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Some Enestrom- Kakeya Type Results

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Abstract: In this paper we prove some Enestrom-Kakeya type results on the location of zeros of a polynomial.

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I. INTRODUCTION

Enestrom and Kakeya proved independently a very important result on the location of zeros of a polynomial with real positive coefficients. It is known as Enestrom –Kakeya theorem and is stated as follows[2,3]:

A. *Theorem A*

Let $P(z) = \sum_{j=0}^n a_j z^j$ be a polynomial of degree n such that

$$a_n \geq a_{n-1} \geq \dots \geq a_1 \geq a_0 > 0.$$

Then all the zeros of $P(z)$ lie in $|z| \leq 1$.

II. MAIN RESULTS

In this paper we change the condition on the coefficients of the polynomial and prove the following results:

A. *Theorem 1*

Let $P(z) = \sum_{j=0}^n a_j z^j$ be a polynomial of degree n with real positive coefficients satisfying

$$a_n \leq a_{n-1} \geq a_{n-2} \leq a_{n-3} \geq \dots \leq a_1 \geq a_0$$

if n is odd and

$$a_n \leq a_{n-1} \geq a_{n-2} \leq a_{n-3} \geq \dots \leq a_1 \geq a_0$$

if n is even.

Then for odd n , all the zeros of $P(z)$ lie in

$$|z| \leq \frac{2(L - M) - a_n}{a_n}$$

with $L - M > a_n$, where

$$L = a_{n-1} + a_{n-3} + \dots + a_0,$$

$$M = a_{n-2} + a_{n-4} + \dots + a_1.$$

And for even n , all the zeros of $P(z)$ lie in

$$|z| \leq \frac{2(L' - M') - a_n}{a_n}$$

where

$$L' = a_{n-1} + a_{n-3} + \dots + a_1,$$

$$M' = a_{n-2} + a_{n-4} + \dots + a_2.$$

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with $L' - M' > a_n$.

1) *Remark 1:* If we consider the polynomial

$$P(z) = z^5 + 3z^4 + 3z^3 + 4z^2 + 4z + 5,$$

then Enestrom-akeya theorem is not applicable. The classical Cauchy's theorem gives the bound for all the zeros of $P(z)$ as 6, whereas by Theorem 1, the bound is easily seen to be $3 < 6$.

If the coefficients of the polynomial $P(z)$ are complex, we prove the following result:

B. *Theorem 2*

Let $P(z) = \sum_{j=0}^n a_j z^j$ be a complex polynomial of degree n with $\text{Re}(a_j) = \alpha_j$, $\text{Im}(a_j) = \beta_j$, $j = 0, 1, 2, \dots, n$ and

$\alpha_j > 0, \forall j$ such that

$$\alpha_n \leq \alpha_{n-1} \geq \alpha_{n-2} \leq \alpha_{n-3} \geq \dots \geq \alpha_1 \leq \alpha_0$$

if n is odd and

$$\alpha_n \leq \alpha_{n-1} \geq \alpha_{n-2} \leq \alpha_{n-3} \geq \dots \leq \alpha_1 \geq \alpha_0$$

if n is even.

Then for odd n , all the zeros of $P(z)$ lie in

$$|z| \leq \frac{2(L - M) - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|}$$

where

$$L = \alpha_{n-1} + \alpha_{n-3} + \dots + \alpha_0,$$

$$M = \alpha_{n-2} + \alpha_{n-4} + \dots + \alpha_1$$

with $2(L - M) - \alpha_n + 2 \sum_{j=0}^n |\beta_j| > |a_n|$.

And for even n , all the zeros of $P(z)$ lie in

$$|z| \leq \frac{2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|}$$

where

$$L' = \alpha_{n-1} + \alpha_{n-3} + \dots + \alpha_1,$$

$$M' = \alpha_{n-2} + \alpha_{n-4} + \dots + \alpha_2.$$

with $2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j| > |a_n|$.

1) *Remark 2:* Theorem 2 is a generalization of Theorem 1 and reduces to it by taking $\beta_j = 0, \forall j$.

Applying Theorem 2 to the polynomial $-iP(z)$, we get the following result:

a) *Corollary 1:* Let $P(z) = \sum_{j=0}^n a_j z^j$ be a complex polynomial of degree n with $\text{Re}(a_j) = \alpha_j$,

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$\text{Im}(a_j) = \beta_j, j = 0,1,2,\dots,n$ and $\beta_j > 0, \forall j$ such that

$$\beta_n \leq \beta_{n-1} \geq \beta_{n-2} \leq \beta_{n-3} \geq \dots \leq \beta_1 \geq \beta_0$$

if n is odd and

$$\beta_n \leq \beta_{n-1} \geq \beta_{n-2} \leq \beta_{n-3} \geq \dots \leq \beta_1 \geq \beta_0$$

if n is even.

Then for odd n , all the zeros of $P(z)$ lie in

$$|z| \leq \frac{2(L_1 - M_1) - \beta_n + 2\sum_{j=0}^n |\alpha_j|}{|a_n|}$$

where

$$L = \beta_{n-1} + \beta_{n-3} + \dots + \beta_0,$$

$$M = \beta_{n-2} + \beta_{n-4} + \dots + \beta_1$$

with $2(L_1 - M_1) - \beta_n + 2\sum_{j=0}^n |\alpha_j| > |a_n|$.

And for even n , all the zeros of $P(z)$ lie in

$$|z| \leq \frac{2(L'_1 - M'_1) - \beta_n + 2\sum_{j=0}^n |\alpha_j|}{|a_n|}$$

where

$$L'_1 = \beta_{n-1} + \beta_{n-3} + \dots + \beta_1,$$

$$M'_1 = \beta_{n-2} + \beta_{n-4} + \dots + \beta_2.$$

with $2(L'_1 - M'_1) - \beta_n + 2\sum_{j=0}^n |\alpha_j| > |a_n|$.

Next we prove the following result:

C. Theorem 3

Let $P(z) = \sum_{j=0}^n a_j z^j$ be a complex polynomial of degree n such that for some real α, β ,

$$|\arg a_j - \beta| \leq \alpha \leq \frac{\pi}{2}, j = 0,1,2,\dots,n$$

and

$$|a_n| \leq |a_{n-1}| \geq |a_{n-2}| \leq |a_{n-3}| \geq \dots \leq |a_1| \geq |a_0|$$

if n is odd and

$$|a_n| \leq |a_{n-1}| \geq |a_{n-2}| \leq |a_{n-3}| \geq \dots \leq |a_1| \geq |a_0|$$

if n is even.

Then for odd n , all the zeros of $P(z)$ lie in

$$|z| \leq \frac{1}{|a_n|} [|a_n| (\sin \alpha - \cos \alpha) + 2(\cos \alpha + \sin \alpha) (|a_{n-1}| + |a_{n-3}| + \dots + |a_2|)]$$

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$$-2(\cos \alpha - \sin \alpha)(|a_{n-2}| + |a_{n-4}| + \dots + |a_1|) + (\cos \alpha + \sin \alpha + 1)|a_0|$$

and for even n, all the zeros of P(z) lie in

$$|z| \leq \frac{1}{|a_n|} [|a_n|(\sin \alpha - \cos \alpha) + 2(\cos \alpha + \sin \alpha)(|a_{n-1}| + |a_{n-3}| + \dots + |a_1|) \\ - 2(\cos \alpha - \sin \alpha)(|a_{n-2}| + |a_{n-4}| + \dots + |a_2|) + (\sin \alpha - \cos \alpha + 1)|a_0|]$$

1) Remark 3: If we take $\alpha = 0, \beta = 0, a_j > 0, \forall j$, Theorem 3 reduces to Theorem 1.

III. LEMMA

For the proofs of the above results, we need the following lemma:

A. Lemma 1

For any two complex numbers b_1, b_2 such that $|b_1| \geq |b_2|$ and $|\arg b_j - \beta| \leq \alpha \leq \frac{\pi}{2}, j = 1, 2$

for some real α, β ,

$$|b_1 - b_2| \leq (|b_1| - |b_2|) \cos \alpha + (|b_1| + |b_2|) \sin \alpha.$$

The above lemma is due to Govil and Rahman [1].

IV. PROOFS OF THEOREMS

A. Proof of Theorem 2

Consider the polynomial

$$F(z) = (1 - z)P(z) \\ = (1 - z)(a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0) \\ = -a_n z^{n+1} + (a_n - a_{n-1})z^n + \dots + (a_{\lambda+1} - a_\lambda)z^{\lambda+1} + (a_\lambda - a_{\lambda-1})z^\lambda \\ + \dots + (a_1 - a_0)z + a_0 \\ = -a_n z^{n+1} + (\alpha_n - \alpha_{n-1})z^n + (\alpha_{n-1} - \alpha_{n-2})z^{n-1} + \dots + (\alpha_1 - \alpha_0)z + \alpha_0 \\ + i\{(\beta_n - \beta_{n-1})z^n + (\beta_{n-1} - \beta_{n-2})z^{n-1} + \dots + (\beta_1 - \beta_0)z + \beta_0\}.$$

For $|z| > 1$ so that $\frac{1}{|z|^j} < 1, j = 1, 2, \dots, n$, and odd n, we have, by using the hypothesis

$$|F(z)| \geq |a_n||z|^{n+1} - [|\alpha_n - \alpha_{n-1}||z|^n + |\alpha_{n-1} - \alpha_{n-2}||z|^{n-1} + \dots + |\alpha_1 - \alpha_0||z| + |\alpha_0| \\ + |\beta_n - \beta_{n-1}||z|^n + |\beta_{n-1} - \beta_{n-2}||z|^{n-1} + \dots + |\beta_1 - \beta_0||z| + |\beta_0|] \\ = |z|^n [|a_n||z| - \{ |\alpha_n - \alpha_{n-1}| + \frac{|\alpha_{n-1} - \alpha_{n-2}|}{|z|} + \frac{|\alpha_{n-2} - \alpha_{n-3}|}{|z|^2} + \dots + \frac{|\alpha_1 - \alpha_0|}{|z|^{n-1}} + \frac{|\alpha_0|}{|z|^n} \\ + |\beta_n - \beta_{n-1}| + \frac{|\beta_{n-1} - \beta_{n-2}|}{|z|} + \dots + \frac{|\beta_1 - \beta_0|}{|z|^{n-1}} + \frac{|\beta_0|}{|z|^n} \}] \\ > |z|^n [|a_n||z| - \{ |\alpha_n - \alpha_{n-1}| + |\alpha_{n-1} - \alpha_{n-2}| + |\alpha_{n-2} - \alpha_{n-3}| + \dots + |\alpha_1 - \alpha_0| + |\alpha_0| \\ + |\beta_n - \beta_{n-1}| + |\beta_{n-1} - \beta_{n-2}| + \dots + |\beta_1 - \beta_0| + |\beta_0| \}]$$

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$$\begin{aligned}
 & + |\beta_n - \beta_{n-1}| + |\beta_{n-1} - \beta_{n-2}| + \dots + |\beta_1 - \beta_0| + |\beta_0| \}] \\
 \geq & |z|^n [|a_n| |z| - \{ |\alpha_n - \alpha_{n-1}| + |\alpha_{n-1} - \alpha_{n-2}| + |\alpha_{n-2} - \alpha_{n-3}| + \dots + |\alpha_1 - \alpha_0| + |\alpha_0| \\
 & + |\beta_n| + |\beta_{n-1}| + |\beta_{n-1}| + |\beta_{n-2}| + \dots + |\beta_1| + |\beta_0| + |\beta_0| \}] \\
 \geq & |z|^n [|a_n| |z| - \{ \alpha_{n-1} - \alpha_n + \alpha_{n-1} - \alpha_{n-2} + \alpha_{n-3} - \alpha_{n-2} + \dots + \alpha_2 - \alpha_1 + \alpha_0 - \alpha_1 \\
 & + \alpha_0 + 2 \sum_{j=0}^n |\beta_j| \}] \\
 = & |z|^n [|a_n| |z| - \{ 2(\alpha_{n-1} + \alpha_{n-3} + \dots + \alpha_0) - 2(\alpha_{n-2} + \alpha_{n-4} + \dots + \alpha_1) - \alpha_n + 2 \sum_{j=0}^n |\beta_j| \}] \\
 = & |z|^n [|a_n| |z| - \{ 2L - 2M - \alpha_n + 2 \sum_{j=0}^n |\beta_j| \}] \\
 > & 0
 \end{aligned}$$

if

$$|z| > \frac{2L - 2M - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|}$$

provided $2L - 2M - \alpha_n + 2 \sum_{j=0}^n |\beta_j| > |a_n|$.

This shows that those zeros of F(z) whose modulus is greater than 1 lie in

$$|z| \leq \frac{2(L - M) - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|} .$$

Since $2L - 2M - \alpha_n + 2 \sum_{j=0}^n |\beta_j| > |a_n|$, it follows that those zeros of F(z) whose modulus is less than or equal to 1 already lie in

$$|z| \leq \frac{2(L - M) - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|} .$$

Since the zeros of P(z) are also the zeros of F(z), it follows that all the zeros of P(z) lie in

$$|z| \leq \frac{2(L - M) - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|}$$

in case n is odd.

If n is even, then for $|z| > 1$ so that $\frac{1}{|z|^j} < 1, j = 1, 2, \dots, n$, we have, as in the above case, by using the hypothesis

$$|F(z)| \geq |z|^n [|a_n| |z| - \{ |\alpha_n - \alpha_{n-1}| + |\alpha_{n-1} - \alpha_{n-2}| + |\alpha_{n-2} - \alpha_{n-3}| + \dots + |\alpha_1 - \alpha_0| + |\alpha_0|$$

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$$\begin{aligned}
 & + |\beta_n| + |\beta_{n-1}| + |\beta_{n-1}| + |\beta_{n-2}| + \dots + |\beta_1| + |\beta_0| + |\beta_0| \}] \\
 \geq & |z|^n [|a_n| |z| - \{ \alpha_{n-1} - \alpha_n + \alpha_{n-1} - \alpha_{n-2} + \alpha_{n-3} - \alpha_{n-2} \dots + \alpha_1 - \alpha_2 + \alpha_1 - \alpha_0 + \alpha_0 \\
 & + 2 \sum_{j=0}^n |\beta_j| \}] \\
 = & |z|^n [|a_n| |z| - \{ 2(\alpha_{n-1} + \alpha_{n-3} + \dots + \alpha_1) - 2(\alpha_{n-2} + \alpha_{n-4} + \dots + \alpha_2) - \alpha_n + 2 \sum_{j=0}^n |\beta_j| \}] \\
 = & |z|^n [|a_n| |z| - \{ 2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j| \}] \\
 > & 0
 \end{aligned}$$

if

$$|z| > \frac{2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|}$$

provided $2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j| > |a_n|$.

This shows that those zeros of F(z) whose modulus is greater than 1 lie in

$$|z| \leq \frac{2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|}.$$

Since $2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j| > |a_n|$, it follows that those zeros of F(z) whose modulus is less than or equal to 1 already lie in

$$|z| \leq \frac{2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|}.$$

Since the zeros of P(z) are also the zeros of F(z), it follows that all the zeros of P(z) lie in

$$|z| \leq \frac{2(L' - M') - \alpha_n + 2 \sum_{j=0}^n |\beta_j|}{|a_n|}$$

in case n is even.

That completes the proof of Theorem 2.

B. Proof of Theorem 3

Consider the polynomial

$$\begin{aligned}
 F(z) &= (1 - z)P(z) \\
 &= -a_n z^{n+1} + (a_n - a_{n-1})z^n + (a_{n-1} - a_{n-2})z^{n-1} + \dots + (a_1 - a_0)z + a_0
 \end{aligned}$$

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For $|z| > 1$ so that $\frac{1}{|z|^j} < 1, j = 1, 2, \dots, n$, and odd n , we have, as seen earlier, by using the hypothesis

$$\begin{aligned} |F(z)| &\geq |z|^n [|a_n||z| - \{|a_n - a_{n-1}| + |a_{n-1} - a_{n-2}| + |a_{n-2} - a_{n-3}| + \dots + |a_1 - a_0| + |a_0| \\ &\geq |z|^n [|a_n||z| - \{(|a_{n-1}| - |a_n|)\cos\alpha + (|a_{n-1}| + |a_n|)\sin\alpha + (|a_{n-1}| - |a_{n-2}|)\cos\alpha \\ &\quad + (|a_{n-1}| + |a_{n-2}|)\sin\alpha + (|a_{n-3}| - |a_{n-2}|)\cos\alpha + (|a_{n-3}| + |a_{n-2}|)\sin\alpha \\ &\quad + \dots + (|a_2| - |a_1|)\cos\alpha + (|a_2| + |a_1|)\sin\alpha + (|a_0| - |a_1|)\cos\alpha \\ &\quad + (|a_0| + |a_1|)\sin\alpha + |a_0\}|] \\ &= |z|^n [|a_n||z| - \{|a_n|(\sin\alpha - \cos\alpha) + 2(\cos\alpha + \sin\alpha)(|a_{n-1}| + |a_{n-3}| + \dots + |a_2|) \\ &\quad - 2(\cos\alpha - \sin\alpha)(|a_{n-2}| + |a_{n-4}| + \dots + |a_1|) + (\cos\alpha + \sin\alpha + 1)|a_0\}|] \\ &> 0 \end{aligned}$$

if

$$\begin{aligned} |z| > \frac{1}{|a_n|} [&|a_n|(\sin\alpha - \cos\alpha) + 2(\cos\alpha + \sin\alpha)(|a_{n-1}| + |a_{n-3}| + \dots + |a_2|) \\ &- 2(\cos\alpha - \sin\alpha)(|a_{n-2}| + |a_{n-4}| + \dots + |a_1|) + (\cos\alpha + \sin\alpha + 1)|a_0\}]. \end{aligned}$$

This shows that those zeros of $F(z)$ whose modulus is greater than 1 lie in

$$\begin{aligned} |z| \leq \frac{1}{|a_n|} [&|a_n|(\sin\alpha - \cos\alpha) + 2(\cos\alpha + \sin\alpha)(|a_{n-1}| + |a_{n-3}| + \dots + |a_2|) \\ &- 2(\cos\alpha - \sin\alpha)(|a_{n-2}| + |a_{n-4}| + \dots + |a_1|) + (\cos\alpha + \sin\alpha + 1)|a_0\}]. \end{aligned}$$

Since those zeros of $F(z)$ whose modulus is less than or equal to 1 already satisfy the above inequality, it follows that all the zeros of $F(z)$ and hence $P(z)$ lie in

$$\begin{aligned} |z| \leq \frac{1}{|a_n|} [&|a_n|(\sin\alpha - \cos\alpha) + 2(\cos\alpha + \sin\alpha)(|a_{n-1}| + |a_{n-3}| + \dots + |a_2|) \\ &- 2(\cos\alpha - \sin\alpha)(|a_{n-2}| + |a_{n-4}| + \dots + |a_1|) + (\cos\alpha + \sin\alpha + 1)|a_0\}]. \end{aligned}$$

in case n is odd.

If n is even, we have as in the above case, for $|z| > 1$,

$$\begin{aligned} |F(z)| &\geq |z|^n [|a_n||z| - \{(|a_{n-1}| - |a_n|)\cos\alpha + (|a_{n-1}| + |a_n|)\sin\alpha + (|a_{n-1}| - |a_{n-2}|)\cos\alpha \\ &\quad + (|a_{n-1}| + |a_{n-2}|)\sin\alpha + (|a_{n-3}| - |a_{n-2}|)\cos\alpha + (|a_{n-3}| + |a_{n-2}|)\sin\alpha \\ &\quad + \dots + (|a_1| - |a_2|)\cos\alpha + (|a_1| + |a_2|)\sin\alpha + (|a_1| - |a_0|)\cos\alpha \\ &\quad + (|a_1| + |a_0|)\sin\alpha + |a_0\}|] \\ &= |z|^n [|a_n||z| - \{|a_n|(\sin\alpha - \cos\alpha) + 2(\cos\alpha + \sin\alpha)(|a_{n-1}| + |a_{n-3}| + \dots + |a_1|) \\ &\quad - 2(\cos\alpha - \sin\alpha)(|a_{n-2}| + |a_{n-4}| + \dots + |a_2|) + (\sin\alpha - \cos\alpha + 1)|a_0\}|] \\ &> 0 \end{aligned}$$

if

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$$|z| > \frac{1}{|a_n|} [|a_n| (\sin \alpha - \cos \alpha) + 2(\cos \alpha + \sin \alpha) (|a_{n-1}| + |v_{n-3}| + \dots + |a_1|) \\ - 2(\cos \alpha - \sin \alpha) (|a_{n-2}| + |a_{n-4}| + \dots + |a_2|) + (\sin \alpha - \cos \alpha + 1) |a_0|].$$

This shows that those zeros of F(z) whose modulus is greater than 1 lie in

$$|z| \leq \frac{1}{|a_n|} [|a_n| (\sin \alpha - \cos \alpha) + 2(\cos \alpha + \sin \alpha) (|a_{n-1}| + |v_{n-3}| + \dots + |a_1|) \\ - 2(\cos \alpha - \sin \alpha) (|a_{n-2}| + |a_{n-4}| + \dots + |a_2|) + (\sin \alpha - \cos \alpha + 1) |a_0|].$$

Since those zeros of F(z) whose modulus is less than or equal to 1 already satisfy the above inequality, it follows that all the zeros of F(z) and hence P(z) lie in

$$|z| \leq \frac{1}{|a_n|} [|a_n| (\sin \alpha - \cos \alpha) + 2(\cos \alpha + \sin \alpha) (|a_{n-1}| + |v_{n-3}| + \dots + |a_1|) \\ - 2(\cos \alpha - \sin \alpha) (|a_{n-2}| + |a_{n-4}| + \dots + |a_2|) + (\sin \alpha - \cos \alpha + 1) |a_0|].$$

in case n is even.

That completes the proof of Theorem 3.

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