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Application of Fuzzy Soft Set in Job Scheduling Problem

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Abstract: The concept fuzzy soft set, a new mathematical tool for dealing with ncertainty, which always possesses parametrization tools has potential applications in many different fields, including the decision making , game theory, operations research, Riemann integration, probability theory, and measurement theory. The main aim of this paper is to apply the decision method, which is based on Sanchez's method to solve the job scheduling problem under uncertain information, by using the notions of fuzzy soft set and fuzzy numbers.
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I. INTRODUCTION

We cannot solve the problems by using mathematical tools generally in the social life since in mathematics the concepts are precise. Some theories are developed to eliminate this lack for vagueness such as theory of fuzzy sets[13], theory of rough sets[11], theory of intuitionistic fuzzy sets[1],theory of vague sets[4],theory of interval mathematics[5], but all these theories have own their difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory. Molodtsov[10] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which cannot be handled by traditional mathematical tools and which is from free the problems mentioned above. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so forth.

Maji et al.[7,9] have further studied the theory of soft sets and used this theory to solve some decision-making problems. They have also introduced the concept of fuzzy soft set[8], a more general concept, which is a combination of fuzzy set and soft set and studied its properties, and also Roy and Maji used this theory to solve some decision-making problems[12]. This paper studies job shop scheduling problem under uncertain information such as machine breakdowns, processing times variations, cancellations or arrivals of new jobs and consumption costs etc. We solve this Job Scheduling Problem with the use of fuzzy soft sets and fuzzy numbers by the method, analogues to Sanchez's method[3] for diagnosis.

II. PRELIMINARIES

In this section we recall some definitions regarding fuzzy soft sets and fuzzy numbers which are required for this paper.

Throughout our discussion, $U = \{x_1, x_2, \ldots, x_n\}$ would refer to an initial universal set, $E = \{e_1, e_2, \ldots, e_n\}$ the set of all parameters for U and I_U, the set of all fuzzy subsets of U. Also by (U, E) we mean the universal set U and the parameter set E.Let $A, B \subset E$.

Definition 2.1 A pair (F, E) is called a soft set on an initial universe X if and only if F is a mapping of E into the set of all subsets of the set X. In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E, F(e)$, may be considered as the set of e-elements of the soft set (F, E), or as the set of e-approximate elements of the soft set.

Definition 2.2 [10] A pair (F, A) is called a fuzzy soft set over U where F is mapping given by $F : A \to I^U$, the set of all fuzzy subsets of U.

Definition 2.3 [8] For two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft universe (U, E), (F, A) is a fuzzy soft subset of (G, B), if

i. $A \subset B$
ii. $F(e) \subset G(e), \forall e \in E$ and is written as $(F, A) \tilde{\subseteq} (G, B)$.

Definition 2.4 [8] The union of two soft sets of (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A - B$, $H(e) = G(e)$ if $e \in B - A$ and $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$. Denoted as $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 2.5 [8] The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X, denoted by $(F, A) \tilde{\cap} (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.6 [8] The complement of a fuzzy soft set (F, A) is denoted by $1 - (F, A)$ and is defined by $1 - (F, A) = (1 - F, A)$ where $1 - F : A \to I^X$ is a mapping given by $(1 - F)(e) = [1 - F(e)]$, $\forall e \in E$.

Definition 2.7 [8] A fuzzy soft set (F, A) is said to be absolute fuzzy soft set over X, denoted by, $\tilde{A}$ if $F(e) = 1, \forall e \in E$.

Definition 2.8 [8] A fuzzy soft set (F, A) is said to be null fuzzy soft set over X, denoted by $\tilde{\varphi}$, if $F(e) = 0, \forall e \in E$.

Example 2.1

Let $\mathrm{X} = \{x_1, x_2, x_3\}$ be the set of drivers under consideration and $\mathrm{E} = \{\mathrm{e}_1 (\text{very speed}), \mathrm{e}_2 (\text{normal speed}), \mathrm{e}_3 (\text{low speed})\}$ be the level of driving speed of the drivers.

Consider the two fuzzy soft sets (F, A) and (G, B), where $A = \{e_2, e_3\}$ and $B = \{e_1, e_2, e_3\}$ in E, given by

$$(F, A) = \{F(e_2) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.3)\},$$
$$F(e_3) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.5)\}\}$$

And

$$(G, B) = \{G(e_1) = \{(x_1, 0.6), (x_2, 0.2), (x_3, 0.8)\}$$
$$G(e_2) = \{(x_1, 0.7), (x_2, 0.7), (x_3, 0.5)\},$$
$$G(e_3) = \{(x_1, 0.3), (x_2, 0.6), (x_3, 0.8)\}\}$$

It is also clear that $(F, A) \tilde{\subseteq} (G, B)$.

Then

$$1 - (F, A) = \{(1 - F)(e_2) = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.7)\},$$
$$(1 - F)(e_3) = \{(x_1, 0.9), (x_2, 0.8), (x_3, 0.5)\}\}$$

and

$$1 - (G, B) = \{(1 - G)(e_1) = \{(x_1, 0.4), (x_2, 0.8), (x_3, 0.2)\}, (1 - G)(e_2) = \{(x_1, 0.3), (x_2, 0.3), (x_3, 0.5)\},$$
$$(1 - G)(e_3) = \{(x_1, 0.7), (x_2, 0.4), (x_3, 0.2)\}\}$$

Definition 2.10 [13] A fuzzy set λ on the universe of discourse X is convex if and only if for x, y in X,

$$\lambda[ax + (1 - a)y] \geq \lambda(x) \wedge \lambda(y)$$

where $a \in [0, 1]$.

Definition 2.12 [13] A fuzzy set λ on the universe of discourse X is called normal fuzzy set if there exists $x \in X$ such that $\lambda(x) = 1$.Definition 2.13 [13] A fuzzy number is a fuzzy set defined on the universe of discourse X which is both convex and normal.

A fuzzy number λ on the universe of discourse X may be characterized by a triangular fuzzy number which is parameterized by a triplet $\tilde{b} = (a, b, c)$. The membership function of this fuzzy number is defined as

$$\lambda(x) = \begin{cases} 0, for\ x < a, \\ \dfrac{x - a}{b - a}, for\ a \leq x \leq b, \\ \dfrac{c - x}{c - b}, for\ b \leq x \leq c, \\ 1, for\ x > c \end{cases}$$

Let λ_1 and λ_2 the two triangular fuzzy numbers parameterized by the triplets $\tilde{b_1} = (a_1, b_1, c_1)$ and $\tilde{b_2} = (a_2, b_2, c_2)$ respectively.

The addition and multiplication of the fuzzy numbers λ_1 and λ_2 are given by,

$$\lambda_1 \oplus \lambda_2 = \tilde{b_1} \oplus \tilde{b_2}$$
$$= (a_1, b_1, c_1) \oplus (a_2, b_2, c_2)$$
$$= (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

and

$$\lambda_1 \otimes \lambda_2 = \tilde{b_1} \otimes \tilde{b_2}$$
$$= (a_1, b_1, c_1) \otimes (a_2, b_2, c_2)$$
$$= (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2)$$

Chen (1999) proposed that the defuzzification of the triangular fuzzy number $\tilde{b} = (a, b, c)$ is the bisection of the trapezoidal fuzzy

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number (a, b, b, c) as $(a + 2b + c)/4$.

III. APPLICATION OF FUZZY SOFT SET IN JOB SCHEDULING PROBLEM

In this section, an application of fuzzy soft set in job scheduling problem under uncertain information with the use of the method analogues to Sanchez's notion by Das & Borgohan (2010) of medical diagnosis is presented. The attempt is going to assign a particular job in a particular machine, satisfying the level of attributes in order to get the desired optimization.

A. Methodology

Now take $J = \{j_1, j_2, \ldots, j_n\}$ as the universal set where $j_1, j_2, \ldots, j_n$ represents the jobs. And consider the set $C = \{c_1, c_2, \ldots, c_p\}$ as universal set where $c_1, c_2, \ldots, c_p$ represents attributes related to the job and the set $M = \{m_1, m_2, \ldots, m_r\}$ represents the set of r machines. For this, construct a fuzzy soft set (F, J) over C where F is a mapping $F : J \rightarrow I^C$. This fuzzy soft set gives a relation matrix (weighed matrix) P, called job-attribute matrix, where each element denote the weight of the job related to the level of attribute. Then construct the another fuzzy soft set (G, C) over M, where G is a mapping $G : C \rightarrow I^M$. Let Q be the a relation matrix (weighed matrix) given by this soft set, called attribute-machine matrix, where each element denote the weight of the level of attribute for a particular machine. Here the elements in the relation matrix are taken as triangular fuzzy number $\tilde{p}$ parameterized by the triplet $(p - 1, p, p + 1)$.

B. Algorithm

The following algorithm may be followed for the desired job assignment.

Step 1 : Input the fuzzy soft set (F, J) to obtain the job-attribute matrix, P.
Step 2 : Input the fuzzy soft set (G, C) to obtain the attribute-machine matrix, Q.
Step 3 : Perform the transformation operation $P \otimes Q$ to get the job assignment matrix R^*.
Step 4 : Defuzzify all the elements of the matrix R^* to obtain the matrix R^{**}.
Step 5 : Find k in R^{**} for which $a_{ik} = \underbrace{max}_{j}\ a_{ij}$

Then conclude that the job j_j is assigned to the machine m_k. In case $\underbrace{max}_{1 \le j \le s}\ a_{ij}$ occurs for more than one value of j, then go to step 1 and repeat the process by checking the level of attributes to break the tie.

C. Numerical Example

Suppose there are four jobs $J = \{j_1, j_2, j_3, j_4\}$. Next consider the attributes set $C = \{c_1, c_2, c_3\}$ where c_1, c_2, c_3 represents attributes namely the required processing time, number of labors needed, and the total expenses. And also consider the set $M = \{m_1, m_2, m_3, m_4\}$ represents the set of four machines.

Suppose that $F : J \rightarrow I^C$ is defined as

$$F(j_1) = \{c_1/\tilde{7}, c_2/\tilde{1}, c_3/\tilde{8}\}$$
$$F(j_2) = \{c_1/\tilde{3}, c_2/\tilde{9}, c_3/\tilde{2}\}$$
$$F(j_3) = \{c_1/\tilde{9}, c_2/\tilde{2}, c_3/\tilde{6}\}$$
$$F(j_4) = \{c_1/\tilde{1}, c_2/\tilde{2}, c_3/\tilde{8}\}$$

Then the fuzzy soft set (F, J) is a parameterized family of all fuzzy sets over C. This fuzzy soft set (F, J) represents the relation matrix, job-attribute matrix P and is given by

$$P = \begin{array}{c c} & \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{array}{c} j_1 \\ j_2 \\ j_3 \\ j_4 \end{array} & \begin{pmatrix} \tilde{7} & \tilde{1} & \tilde{8} \\ \tilde{3} & \tilde{9} & \tilde{2} \\ \tilde{9} & \tilde{2} & \tilde{6} \\ \tilde{1} & \tilde{2} & \tilde{8} \end{pmatrix} \end{array}$$

Next suppose that $G : C \rightarrow I^M$ is defined as

$$G(c_1) = \{m_1/\tilde{1}, m_2/\tilde{3}, m_3/\tilde{5}, m_4/\tilde{4}\}$$

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$$G(c_2) = \{m_1/\tilde{4}, m_2/\tilde{6}, m_3/\tilde{7}, m_4/\tilde{8}\}$$
$$G(c_3) = \{m_1/\tilde{6}, m_2/\tilde{5}, m_3/\tilde{3}, m_4/\tilde{2}\}$$

Then the fuzzy soft set (G, C) is a parameterized family of all fuzzy sets over M. This fuzzy soft set (G, C) represents the relation matrix, attribute-machine matrix Q and is given by

$$Q = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \end{array} \\ \begin{pmatrix} \tilde{1} & \tilde{3} & \tilde{5} & \tilde{4} \\ \tilde{4} & \tilde{6} & \tilde{7} & \tilde{8} \\ \tilde{6} & \tilde{5} & \tilde{3} & \tilde{2} \end{pmatrix} \end{array}$$

Then performing the transformation operation $P \otimes Q$ to get the job assignment matrix R^* as

$$R^* = \begin{array}{c} \\ j_1 \\ j_2 \\ j_3 \\ j_4 \end{array} \begin{array}{c} \begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \end{array} \\ \begin{pmatrix} \widetilde{59} & \widetilde{67} & \widetilde{66} & \widetilde{52} \\ \widetilde{51} & \widetilde{73} & \widetilde{84} & \widetilde{88} \\ \widetilde{53} & \widetilde{69} & \widetilde{77} & \widetilde{64} \\ \widetilde{57} & \widetilde{55} & \widetilde{43} & \widetilde{36} \end{pmatrix} \end{array}$$

where,

$$\widetilde{59} = (35, 59, 89),\ \widetilde{67} = (40, 67, 100),\ \widetilde{66} = (38, 66, 100),\ \widetilde{52} = (25, 52, 85),$$
$$\widetilde{51} = (29, 51, 79), \widetilde{73} = (48, 73, 104), \widetilde{84} = (58, 84, 116),\ \widetilde{88} = (63, 88, 119),$$
$$\widetilde{53} = (28, 53, 84), \widetilde{69} = (41, 69, 103), \widetilde{77} = (48, 77, 112), \widetilde{64} = (36, 64, 98),$$
$$\widetilde{57} = (38, 57, 82), \widetilde{55} = (33, 55, 83), \widetilde{43} = (20, 43, 72), \widetilde{36} = (14, 36, 64).$$

Now defuzzifying the above atrix, it is found that

$$R^{**} = \begin{array}{c} \\ j_1 \\ j_2 \\ j_3 \\ j_4 \end{array} \begin{array}{c} \begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \end{array} \\ \begin{pmatrix} 60.5 & 68.5 & 67.5 & 53.5 \\ 52.5 & 74.5 & 85.5 & 89.5 \\ 54.5 & 70.5 & 78.5 & 65.5 \\ 58.5 & 56.5 & 44.5 & 37.5 \end{pmatrix} \end{array}$$

Then it is clear that from above matrix the job j_1 is assigned to machine m_2, the job j_2 is assigned to machine m_4, the job j_3 is assigned to machine m_3 and the job j_4 is assigned to machine m_1.

IV. CONCLUSION

A soft set is a mapping from parameter to the crisp subset of universe. Because of the fuzzy characters of the parameters some situations may be more complicated in real world. Here we applied the decision method using the fuzzy soft set and fuzzy numbers based on Sanchez's method, in job scheduling problem with an example. We hope that this approach will be useful to handle different uncertain problem.

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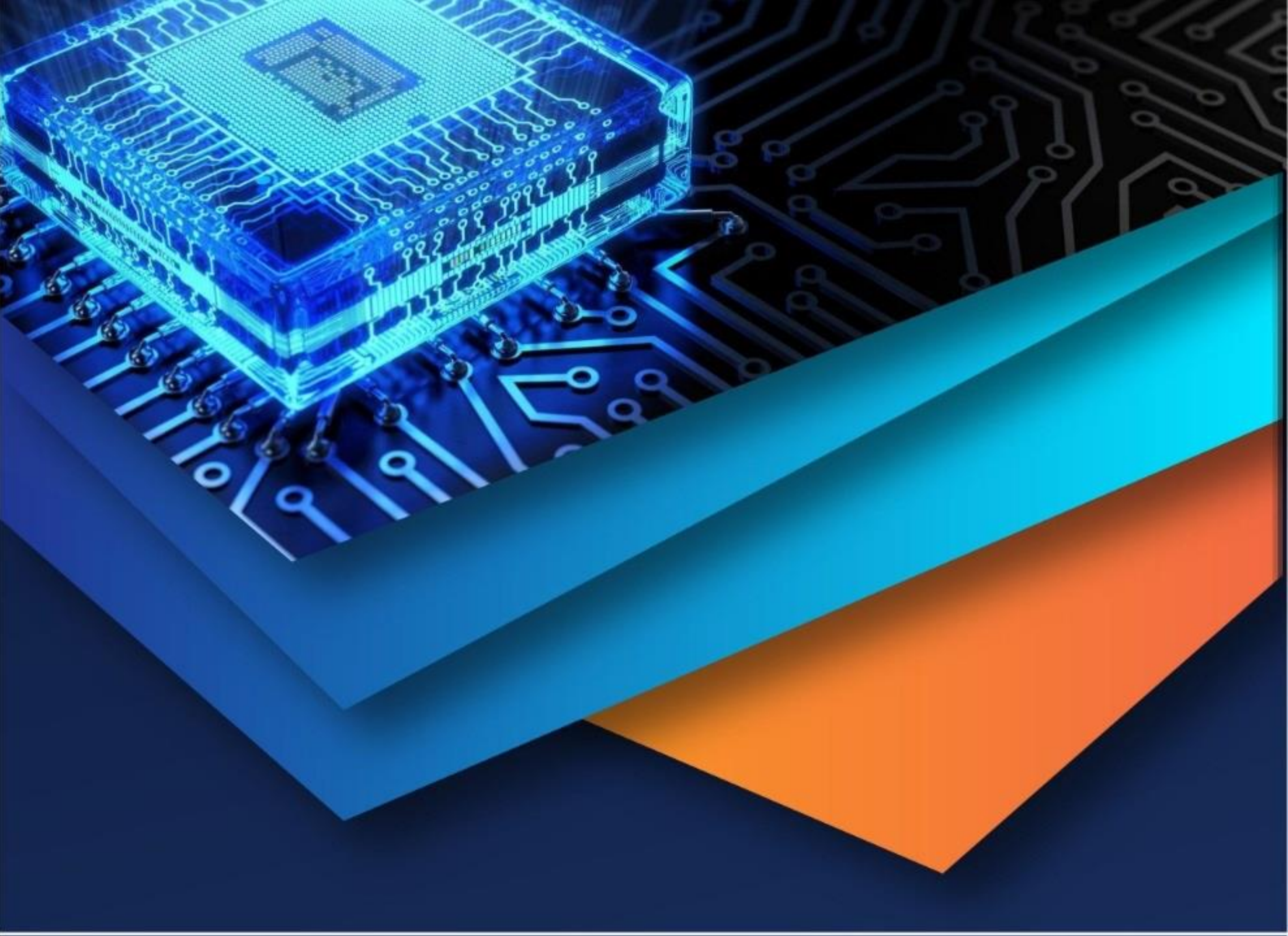

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