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# Numerical Analysis of Three-Dimensional Squeezing Nanofluid Flow in a Rotating Channel on a Lower Stretching Porous Wall

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**Abstract:** Unsteady 3D rotating Nano fluid flow of an Incompressible electronically conducting viscous fluid between two infinite horizontal plane walls investigated Numerically, The governing Navier stokes equations are converted into ordinary differential equations by using similarity transformation. Runge kutta method sixth order by using mathematical software is used to solve the complete system of nonlinear ODE. The non dimensional parameters characteristic parameter of the flow, rotation parameter, the magnetic parameter, nano particle volume fraction, the suction parameter analyzed numerically and graphically with comparison of analytical result.

## I. INTRODUCTION

Nano fluid is a liquid covering nanometer-sized elements (having diameter less than 100nm), named nanoparticles. The nanoparticles are classically made up of metals, oxides and carbides or carbon nanotubes. Owing to the vast number of submissions in chemical as well as motorized engineering procedures such as the production of thin plastic sheets, insulating materials, paper fabrication, and other various procedures, this paper pays specific care to the study of rotating flow over stretching surfaces [1]. Bachok et al. [2] employed a numerical, Keller-Box technique for steady Nano fluid flow over a porous rotating disk. Khairy Zaimi et al. [3] inspected the Flow and Heat Transfer over a Shrinking Sheet in a Nano fluid with Suction at the Boundary. Prasad et al. [4] working a fourth order Runge-Kutta integration system to explore the consequence of variable fluid viscosity, magnetic parameter, Prandtl number, variable thermal conductivity, thermal radiation parameter and heat source/sink parameter, on MHD fluid flow over a stretching sheet. Three-dimensional flow in a channel with a stretching wall was investigated by Borkakoti and Bharali [5]. Domairry and Aziz [6] found an analytic solution for unsteady MHD squeezing flow with suction and injection properties by the use of homotopy perturbation technique. El-Mistikawy and Attia [7] was investigated the influence of an external uniform magnetic field on the flow due to a rotating disk. Syahira Mansur and AnuarIshak [8] studying the heat transfer characteristic of an unsteady boundary layer flow of a nanofluid over a stretching sheet. Mehmood and Ali [9] analyzed a three-dimensional flow in a channel bounded through the lower stretching plate and upper permeable plate and experimental that the viscous drag at lower wall rises due to the incidence of injection at the higher plate. To the best of authors' knowledge, Choi and Eastman [10] stood probably the first investigators who employed a mixture of nanoparticles and base fluid and called this mixture a "Nano fluid." A varied choice of evaluation documents have been issued on Nano fluids in recent years. NavidFreidoonimehr [11] derives a coupled system of non-linear ordinary differential equations to model the three- dimensional flow rotating channel of a nanofluid on a lower permeable stretching wall. The three-dimensional problem of steady fluid deposition on an inclined rotating disk is studied by Sheikholeslami et al. [12] and M. Sheikholeslami [13] at al study is to apply HPM to find approximate solutions of nonlinear differential equations governing the problem of heat transfer in the unsteady squeezing Nano fluid flow between parallel plates.

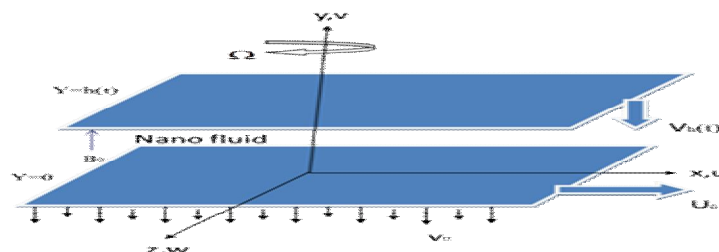


Figure 1: Schematic diagram of the flow configuration and the coordinate system for the considered flow

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### II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider unsteady 3D rotating Nano fluid flow of an incompressible electrically conducting viscous fluid between two infinite horizontal plane walls. The lower plane is placed at  $y=0$  and is stretched with a time-dependent velocity  $U_o(t) = ax/(1-\gamma t)$  in  $x$ -direction. The upper plane is also placed at a variable distance  $h(t) = \sqrt{v_f(1-\gamma t)/a}$  and the fluid is squeezed with a time-dependent velocity  $v_h = dh/dt$  in the negative  $y$ -direction. The fluid and the channel are rotating around  $y$ -axis with an angular velocity  $\Omega = \omega\hat{j}/(1-\gamma t)$  and also the lower plate intakes the flow with the velocity  $-V_0/(1-\gamma t)$ . A magnetic field with density  $B_0/\sqrt{(1-\gamma t)}$  is applied along the  $y$ -axis about the system which is rotating. These velocities and magnetic fields are introduced to obtain similarity solutions by reducing governing equations into ordinary differential equations (ODEs).

The physical model of the considered problem along with the coordinate system is illustrated in Figure 1. The governing equations of continuity and momentum of Nano fluid flow in a rotating frame.

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\rho_{nf} \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} + 2\Omega \times \mathbf{V} \right] = \nabla \cdot \mathbf{T} + \mathbf{J} \times \mathbf{B},$$

Where  $\mathbf{T}$  is the Cauchy stress tensor,  $\mathbf{J}$  the magnetic flux, and  $\mathbf{B}$  the current density. The above governing equations can be also described by the following set of Navier-Stokes equations [14, 15]:

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + 2 \frac{\omega}{1-\gamma t} \omega &= -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + v_{nf} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \frac{\sigma B_0^2}{\rho_{nf} (1-\gamma t)} u + g\beta (T - T_\infty) + g\beta^* (C - C_\infty) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + v_{nf} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] \\ \frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} - 2 \frac{\omega}{1-\gamma t} \omega &= v_{nf} \left[ \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right] - \frac{\sigma B_0^2}{\rho_{nf} (1-\gamma t)} \omega, \end{aligned} \tag{2}$$

Where  $\rho_{nf}$  is the Nano fluid density,  $v_{nf} (= \mu_{nf} / \rho_{nf})$  is the Nano fluid kinematic viscosity, where  $\mu_{nf}$  has been proposed by Brinkman,  $\sigma$  is the electrical conductivity,  $B_0$  is the magnetic field, and  $\gamma$  is the characteristic parameter with dimension of  $(time)^{-1}$  and  $\gamma t < 1$ . The above Nano fluid constants are defined as follows:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{3}$$

Where  $\mu_f$  is the viscosity of the fluid fraction,  $\phi$  is the nanoparticle volume fraction, and  $\rho_f$  and  $\rho_s$  are the densities of the fluid and of the solid fractions, respectively. The thermo physical properties of the base fluid (water) and different nano particles. The appropriate boundary conditions are introduced as follows:

$$\left. \begin{aligned} u(x, y, t) &= U_0 = \frac{ax}{1-\gamma t} \\ v(x, y, t) &= -\frac{V_0}{1-\gamma t}, \\ w(x, y, t) &= 0 \end{aligned} \right\} \text{at } y = 0$$

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$$\left. \begin{aligned} u(x, y, t) &= 0 \\ v(x, y, t) &= V_h = \frac{dh}{dt} = \frac{-\gamma}{2} \sqrt{\frac{vf}{a(1-\gamma t)}} \\ w(x, y, t) &= 0 \end{aligned} \right\} at \quad (4)$$

Where  $a$  is the stretching rate of the lower plate. The following appropriate similarity transformations are employed to convert the above governing equations (2)–(4) into a system of ordinary differential equations in terms of a stream function  $\psi$ :

$$\psi = \sqrt{\frac{avf}{1-\gamma t}} x f(\eta), \quad \eta = \frac{y}{h(t)}, \quad (5)$$

$$u = \frac{\partial \psi}{\partial y} = U_0 f'(\eta), \quad v = -\frac{\partial \psi}{\partial x} = -\sqrt{\frac{avf}{1-\gamma t}} f(\eta) \quad (6)$$

$$w = U_0 g(\eta)$$

Substituting the similarity transformations (5) and (6) into (2)-(4), we obtain the following System of nonlinear ordinary differential equations:

$$f''' + \frac{vf}{v_{nf}} (ff'' - (f')^2) - \beta(f' + \frac{\eta}{2} f'') + Gr\theta + Gm\phi - 2\Omega g - \frac{1}{\rho_{nf} / \rho_f} M^2 f' = \frac{(1-\gamma t)^2 v_f}{\rho_{nf} a^2 x v_{nf}} \frac{\partial p}{\partial x}$$

$$-f'' + \frac{vf}{v_{nf}} (-ff' - \frac{\beta}{2} (f + \eta f'')) = \frac{(1-\gamma t)}{\rho_{nf} a^2 x v_{nf}} \frac{\partial p}{\partial \eta} \quad (7)$$

$$g'' + \frac{vf}{v_{nf}} (fg' - f'g - \beta(g + \frac{\eta}{2} g') + 2\Omega f') - \frac{1}{\mu_{nf} / \mu_f} M^2 g = 0 \quad (8)$$

Where  $\beta = \gamma/a$  is the characteristic parameter of the flow,  $\Omega = \omega/a$  is the rotation parameter,  $M^2 = \sigma B_0^2 / \rho_f a$  is the magnetic parameter, and prime denotes differentiation with respect to  $\eta$ . In order to squeeze the flow, we take  $\beta > 0$ , for which the upper plate moves downward with velocity  $V_h < 0$ . For  $\beta < 0$  the upper plate moves apart with respect to the plane  $y = 0$  and  $\beta = 0$  corresponds to the steady state case of the considered problem or stationary upper plate. In order to reduce the number of independent variables and to retain the similarity solution, (7) are simplified by cross differentiation and thus we obtain the following system of differential equations:

$$f^{iv} - (1-\varphi + \varphi(\rho_s / \rho_f))(1-\varphi)^{2.5} \times (ff'' - ff''' + 2\Omega g' + \frac{\beta}{2} (3f'' + \eta f''')) - (1-\varphi)^{2.5} M^2 f'' = 0$$

$$g'' - (1-\varphi + \varphi(\rho_s / \rho_f))(1-\varphi)^{2.5} \times ((fg' - f'g + \beta(g + \frac{\eta}{2} g') + 2\Omega f')) - (1-\varphi)^{2.5} M^2 g = 0 \quad (9)$$

The transformed boundary conditions take the form

$$\begin{aligned} f(0) &= \omega_0, f'(0) = 1, g(0) = 0, \\ f(1) &= \frac{\beta}{2}, f'(1) = 0, g(1) = 0, \\ \omega_0 &= V_0 / ah \end{aligned} \quad (10)$$

Where  $\omega_0 = V_0 / ah$  is the suction parameter In this problem, the physical quantity of interest is the skin friction coefficients  $C_f$

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along the stretching wall at the lower and upper walls which are defined as [13]

$$C_{f,lower} = \frac{\mu_{nf}(\partial u / \partial y)_{y=0}}{\rho_{nf}U_0^2}, \quad (11)$$

$$C_{f,upper} = \frac{\mu_{nf}(\partial u / \partial y)_{y=h(t)}}{\rho_{nf}U_0^2},$$

Substituting (6) into (11) we obtain

$$\bar{C}_{f,lower} =$$

$$C_{f,lower} Re_x = \frac{f''(0)}{(1-\phi + \phi(\rho_s / \rho_f))(1-\phi)^{2.5}},$$

$$\bar{C}_{f,upper} =$$

$$C_{f,upper} Re_x = \frac{f''(1)}{(1-\phi + \phi(\rho_s / \rho_f))(1-\phi)^{2.5}}, \quad (12)$$

Where  $Re_x = \rho_f U_0 h / \mu_f$  is the local Reynolds number. In this section we have derived the coupled system of nonlinear ordinary differential equations (9) to model the three-dimensional flow of a Nano fluid in a rotating channel on a lower permeable stretching wall. We next apply the DTM to solve the coupled system.

### III. NUMERICAL ANALYSIS

In this study is to analyzed Unsteady 3D rotating Nano fluid flow of an Incompressible electronically conducting viscous fluid between two infinite horizontal plane walls investigated Numerically, The governing Navier stokes equations are converted into ordinary differential equations by using similarity transformation. Runge kutta method sixth order by using mathematical software is used to solve the complete system of nonlinear ODE. The non dimensional parameters characteristic parameter of the flow, rotation parameter, the magnetic parameter, nano particle volume fraction, the suction parameter analyzed numerically and graphically with comparison of analytical result.

### IV. RESULTS AND DISCUSSION

In this section we analyzed the clear insight of the Problem. The non-dimensional parameters are Rotation parameter( $\Omega$ ), Characteristic parameter( $\beta$ ), Magnetic parameter( $M$ ), Nano particle volume friction( $\phi$ ) and Suction parameter ( $\omega_0$ ) The Normal velocity  $f(\eta)$  and the transverse velocity  $g(\eta)$  profiles are given graphically

Fig 1 exhibits the schematic diagram of the flow configuration in the physical problem. Fig 2 & Fig 3 shows that the effect of Rotation parameter ( $\Omega$ ) and the Magnetic parameter ( $M$ ) on the transverse velocity profiles ( $g(\eta)$ ).It is observed that, the velocity profiles increases with Increasing values of  $\Omega$  and  $M$

Fig4 & Fig5 shows that the Effect of suction parameter ( $\omega_0$ ) and the nano particle volume fraction ( $\phi$ ) on the transverse velocity profiles  $g(\eta)$ . It is observed that the velocity profile decreases with increasing values of  $\omega_0$  and  $\phi$

Fig6 & Fig7 shows that the effect of Rotation parameter ( $\Omega$ ) and the Magnetic parameter ( $M$ ) On the Normal velocity profiles  $f(\eta)$ .It is observed that the velocity profile decrease with an increasing values of  $\Omega$  and  $M$

Fig 8 & Fig 9 shows that the effect of characteristic parameter ( $\beta$ ) on the normal velocity ( $\eta$ ) and the transverse velocity  $g(\eta)$  profiles both are enhances with increasing values of  $\beta$  ,Fig 10 & Fig 11 shows that the effect of suction parameter ( $\omega_0$ ) and Nano particle volume fraction ( $\phi$ ) on the normal velocity ( $\eta$ ) profiles, It is observed that the velocity profiles increases that increasing values of  $\phi$  and  $\omega_0$



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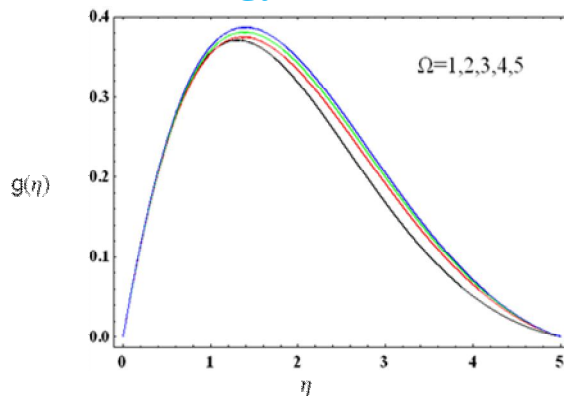


Figure 2: Effect of  $\Omega$  on the transverse velocity of  $g(\eta)$   
when  $\beta=1$ ,  $\varphi=0.1$ , and  $M=\omega_0=0.5$ .

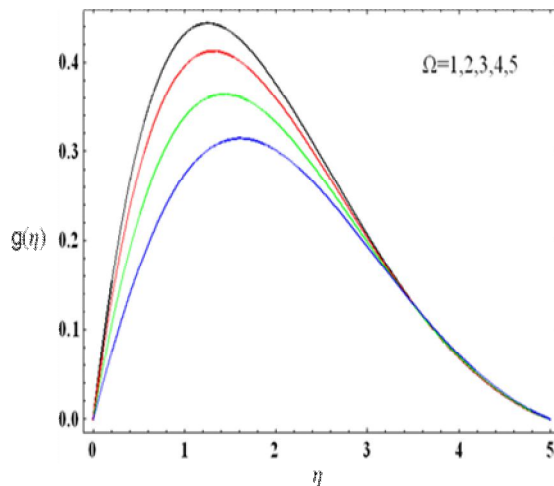


Figure 3: Effect of  $\Omega$  on the transverse velocity of  $g(\eta)$   
when  $\beta=1$ ,  $\varphi=0.1$ , and  $M=\omega_0=0.5$ .

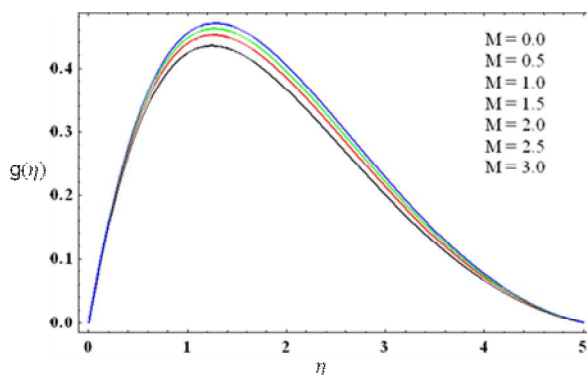


Figure 4: Effect of  $M$  on the velocity component of  $g(\eta)$   
when  $\beta=\Omega=1$ ,  $\varphi=0.1$ , and  $\omega_0=0.5$ .

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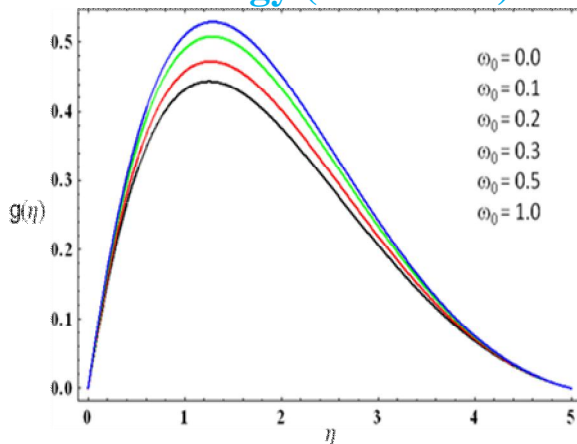


Figure 5: Effect of  $\omega_0$  on the velocity component of  $g(\eta)$   
 when  $\beta = \Omega = 1$ ,  $M = 0.5$ , and  $\varphi = 0.1$ .

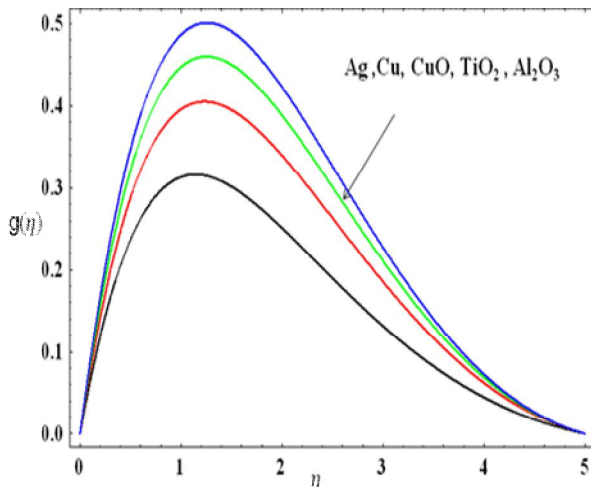


Figure 6: Effect of Nano particle types on the velocity component of  $g(\eta)$   
 when  $\beta = \Omega = 1$ ,  $M = \omega_0 = 0.5$ , and  $\varphi = 0.1$ .

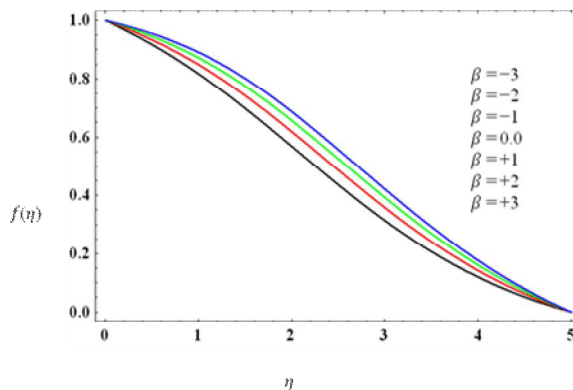


Figure 7: Effect of  $\beta$  on the velocity component of  $f(\eta)$   
 when  $\Omega = 1$ ,  $\varphi = 0.1$ , and  $M = \omega_0 = 0.5$ .

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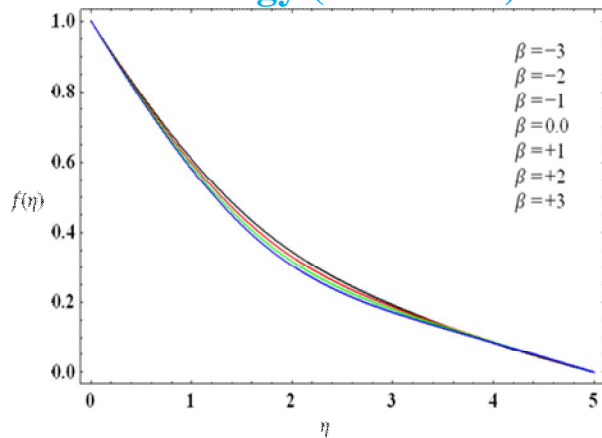


Figure 8: Effect of  $\beta$  on the velocity component of  $g(\eta)$   
 when  $\Omega=1$ ,  $\varphi = 0.1$ , and  $M = \omega_0 = 0.5$ .

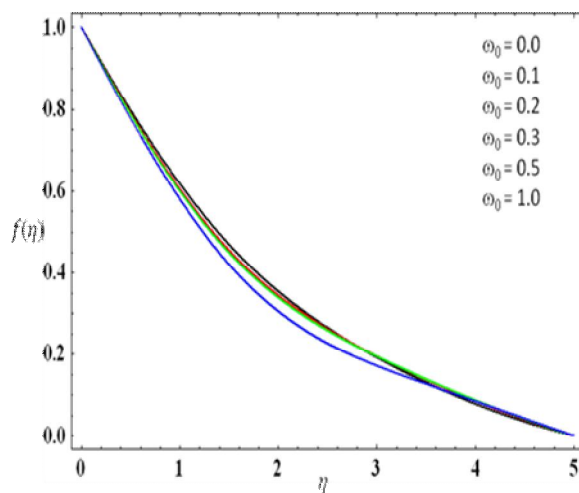


Figure 10: Effect of  $\omega_0$  on the velocity component of  $f(\eta)$   
 when  $\beta = \Omega = 1$ ,  $M = 0.5$ , and  $\varphi = 0.1$ .

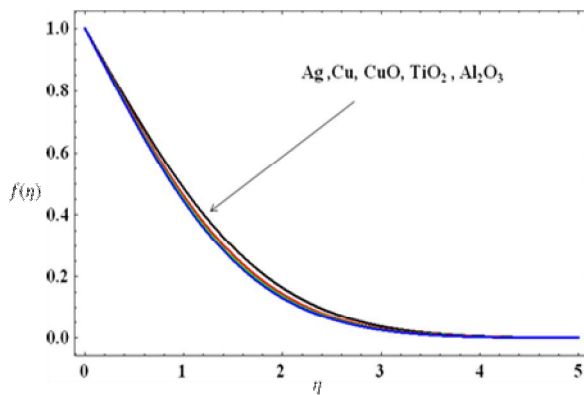


Figure 11: Effect of Nano particle types on the velocity component of  $f(\eta)$   
 when  $\beta = \Omega = 1$ ,  $M = \omega_0 = 0.5$ , and  $\varphi = 0.1$ .

## V. CONCLUSION



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In this paper, we studied the learning on Numerical Analysis of Three-Dimensional Squeezing Nanofluid Flow in a Rotating Channel on a Lower Stretching Porous Wall. The non dimensional parameters characteristic parameter of the flow, the magnetic parameter, nano particle volume fraction, rotation parameter, the suction parameter analyzed numerically and graphically with comparison of analytical result.

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