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Integrability of Trigonometric Series with Coefficients Satisfying Certain Conditions

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Abstract: Let $1 \leq P < \infty$ and $-1 < \alpha P < P - 1$, suppose that $\{a_n\}$ is a sequence of numbers such that $a_n \in A_j$ or

$a_n \in A_{-j}$ and $\left\{ \sum_{n=1}^{\infty} n^{P-\alpha P-2} L(n)(a_n)^P \right\}^{1/P} < \infty$, then we will prove that $L^{1/P} \left(\frac{1}{x} \right) f(x) \in L(P, \alpha)$

And $\left\| L^{1/P} \left(\frac{1}{x} \right) f(x) \right\|_{P, \alpha}^P \leq B(\alpha, P, j) \sum_{n=1}^{\infty} n^{P-\alpha P-2} L(n)(a_n)^P$

& ii) Let $\{a_n\}$ be a sequence of numbers such that $a_n \in A_j$ or $a_n \in A_{-j}$. If $1 \leq P < \infty$ and $-1 < \alpha P < P - 1$, then a necessary and sufficient condition that $L^{1/P} \left(\frac{1}{x} \right) f(x) \in L(P, \alpha)$ is that $\sum_{n=1}^{\infty} n^{P-\alpha P-2} L(n)a_n^P < \infty$.

I. INTRODUCTION

A. Definitions

A function $\phi(x)$ is said to belong to class $L(P, \alpha)$ if $\int_0^{\pi} |\phi(x)|^P (\sin x)^{\alpha P} dx < \infty$, α is a real number and $P > 0$, it is easy to see

that

$L(P, \alpha) \Rightarrow L^P$ for $\alpha < 0$ And

$L^P \Rightarrow L(P, \alpha)$ for $\alpha > 0$ And $L(P, \alpha) = L^P$ if $\alpha = 0$.

We define norm of a function $\phi(x) \in L(P, \alpha)$ as: $\|\phi(x)\|_{P, \alpha} = \left\{ \int_0^{\pi} |\phi(x)|^P (\sin x)^{\alpha P} dx \right\}^{1/P}$

A positive continuous function $L(x)$ is said to be "slowly increasing", in the sense of Karamata [4] if

$$\lim_{x \rightarrow \infty} \frac{L(kx)}{L(x)} = 1 \text{ For every } k > 0.$$

A sequence $\{a_n\}$ of non-negative number is said to be quasi-monotone [7, 9] if for some constant $\alpha \geq 0$

$$a_{n+1} \leq a_n \left(1 + \frac{\alpha}{n} \right) \text{ for all } n > n_0(\alpha).$$

An equivalent definition is that $n^{-\beta} a_n \downarrow 0$ for some $\beta > 0$. We shall say that the coefficients of trigonometric series,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \text{ and } g(x) = \sum_{n=1}^{\infty} a_n \sin nx \text{ belong to the class } A_j \text{ if for some } j \geq 0, \text{ the number } n^{-j} a_n, a_n \geq 0,$$

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decreases and to the class A_{-j} if, for some $j > 0$, the number $n^j a_n$, $a_n \geq 0$ increases. The coefficients decrease monotonically to zero belongs to the class A_0 .

B. Some known results

Theorem_ If $0 < \nu < 1$, $a_n \downarrow 0$, then $x^{-\nu} L\left(\frac{1}{x}\right) f(x) \in L(0, \pi)$ if and only if $\sum_{n=1}^{\infty} n^{\nu-1} L(n) a_n$ is convergent.

Theorem_ If $a_n \downarrow 0$, $P \geq 1$, and $-1 < \nu < 0$, then the necessary and sufficient condition that $\sum_{n=1}^{\infty} n^{-1+P\nu+P} L(n) a_n^P$ should converge, is that $x^{-1-P\nu} L\left(\frac{1}{x}\right) f^P(x) \in L(0, \pi)$.

Theorem_ Let $\{a_n\}$ is quasi-monotone if $\alpha < 1$ and such that $0 < M_1 \leq n^\beta L_1(n) a_n \leq M_2$ with $\beta > 0$, if $P \geq 1$ and $1 - P < \lambda < 1$. Then,

$f(\lambda, L, P) = x^{-\lambda} L_2\left(\frac{1}{x}\right) f^P(x)$ is integrable in $(0, \pi)$ if and only if $\sum_{n=1}^{\infty} n^{\lambda+P-2} L_2(n) a_n^P < \infty$, where L_1 and L_2 are slowly increasing function in the sense of *Karamata*.

Theorem_ Let a_n be positive and tends to zero. Let $a_n n^{-k}$ be monotonically decreasing for some non-negative integer k . Let $1 \leq P < \infty$ and $-1 < \alpha P < P - 1$, then a necessary and sufficient condition that $f(x) \in L(P, \alpha)$, where

$$f(x) \sim \sum_{n=1}^{\infty} a_n \cos nx \text{ is that}$$

$$\sum_{n=1}^{\infty} n^{P-\alpha P-2} a_n^P < \infty.$$

Theorem_ Let $\{a_n\}$ be a positive sequence tending to zero and $\{a_n n^{-k}\}$ be monotonically decreasing for some non-negative integer k . If $1 \leq P < \infty$ and $-1 < \alpha P < P - 1$ then the necessary and sufficient condition that $L^{1/P}\left(\frac{1}{x}\right) f(x) \in L(P, \alpha)$ is

$$\text{that, } \sum_{n=1}^{\infty} n^{P-\alpha P-2} L(n) a_n^P < \infty, \text{ where } f(x) \sim \sum_{n=1}^{\infty} a_n \cos nx.$$

C. Our theorems

We shall prove the following theorems

Theorem_ Let $1 \leq P < \infty$ and $-1 < \alpha P < P - 1$, suppose that $\{a_n\}$ is a sequence of numbers such that $a_n \in A_j$ or $a_n \in A_{-j}$

$$\text{and } \left\{ \sum_{n=1}^{\infty} n^{P-\alpha P-2} L(n) (a_n)^P \right\}^{1/P} < \infty,$$

$$\text{Then, } L^{1/P}\left(\frac{1}{x}\right) f(x) \in L(P, \alpha)$$

$$\text{And } \left\| L^{1/P}\left(\frac{1}{x}\right) f(x) \right\|_{P, \alpha}^P \leq B(\alpha, P, j) \sum_{n=1}^{\infty} n^{P-\alpha P-2} L(n) (a_n)^P$$

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Theorem Let $\{a_n\}$ be a sequence of numbers such that $a_n \in A_j$ or $a_n \in A_{-j}$. If $1 \leq P < \infty$ and $-1 < \alpha P < P - 1$, then a necessary and sufficient condition that $L^{1/P}\left(\frac{1}{x}\right)f(x) \in L(P, \alpha)$ is that $\sum_{n=1}^{\infty} n^{P-\alpha P-2} L(n)a_n^P < \infty$.

D. Lemmas

The following lemmas will be required for the proof of our theorems

1) **Lemma:** Let $f(x) \geq 0$ for $(x) \geq 0$ and $f(x)$ be the integral of $f(x)$. If $1 \leq P < q$ and $r > -1$ then

$$\left\{ \int_0^{\infty} t^{-1-qr} \left(L\left(\frac{1}{t}\right) \frac{f(t)}{t} \right)^q dt \right\}^{1/q} \leq B \left\{ \int_0^{\infty} t^{-1-Pr} \left(L\left(\frac{1}{t}\right) f(t) \right)^P dt \right\}^{1/P}$$

$$\left\{ \int_0^{\infty} t^{-1-qr} \left(L(t) \frac{f(t)}{t} \right)^q dt \right\}^{1/q} \leq B \left\{ \int_0^{\infty} t^{-1-Pr} (L(t)f(t))^P dt \right\}^{1/P}$$

2) **Lemma :** Let $\sum_{n=1}^{\infty} a_n$ be a series of positive terms and $A_n = \sum_{k=n}^{\infty} a_k$ and suppose that

$$\sum_{n=1}^{\infty} n^{-c} L(n)(na_n)^P < \infty, (P \geq 1, c < 1), \text{ then } \sum_{n=1}^{\infty} n^{-c} L(n)A_n^P \leq B \sum_{n=1}^{\infty} n^{-c} L(n)(na_n)^P, \text{ where B is some constant depending upon c and P.}$$

3) **Lemma:** For any non-negative ν and n , we have

$$\left| \sum_{k=2^{\nu}(n+1)}^{2^{\nu+1}(n+1)-1} a_k \phi(kx) \right| \leq \begin{cases} \frac{2j}{|\sin \frac{x}{2}|} a_{2^{\nu}(n+1)}, & \text{if } a_k \in A_j \\ \frac{2j}{|\sin \frac{x}{2}|} a_{2^{\nu+1}(n+1)-1}, & \text{if } a_k \in A_{-j} \end{cases}$$

$x \neq 2k\pi, k = 0, \pm 1, \pm 2, \dots$. Where $\phi(x) = \cos x$ or $\phi(x) = \sin x$

4) **Lemma:** If $a_k \in A_j$ or $a_k \in A_{-j}$ then

$$\sum_{\nu=1}^{\infty} \left[2^{\nu}(n+1) \right]^{1+\alpha} a_{2^{\nu}(n+1)} \leq c_1(\alpha, j) \sum_{k=n+1}^{\infty} k^{\alpha} a_k \text{ if } a_k \in A_j,$$

$$\sum_{\nu=0}^{\infty} \left[2^{\nu}(n+1) \right]^{1+\alpha} a_{2^{\nu+1}(n+1)-1} \leq c_2(\alpha, j) \sum_{k=n+1}^{\infty} k^{\alpha} a_k \text{ if } a_k \in A_{-j},$$

Where,

$$C_1(\alpha, j) = \begin{cases} 2 & \text{for } \alpha + j \leq 0 \\ 2^{1+\alpha+j} & \text{for } \alpha + j > 0 \end{cases}$$

$$C_2(\alpha, j) = \begin{cases} 1 & \text{for } \alpha - j \geq 0 \\ 2^{j-1} & \text{for } \alpha - j < 0 \end{cases}$$

5) **Lemma:** Let $\{a_n\}$ be a sequence of non-negative terms and

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$$A(n) = \sum_{k=\left(\frac{n}{2}\right)+1}^n, \quad A^*(n) = \sum_n^{2n} a_k \text{ Then}$$

$$na_n \leq B(j)A(n) \quad \text{if } a_k \in A_j$$

$$na_n \leq B(j)A^*(n) \quad \text{if } a_k \in A_{-j}$$

Where $B(j)$ is some positive constant depending on j .

Proof:

$$\sum_{k=s}^m \frac{a_k}{k^j} K^j \geq \frac{a_m}{m^j} s^j (m-s+1), \quad \text{if } a_k \in A_j,$$

$$\sum_{k=s}^m a_k K^j k^{-j} \geq a_s s^j m^j (m-s+1), \quad \text{if } a_k \in A_{-j},$$

We set $s = \left\lfloor \frac{n}{2} \right\rfloor + 1, m = n$ in (i) and $s = n, m = 2n$ in (ii), we have

$$na_n \leq \frac{A_n}{\left[\left\lfloor \frac{n}{2} \right\rfloor + 1\right]^j} n^{j+1} \leq B(j)A(n)$$

And $na_n \leq B(j)A^*(n)$.

This completes the proof of the lemma.

E. Proof of Theorem

Since we are given that $\sum_{n=1}^{\infty} n^{p-\alpha p-2} L(n) a_n^p < \infty$, it follows by virtue of *Lemma2* (putting $c = -p + \alpha p + 2$ and $a_k = \frac{a_k}{k}$)

that $\sum_{n=1}^{\infty} n^{p-\alpha p-2} L(n) \left(\sum_{k=n}^{\infty} \frac{a_k}{k} \right)^p < \infty$, further on putting $n=1$, we have $\sum_{k=1}^{\infty} \frac{a_k}{k} < \infty$.

$$\begin{aligned} \text{Put } R_n(x) &= \left| \sum_{k=n+1}^{\infty} a_k \cos kx \right| \leq \left| \sum_{\nu=0}^{\infty} \sum_{k=2^{\nu}(n+1)}^{2^{\nu+1}(n+1)-1} a_k \cos kx \right| \\ &\leq \frac{2j}{\left| \sin \frac{x}{2} \right|} \begin{cases} \sum_{\nu=0}^{\infty} a_{2^{\nu}(n+1)}, & \text{if } a_k \in A_j \\ \sum_{\nu=0}^{\infty} a_{2^{\nu+1}(n+1)}, & \text{if } a_k \in A_{-j} \end{cases} \end{aligned}$$

$x \neq 2k\pi, k = 0, \pm 1, \pm 2, \dots$ by virtue of *lemma3* (choosing $\phi(x) = \cos x$)

Now using the particular case, when $\alpha = -1$ of *lemma 4*, we obtain

$$R_n(x) \leq \frac{B^j}{\left| \sin \frac{x}{2} \right|} \left[a_{n+1} + \sum_{k=n+1}^{\infty} \frac{a_k}{k} \right], x \neq 2k\pi$$

Therefore series $\sum_{n=1}^{\infty} a_n \cos nx$ converges uniformly and is a Fourier series of the function $f(x)$ which is continuous in $(0, 2\pi)$. We

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$$\begin{aligned} \text{have } |f(x)| &= \left| \sum_{k=1}^{\infty} a_k \cos kx \right| = \left| \sum_{k=1}^n a_k \cos kx + \sum_{k=n+1}^{\infty} a_k \cos kx \right| \\ &\leq \sum_{k=1}^n a_k + \frac{B^j}{x} \left[a_{n+1} + \sum_{k=n+1}^{\infty} \frac{a_k}{k} \right] \\ &\leq \sum_{k=1}^n a_k + O\left(\frac{1}{x}\right) a_{n+1} + O\left(\frac{1}{x}\right) \sum_{k=n+1}^{\infty} \frac{a_k}{k} \\ &\leq s_n + O\left(\frac{1}{x}\right) a_{n+1} + O\left(\frac{1}{x}\right) \left(\sum_{k=n+1}^{\infty} \frac{a_k}{k} \right) \end{aligned}$$

Where $s_n = \sum_{k=1}^n a_k$

$$\begin{aligned} \text{Now } \int_0^{\pi/2} L\left(\frac{1}{x}\right) |f(x)|^p (\sin x)^{\alpha p} dx &= \sum_{n=2}^{\infty} \int_{\pi/n+1}^{\pi/n} L\left(\frac{1}{x}\right) |f(x)|^p (\sin x)^{\alpha p} dx \\ &\leq \sum_{n=2}^{\infty} \int_{\pi/n+1}^{\pi/n} L\left(\frac{1}{x}\right) \left\{ s_n + O\left(\frac{1}{x}\right) a_{n+1} + O\left(\frac{1}{x}\right) \left(\sum_{k=n+1}^{\infty} \frac{a_k}{k} \right) \right\}^p (\sin x)^{\alpha p} dx \\ &\leq B(p) \sum_{n=2}^{\infty} \int_{\pi/n+1}^{\pi/n} L\left(\frac{1}{x}\right) s_n^p (\sin x)^{\alpha p} dx + B(p, j) \sum_{n=2}^{\infty} \int_{\pi/n+1}^{\pi/n} L\left(\frac{1}{x}\right) (a_{n+1})^p (\sin x)^{\alpha p - p} dx \\ &\quad + B(p, j) \sum_{n=2}^{\infty} \int_{\pi/n+1}^{\pi/n} L\left(\frac{1}{x}\right) \left(\sum_{k=n+1}^{\infty} \frac{a_k}{k} \right)^p (\sin x)^{\alpha p - p} dx \\ &\leq B(\alpha, p) \sum_{n=2}^{\infty} n^{-2-\alpha p} L(n) s_n^p + B(\alpha, p, j) \sum_{n=2}^{\infty} n^{p-\alpha p-2} L(n) (a_{n+1})^p \\ &\quad + B(\alpha, p, j) \sum_{n=2}^{\infty} n^{p-\alpha p-2} L(n) \left(\sum_{k=n+1}^{\infty} \frac{a_k}{k} \right)^p \end{aligned}$$

$= j_1 + j_2 + j_3$ (say)

Now put $a_{(x)} = a_n$ for $n-1 \leq x < n$ ($n = 1, 2, \dots$)

And $A(x) = \int_0^{\pi} a(t) dt$ then, we have,

$$j_1 \leq B(\alpha, p) \sum_{n=1}^{\infty} \int_n^{n+1} x^{-2-\alpha p} L(x) A^p(x) dx = B(\alpha, p) \int_1^{\infty} x^{-2-\alpha p + p} \left\{ \frac{L(x)^{1/p} A(x)}{x} \right\}^p dx$$

On applying lemma (ii) (taking $q = p$ and $-1 - qr = p - \alpha p - 2$) we get,

$$j_1 \leq B(\alpha, p) \int_1^{\infty} x^{-2-\alpha p + p} L(x) (a(x))^p dx$$

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$$= B(\alpha, p) \sum_{n=2}^{\infty} \int_{n-1}^n x^{-2-\alpha p+p} L(x) (a(x))^p (x) dx$$

$$\leq B(\alpha, p) \sum_{n=1}^{\infty} n^{p-2-\alpha p} L(n) (a_n)^p$$

$< \infty$, by hypothesis of
 heses.

By virtue of lemma 2 the theorem,

$$j_2 = O(1) \text{ by the hypotand by the hypothesis,}$$

$$j_3 = O(1)$$

A similar method may be used to estimate

$$\int_{\frac{\pi}{2}}^{\pi} |f(x)|^p (\sin x)^{\alpha p} dx$$

This finishes proof of theorem.

F. Proof of Theorem

Necessity: Suppose that $L^{1/p} \left(\frac{1}{x} \right) f(x) \in L(P, \alpha)$, then we have to prove, $\sum_{n=1}^{\infty} n^{p-2-\alpha p} L(n) (a_n)^p < \infty$

$$\text{Let } f_1(x) = \int_0^x f(u) du, f_2(x) = \int_0^x f_1(u) du$$

$$\text{Then } f_2(x) = \int_0^{\infty} \left(\sum_{k=1}^{\infty} \frac{a_k}{k} \sin ku \right) du$$

$$= \sum_{k=1}^{\infty} \frac{a_k}{k} \int_0^x \sin ku du$$

$$= \sum_{k=1}^{\infty} a_k (1 - \cos kx) k^{-2}$$

$$\geq \sum_{k=s}^m a_k (1 - \cos kx) k^{-2} \tag{A}$$

For any positive integers s and m ,

Case I: When $a_k \in A_j$

Now set $x = \left[\frac{n}{2} \right] + 1$ and $m = n$ and using the inequality $1 - \cos nx \geq A \frac{nx^2}{2}$ for $\frac{\pi}{4(n+1)} \leq x \leq \frac{\pi}{4n}$, we have

$A_n \leq Bn^2 f_2(x)$, where B is some constant. By virtue of Lemma 5(i), it follows

$$na_n \leq B(j)A_n \leq B(j)n^2 f_2(x)$$

$$\text{Now, } \sum_{n=1}^{\infty} n^{-2-\alpha p} L(n) (na_n)^p$$

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$$\begin{aligned} &\leq \mathbf{B}(j) \sum_{n=1}^{\infty} n^{2p-2-\alpha p} L(n) \min(f_2(x))^p \quad \left(\frac{\pi}{4(n+1)} \leq x \leq \frac{\pi}{4n} \right) \\ &\leq B(j) \sum_{n=1}^{\infty} \int_{\frac{\pi}{4(n+1)}}^{\frac{\pi}{4n}} (\sin x)^{-2p+\alpha p} L\left(\frac{1}{x}\right) f_2^p(x) dx \\ &\leq B(j) \int_0^{\frac{\pi}{4}} (\sin x)^{-p+\alpha p} \left\{ \frac{L\left(\frac{1}{x}\right)^{1/p} f_2(x)}{x} \right\}^p dx, \\ &\leq B(\alpha, p, j) \int_0^{\frac{\pi}{4}} (\sin x)^{-p+\alpha p} \left\{ L\left(\frac{1}{x}\right)^{1/p} |f_1(x)| \right\}^p dx \\ &\leq B(\alpha, p, j) \int_0^{\frac{\pi}{4}} (\sin x)^{\alpha p} \left\{ L\left(\frac{1}{x}\right)^{1/p} |f(x)| \right\}^p dx \\ &= B(\alpha, p, j) \left\| L^{1/p}\left(\frac{1}{x}\right) f(x) \right\|_{p,\alpha}^p < \infty \end{aligned}$$

This follows by lemma 1 (i) ($q = p$, $\alpha p - p = -1 - pr$ and $\alpha p = -1 - pr$ respectively)

Case II: When $a_k \in A_j$, we set $s = n$ and $m = 2n$ in (A) and obtain

$$A_n^* \leq Bn^2 f_2(x) \quad \text{For } \frac{\pi}{8(n+1)} \leq x \leq \frac{\pi}{8n}$$

By lemma 5(ii) we get $na_n \leq B(j)n^2 f_2(x)$

$$\begin{aligned} \text{Now, } &\sum_{n=1}^{\infty} n^{-2-\alpha p} L(n)(na_n)^p \\ &\leq \mathbf{B}(j) \sum_{n=1}^{\infty} n^{2p-2-\alpha p} L(n) \min(f_2(x))^p, \quad \frac{\pi}{8(n+1)} \leq x \leq \frac{\pi}{8n} \\ &\leq B(j) \sum_{n=1}^{\infty} \int_{\frac{\pi}{8(n+1)}}^{\frac{\pi}{8n}} (\sin x)^{-2p+\alpha p} L\left(\frac{1}{x}\right) f_2^p(x) dx \\ &\leq B(j) \int_0^{\frac{\pi}{8}} (\sin x)^{-p+\alpha p} \left\{ \frac{L\left(\frac{1}{x}\right)^{1/p} f_2(x)}{x} \right\}^p dx < \infty \end{aligned}$$

By some agreement as in the case I this possesses the necessity part of theorem 2.

Sufficiency: Now suppose that $\sum_{n=1}^{\infty} n^{p-2-\alpha p} L(n)(na_n)^p \leq \infty$. Then we have to show $L^{1/p}\left(\frac{1}{x}\right) f(x) \in L(P, \alpha)$.

This follows by theorem 1 and proof of the theorem is thus completed.

II. CONCLUSION

Theorem 1 and theorem 2 also hold for sine series. The proof of sufficiency part for sine series follows exactly in a same way as in

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case of theorem 1 while, for proof of necessity part, some minor changes are required.

For sake of convenience the *theorem 1* is stated and proved otherwise theorem is essentially the same as $\sum - \int$ part of theorem 2.

Our theorem 2 is not only more general than a result of *Askey and Wainger [2]* and theorem of *Khan [5]*, but has a proof applicable in sine and cosine series both.

In the end I wish to express my sincere thanks to “Dr. J.P. Singh” for his kind guidance.

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