



iJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 4 Issue: XII Month of publication: December 2016

DOI:

www.ijraset.com

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Conquences of A Fixed Point Theorem for Quasi-Contractions of D^* -Metric Spaces

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Abstract: The main aim of this paper is to prove the consequences of existence of fixed point on λ -generalized contraction of self mapping functions on D^* -metric space.

Key Words: D^* -metric space, Quasi-contraction, self mappings,

I. INTRODUCTION

The notion of Quasi-contraction defined for selfmaps of metric spaces given by Lj. B. Ciric [3] has been extended to the selfmaps of D^* -metric spaces as follows:

A. Definition

Let f be a selfmap of a D^* -metric space (X, D^*) and $x \in X$, $n \geq 1$ be an integer. The **orbit of x under f of length n** , denoted by $O_f(x : n)$, is defined by

$$O_f(x : n) = \{x, fx, f^2x, \dots, f^n x\}$$

We define the diameter $\delta(A)$ of a set A in a D^* -metric space (X, D^*) by $\delta(A) = \sup_{x, y \in A} \{D^*(x, y, y)\}$

The following Lemmas are use full in proving fixed point theorems of quasi-contractions on D^* -metric spaces:

B. Definition

A selfmap f of a D^* -metric space (X, D^*) is called a **Quasi-contraction**, if there is a number q with $0 \leq q < 1$ such that

$$D^*(fx, fy, fy) \leq q \cdot \max \{ D^*(x, y, y), D^*(x, fx, fx), D^*(y, fy, fy), \\ D^*(x, fy, fy), D^*(y, fx, fx) \}$$

for all $x, y \in X$.

As already noted that for every λ -generalized contraction is a quasi-contraction. However the following example gives a quasi-contraction f on a D^* -metric space (X, D^*) which is not a λ -generalized contraction.

II. PRELIMINARY NOTES

A. Definition

Let f be a selfmap of a D^* -metric space (X, D^*) and $x \in X$, $n \geq 1$ be an integer. The **orbit of x under f of length n** , denoted by $O_f(x : n)$, is defined by

$$O_f(x : n) = \{x, fx, f^2x, \dots, f^n x\}$$

We define the diameter $\delta(A)$ of a set A in a D^* -metric space (X, D^*) by $\delta(A) = \sup_{x, y \in A} \{D^*(x, y, y)\}$

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The following Lemmas are use full in proving fixed point theorems of quasi-contractions on D^* -metric spaces:
 In this section we prove

B. Lemma

Suppose f is a quasi-contraction with constant q on a D^* -metric space (X, D^*) and n be a positive integer. Then for each $x \in X$ and all integers $i, j \in \{1, 2, 3, \dots, n\}$,

$$D^*(f^i x, f^j x, f^j x) \leq q \delta [O_f(x:n)] < \delta [O_f(x:n)].$$

Proof: Let $x \in X$ be arbitrary, $n \geq 1$ be an integer and $i, j \in \{1, 2, 3, \dots, n\}$. Then $f^{i-1}x, f^{j-1}x, f^i x, f^j x \in O_f(x:n)$ and since f is a quasi-contraction,

$$\begin{aligned} D^*(f^i x, f^j x, f^j x) &= D^*(ff^{i-1}x, ff^{j-1}x, ff^{j-1}x) \\ &\leq q \cdot \max \left\{ D^*(f^{i-1}x, f^{j-1}x, f^{j-1}x), D^*(f^{i-1}x, f^i x, f^i x), \right. \\ &\quad D^*(f^{j-1}x, f^j x, f^j x), D^*(f^{i-1}x, f^j x, f^j x), \\ &\quad \left. D^*(f^{j-1}x, f^i x, f^i x) \right\} \\ &\leq q \cdot \text{Sup} \{ D^*(u, v, v) : u, v \in O_f(x:n) \} \\ &= q \delta [O_f(x:n)] \\ &< \delta [O_f(x:n)] \end{aligned}$$

C. Lemma

Suppose f is a quasi-contraction with constant q on a D^* -metric space (X, D^*) and $x \in X$, then for every positive integer n , there exists positive integer $k \leq n$, such that

$$D^*(x, f^k x, f^k x) = \delta [O_f(x:n)]$$

Proof: If possible assume that the result is not true. This implies that there is positive integer m such that for all $k \leq m$, we have $D^*(x, f^k x, f^k x) \neq \delta [O_f(x:m)]$. Since $O_f(x:m)$ contains x and $f^k x$ for $k \leq m$, it follows that

$$D^*(x, f^k x, f^k x) < \delta [O_f(x:m)]$$

Since $O_f(x:m)$ is closed, there exists $i, j \in \{1, 2, 3, \dots, m\}$ such that $D^*(x, f^k x, f^k x) = \delta [O_f(x:m)]$, contradicting the Lemma 2.1. Therefore

$$D^*(x, f^k x, f^k x) = \delta [O_f(x:n)] \text{ for some } k \leq n.$$

D. Lemma

Suppose f is a quasi-contraction with constant q on a D^* -metric space (X, D^*) , then

$$\delta [O_f(x:\infty)] \leq \frac{1}{1-q} D^*(x, fx, fx) \text{ for all } x \in X.$$

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Proof: Let $x \in X$ be arbitrary. Since $O_f(x:1) \subseteq O_f(x:2) \subseteq \dots \subseteq O_f(x:n) \subseteq O_f(x:n+1) \subseteq \dots$, we get that

$$\delta[O_f(x:1)] \leq \delta[O_f(x:2)] \leq \dots \leq \delta[O_f(x:n)] \leq \delta[O_f(x:n+1)] \leq \dots, \text{showing}$$

$$\lim_{n \rightarrow \infty} \delta[O_f(x:n)] = \text{Sup} \{ \delta[O_f(x:n)] : n = 1, 2, 3, \dots \}.$$

Therefore to prove the Lemma, it is enough to show

III. MAIN RESULT

A. Theorem

Suppose f is a selfmap of a D^* -metric space (X, D^*) and X is f -orbitally complete. If there is a positive integer k such that f^k is a quasi-contraction with constant q . Then f has a unique fixed point $u \in X$. In fact,

$$u = \lim_{n \rightarrow \infty} f^n x \text{ for any } x \in X$$

and

$$D^*(f^n x, u, u) \leq \frac{q^n}{1-q} a(x) \text{ for all } x \in X, n \geq 1,$$

where $a(x) = \max \{ D^*(f^i x, f^{i+k} x, f^{i+k} x) : i = 1, 2, 3, \dots \}$ and $m = \left[\frac{n}{k} \right]$, the greatest integer not exceeding $\frac{n}{k}$.

Proof: Suppose f^k is a quasi-contraction of a D^* -metric space (X, D^*) . It has unique fixed point by Theorem 3.1. Let u be a fixed point of f^k . Then we claim that fu is also a fixed point of f^k . In fact,

$$f^k(fu) = f^{k+1}u = f(f^k u) = fu$$

By the uniqueness of fixed point of f^k , it follows that $fu = u$, showing that u is a fixed point of f . Uniqueness of the fixed point of f can be proved as in the Theorem 3.1.

To prove (3.3), let n be any integer. Then by the division algorithm, we have, $n = mk + j$, $0 \leq j < k$, $m \geq 0$

Therefore $x \in X$, $f^n x = (f^k)^m f^j x$, since f^k is a quasi-contraction,

$$\begin{aligned} D^*(f^n x, u, u) &\leq \frac{q^m}{1-q} D^*(f^j x, f^k f^j x, f^k f^j x) \\ &\leq \frac{q^m}{1-q} \cdot \max \{ D^*(f^i x, f^k f^i x, f^k f^i x) : i = 0, 1, 2, \dots, k-1 \} \\ &\leq \frac{q^m}{1-q} \cdot \max \{ D^*(f^i x, f^{k+i} x, f^{k+i} x) : i = 0, 1, 2, \dots, k-1 \} \end{aligned}$$

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proving (3.3). Letting $m \rightarrow \infty$, we get that $\lim_{n \rightarrow \infty} f^n x = u$, since $q^m \rightarrow 0$ as $m \rightarrow \infty$, proving (3.2). This completes the proof of the theorem.

B. Theorem

Let f be a quasi-contraction with constant q on a metric space (X, d) and X be f -orbitally complete, then f has a unique fixed point $u \in X$. In fact,

$$u = \lim_{n \rightarrow \infty} f^n x \text{ for all } x \in X$$

and

$$d(f^n x, u) \leq \frac{q^n}{1-q} d(x, fx) \text{ for all } x \in X, n \geq 1.$$

Proof: If (X, d) is a f -orbitally complete metric space, then it can be proved that (X, D_1^*) is a f -orbitally complete D^* -metric space and hence f -orbitally complete for any selfmap f of X . Also if f is a quasi-contraction with constant q of (X, d) , then the condition of quasi-contraction can be written as

$$D_1^*(fx, fy, fy) \leq q \cdot \max \{ D_1^*(x, y, y), D_1^*(x, fx, fx), D_1^*(y, fy, fy), \\ D_1^*(x, fy, fy), D_1^*(y, fx, fx) \}$$

for all $x, y \in X$, since $D_1^*(x, y, y) = d(x, y)$; so that f is a quasi-contraction on (X, D_1^*) . Thus f is a quasi-contraction on the f -orbitally complete D^* -metric space (X, D_1^*) and hence the conclusions of Theorem 3.1 hold for f ; which are the conclusions of the theorem.

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