently from the repair time. The assumption about the failure rates is as in Model 1. Namely within the interval $[T_n, T_{n+1})$ the continuous durations of each job are conditionally independent given X_n and exponentially distributed with parameter

$$\mu_{x_n} \in \{\mu_k, k = 0,1,\dots,m+w\} \subset \mathbb{R}_+ \setminus \{0\} \tag{3}$$

As mentioned, this model is identical to the doubly controlled m −channel loss queueing system studied by the author [13], where Z_t denotes the number of customers in the system at time t. To explain equivalence between both systems, we use the mutual notation below. At time T_n a customer departs from a source and at time T_{n+1} arrives at the system. The customer is served by one of the free parallel channels available, or is lost by the system if the servicing facility is busy. The length of the interval $[T_n, T_{n+1})$ is distributed as in (1) and (2), and the servicing policy is determined by (3). Within the interval $[T_n, T_{n+1})$ the service durations of customers in each of the channels are conditionally independent given X_n and exponentially distributed with parameter μ_{X_n}.

3) *Connection between the Models:* It can be shown that $\tau_1, \tau_2, \dots$ is a sequence of stopping times relative to the canonic filtering $\sigma(Z_u^1; u \leq t)$, that $T_1, T_2, \dots$ is a sequence of stopping times relative to $\sigma(Z_u; u \leq t)$, and that the processes $\left(\Omega_1, \mathfrak{U}_1, (P^x)_{x\in E_1}, (Z_t^1; t \geq 0)\right) \to E_1 = \{0,1,\dots,m+1\}$ and $\left(\Omega, \mathfrak{U}, (P^x)_{x\in E}, (Z_t; t \geq 0)\right) \to E = \{0,1,\dots,m\}$ are semi-regenerative relative to these sequences ($cf.$ definition in [13]). Consequently, $\left(\Omega_1, \mathfrak{U}_1, (P^x)_{x\in E}, (\xi_n; n = 1,2,\dots)\right) \to E$ and $\left(\Omega, \mathfrak{U}, (P^x)_{x\in E}, (X_n; n = 1,2,\dots)\right) \to E$ are embedded Markov chains (MC). Since the idleness of the repairman in the first model is distributed exponentially, it is easy to see that both MC's are stochastically equivalent and they are obviously ergodic. Let $\left(\Omega_1, \mathfrak{U}_1, (P^x)_{x\in E}, (Y_t^1; t \geq 0)\right) \to E_1$ and $\left(\Omega, \mathfrak{U}, (P^x)_{x\in E}, (Y_t; t \geq 0)\right) \to E$ be the semi-Markov processes associated with the sequences of stopping times above. Both are ergodic and their limiting probabilities are expressed through the invariant probability measure P of the MC(X_n) or the MC(ξ_n). (Since (X_n) and (ξ_n) are stochastically equivalent, only one of them, say (X_n), will be mentioned further.) Next we need the limiting probabilities

$$y_k^1 := \lim_{t\to\infty} P^x\{Y_t^1 = k\} = \frac{P_k M_k}{PM}, k \in E \tag{4}$$

(cf [1]) where M_k can be easily derived as

$$M_k := E^k[T_1] = \begin{cases} a_k & , k = 0,1,\dots,m-1 \\ a + \frac{1}{\mu(m+1)}, & k = m \end{cases} \tag{5}$$

and PM is the scalar product of P and $M = (M_0, M_1, \dots, M_m)^T$ which can be expressed the formula

$$= PA + P_m \frac{1}{\mu(m+1)}. \tag{6}$$

The duality principle between Models 1 and 2 is based on the following consideration. Let $\mathcal{B}_n$ and $\mathcal{I}_n$ denote the nth busy period and the idle period following the nth busy period, respectively, in Model 1. Let $\mathcal{C}_t; t \geq 0$ be the counting process associated with the point process $\{\mathcal{B}_n; n = 1,2,\dots\}$. It is readily seen that the processes Z_t and Z_t^1 during their busy periods are stochastically equivalent or formally $P^x\{Z_t = k\} = P^x\{Z_t^1 = k / \mathcal{I}_{\mathcal{C}_t} > t\}, k \in E$, where the probability $P^x\{Z_t^1 = k / \mathcal{I}_{\mathcal{C}_t} > t\}$ can be expressed as

$$P^x\{Z_t^1 = k / \mathcal{J}_{\mathcal{C}_t} > t\} = \frac{P^x\{Z_t^1 = k,\ Z_t^1 \in \{0,1,\dots,m\}\}}{P^x\{Z_t^1 \in \{0,1,\dots,m\}\}}$$

$$= \begin{cases} 0 & , k = m+1 \\ \dfrac{P^x\{Z_t^1 = k\}}{1 - P^x\{Z_t^1 = m+w+1\}} & , k \le m \end{cases}.$$

Therefore,

$$P^x\{Z_t^1 = k\} = [1 - P^x\{Z_t^1 = m+1\}] P^x\{Z_t = k\}, k = 0,1,\dots,m. \quad (7)$$

We now find $P^x\{Z_t^1 = m+1\}$.

$$P^x\{Z_t^1 = m+1\} = P^x\{Y_t^1 = m\} P^x\{\mathcal{J}_{\mathcal{C}_t} \le t / Y_t^1 = m\} \quad (8)$$

Where

$$\lim_{t\to\infty} P^x\{\mathcal{J}_{\mathcal{C}_t} \le t / Y_t^1 = m\} = \frac{1}{1 + a\mu(m+1)}. \quad (9)$$

Let $\pi_k^1 := \lim_{t\to\infty} P^x\{Z_t^1 = k\}$. Then from (4) - (6), (8) and (9) it follows that

$$\pi_{m+1}^1 = \frac{P_m}{PA\mu(m+1) + P_m}. \quad (10)$$

Finally, (7) and (10) yield

$$\pi_k^1 = (1 - \pi_{m+1}^1)\pi_k, k = 0,1,\dots,m \quad (11)$$

Where $\pi_k = \lim_{t\to\infty} P^x\{Z_t = k\}$ was obtained in [13].

4) Model Examples

(i) Recall that in case of Model 2 the repairman leaves the system when all machines are intact. However, the repairman may plan to leave for only a short duration. Assume his expected absence is $= a_m \le \frac{1}{\mu\, m}$. Here both a and μ can be adjusted if necessary. For example,

$$\mu_j = \begin{cases} \mu_0, j = 0,1,\dots,m-1 \\ \mu, \quad\quad j = m \end{cases}$$

While $A_j(x)$ is subject to no restriction.

(ii) An undesirable situation occurs if during the repairman's absence the number of working machines falls below the level $m - r, r = 0,1,\dots,m-1$. We calculate the probability of this event as

$$\gamma_r := \lim_{t\to\infty} P^x\{Y_t = m, Z_t < m - r\} = \sum_{n=0}^{m} \pi_{m,n} - \sum_{n=m-r}^{m} \pi_{m,n}. \quad (12)$$

To derive γ_r we observe that the first sum above is

$$y_m^2 := \lim_{t\to\infty} P^x\{Y_t = m\} = \frac{aP_m}{PA} \quad (13)$$

($cf.$ the similar formula (4))

Also from [13] one can derive by the summation of the equations

$$PAn\,\pi_{jn} = \frac{1}{\mu_j} \sum_{k=0}^{n-1} P_j\, p_{jk}\ , 1 \le n \le \min\{j+1, m\}, j \in E. \quad (14)$$

Therefore, from (12) - (14) and [13] we have

$$\gamma_r = \frac{P_m}{PA}\left[a - \frac{1}{m\mu} \sum_{n=m-r}^{m} \sum_{k=0}^{n-1} P_{m,k}\right]\mathrm{T},$$

(iii) Now we return to the relation between the models. Formulas (10), (11) and [13] can be combined to obtain the corresponding stationary probabilities π_k^1 . An elegant expression follows when $\mu_j = \mu, j \in E$: (15)

$$\pi_k^1 = \frac{(m+1)P_{k-1}}{k[PA\,\mu(m+1) + P_m]}, k = 1,\dots,m+1$$

And finally, when $PA = a (i.e. a_j = a)$, (15) implies Takacs' well known formula [13].

$$\pi_k^1 = \frac{(m+1)P_{k-1}}{k[a\mu(m+1) + P_m]}, k = 1,\dots,m+1 \quad (16)$$

Where the $d.f.$'s $A_j(x)$ are still arbitrary and possibly distinct and thus (16) holds under more general conditions.

5) *Example:* To examine the gender gap in CHD mortality across age groups and to compare the age dependency of the gender gap between blacks and whites, we conducted a prospective cohort study. Baseline examinations were performed between 1958 and 1990, and mean follow up was 13.7 years in general communities in several U.S geographic areas. We included 39,614

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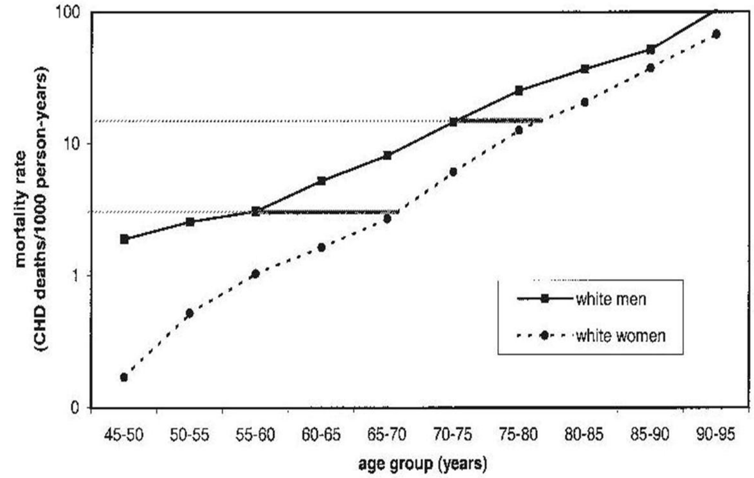
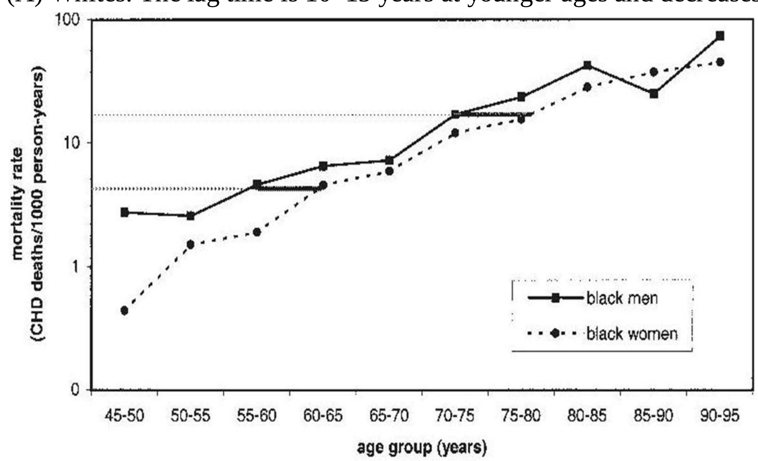
subjects > 30 years and free of cardiovascular disease (CVD) at baseline (18% blacks, 37% men). Completion of follow up was > 97% for all studies. As the main outcome measures, age-specific CHD mortality rates. Primary outcome was CHD mortality, which included acute and old MI, angina pectoris, and other acute, sub-acute, and chronic forms of ischemic heart disease [10], [11] & [14-17].

Figure 1: Age Specific CHD mortality rates in men compared with women

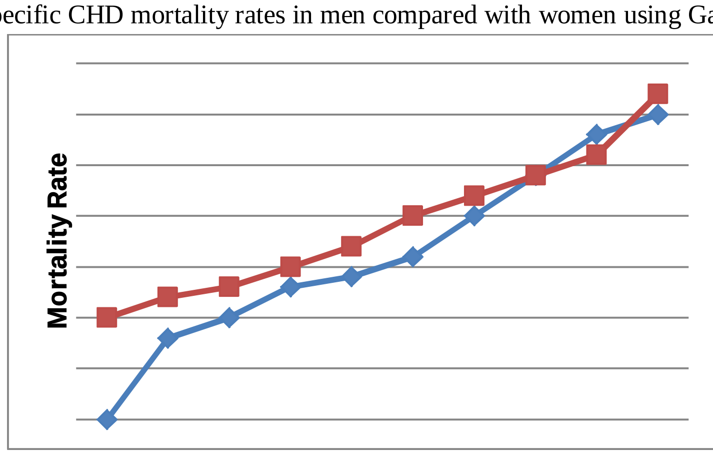


(A) Whites. The lag time is 10–15 years at younger ages and decreases with age



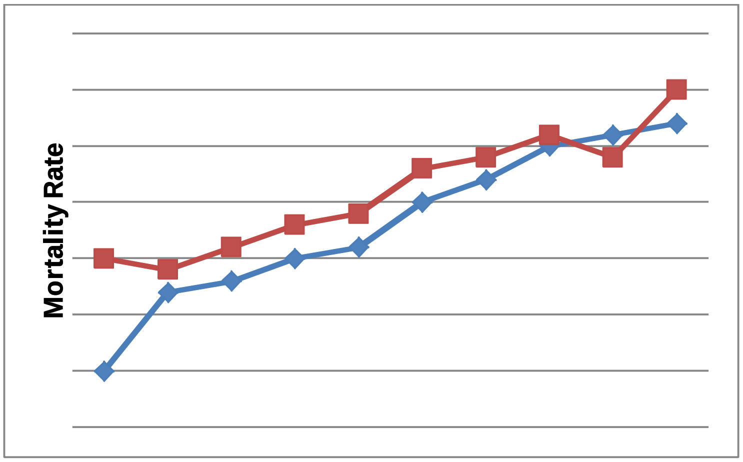
(B) Blacks. The lag time is 5–10 years

Figure 2: Age Specific CHD mortality rates in men compared with women using Gamma Distribution



(A) Whites. The lag time is 10–15 years at younger ages and decreases with age
Red Line: White Men

Blue Line: White Women



(B) Blacks. The lag time is 5–10 years
Red Line: Black Men
Blue Line: Black Women

II. CONCLUSION

The gender difference in CHD mortality was more pronounced in whites than in blacks at younger ages. This discrepancy was not explained by adjustment for CHD risk factors and suggests that other factors may be responsible for the ethnic variation in the gender gap. The failure rates and the repair time controlled with the respect to the state of the system. Tackas formula used to derive steady distribution and comparing the medical report, the mathematical model also give the same report. There is no significance difference between medical and mathematical reports. The medical reports are beautifully fitted with the mathematical model. Hence the mathematical report {Figure (2)} is coincide with the medical report {Figure (1)}.

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