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# Operations on Hesitant Fuzzy Hypergraph

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**Abstract :** The concept of hypergraphs was extended to fuzzy hypergraph. In this paper, we extend the concepts of fuzzy hypergraphs into that of hesitant fuzzy hypergraphs. Based on the definition of hesitant fuzzy graph, order of the hfhg, size of the hfhg, operations like complement, join, union, intersection, ringsum, cartesian product, composition are defined for hesitant fuzzy graphs. The authors further proposed to apply these operations in clustering techniques.

## I. INTRODUCTION

Hypergraph theory, originally developed by C.Berge in 1960, is a generalization of graph theory. The concept of hypergraphs can model more general types of relations than binary relations. The notion of hypergraphs has been extended in fuzzy theory and the concept of fuzzy hypergraphs was proposed by Lee-Kwang and S.M.Chen. The authors have already introduced the concept of Hesitant fuzzy hypergraph. Operations on HFHGs have also been analyzed to a real-life problem with a numerical example.

## II. PRELIMINARIES

### A. Definition 1.1

Let  $X$  be a fixed set, a Hesitant fuzzy set (HFS) on  $X$  is in terms of a function that when applied to  $X$  returns a subset of  $[0, 1]$ . The HFS is defined by a mathematical symbol:

$$A = \{ \langle x, h_A(x) \rangle / x \in X \}$$

where  $h_A(x)$  is a set of some values in  $[0, 1]$ , denoting the possible membership degrees of the element  $x \in X$  to the set  $A$ . Xu and Xia (2011b) called  $h = h_A(x)$  a hesitant fuzzy element (HFE) and  $\Theta$  the set of all HFEs.

### B. Definition 1.2

A Hesitancy Fuzzy Graph is of the form  $G = (V, E)$ , where

$V = \{v_1, v_2, \dots, v_n\}$  such that  $\mu_1: V \rightarrow [0, 1], \gamma_1: V \rightarrow [0, 1]$  and  $\beta_1: V \rightarrow [0, 1]$  denote the degree of membership, non-membership and hesitancy of the element  $v_i \in V$  respectively and  $\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1$  for every  $v_i \in V$ , where  $\beta_1(v_i) = 1 - [\mu_1(v_i) + \gamma_1(v_i)]$  and  $E \subseteq V \times V$  where  $\mu_2: V \times V \rightarrow [0, 1], \gamma_2: V \times V \rightarrow [0, 1]$  and  $\beta_2: V \times V \rightarrow [0, 1]$  are such that,

$$\mu_2(v_i, v_j) \leq \min [\mu_1(v_i), \mu_1(v_j)]$$

$$\gamma_2(v_i, v_j) \leq \max [\gamma_1(v_i), \gamma_1(v_j)]$$

$$\beta_2(v_i, v_j) \leq \min [\beta_1(v_i), \beta_1(v_j)]$$

$$\text{and } 0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) + \beta_2(v_i, v_j) \leq 1 \text{ for every } (v_i, v_j) \in E.$$

### C. Definition 1.3

1) Hypergraph  $H$  is an ordered pair  $H = (V, E)$  where

- $V = \{v_1, v_2, v_3 \dots v_n\}$ , a finite set of vertices.
- $E = \{E_1, E_2, E_3 \dots E_m\}$ , a family of subsets of  $V$ .
- $E_j \neq \emptyset, j = 1, 2, \dots, m$  and  $\cup E_j = V$ .

The set  $V$  is called the set of vertices and  $E$  is the set of edges (or hyper edges).

### D. Definition 1.4

In a hypergraph, two or more vertices  $x_1, x_2, \dots, x_n$  are said to be adjacent if there exist an edge  $E_j$  which contains those vertices. In a hypergraph two edges  $E_i$  &  $E_j, i \neq j$  is said to be adjacent if their intersection is not empty.

### E. Definition 1.5

A regular hyper graph is one in which every vertex is contained in  $k$  edges for some constant  $k$ .

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**F. Definition 1.6**

1) Hesitant Fuzzy HyperGraph (HFHG) is an ordered pair  $H = \langle V, E \rangle$  where

- a)  $V = \{v_1, v_2, v_3 \dots v_n\}$ , a finite set of vertices
- b)  $E = \{E_1, E_2, E_3 \dots E_m\}$ , a family of Hesitant fuzzy subsets of  $V$ .
- c)  $E_j = \{ \langle v_i, \mu_j(v_i), \gamma_j(v_i), \beta_j(v_i) \rangle / \mu_j(v_i), \gamma_j(v_i), \beta_j(v_i) \geq 0 \text{ and } 0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) + \beta_2(v_i, v_j) \leq 1, j = 1, 2, \dots, m \}$
- d)  $E_j \neq \emptyset, j = 1, 2, \dots, m$

Here the edges  $E_j$  are HFHGs.  $\mu_j(v_i), \gamma_j(v_i), \beta_j(v_i)$  denote the degree of membership, non-membership and hesitancy of the element  $v_i \in V$  respectively of the vertex  $v_i$  to edge  $E_j$ .

**G. Definition 1.7**

Let  $H = (X, E)$  be a Regular Hesitant Fuzzy Hypergraph. The **order** of a Regular Hesitant Fuzzy Hypergraph,  $H$  in

$$O(H) = \left( \sum_{x \in X} \mu_{E_i}(x), \sum_{x \in X} \gamma_{E_i}(x), \sum_{x \in X} \beta_{E_i}(x) \right) \text{ for every } x \in X.$$

The **size** of Regular Hesitant Fuzzy Hypergraph is

$$S(H) = \sum_{i=1}^n S(E_i) \text{ where } S(E_i) = \left( \sum_{x \in E_i} \mu_{E_i}(x), \sum_{x \in E_i} \gamma_{E_i}(x), \sum_{x \in E_i} \beta_{E_i}(x) \right)$$

1) **Example 1:** Consider the Hesitant Fuzzy Hypergraph  $H = (X, E)$ , Define  $X = \{a, b, c, d\}$  and

$E = \{E_1, E_2, E_3, E_4\}$ , where

- $E_1 = \{(a, 0.1, 0.7, 0.2) (b, 0.1, 0.7, 0.2)\}$
- $E_2 = \{(b, 0.1, 0.7, 0.2) (c, 0.1, 0.7, 0.2)\}$
- $E_3 = \{(c, 0.1, 0.7, 0.2) (d, 0.1, 0.7, 0.2)\}$
- $E_4 = \{(d, 0.1, 0.7, 0.2) (a, 0.1, 0.7, 0.2)\}$

Here, Order of the HFHG,  $O(H) = (0.4, 2.8, 0.8)$ ,

Size of the HFHG,  $S(H) = (0.8, 5.6, 1.6)$ .

### III. OPERATIONS ON HESITANT FUZZY HYPERGRAPHS

**A. Definition 2.1**

Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two HFHGs. Then the **Union** of  $G_1$  and  $G_2$  is an HFHG  $G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$  defined by

$$\begin{aligned}
 (\mu_1 \cup \mu_1')(v) &= \begin{cases} \mu_1(v) & \text{if } v \in V_1 - V_2 \\ \mu_1'(v) & \text{if } v \in V_2 - V_1 \end{cases} \\
 (\gamma_1 \cup \gamma_1')(v) &= \begin{cases} \gamma_1(v) & \text{if } v \in V_1 - V_2 \\ \gamma_1'(v) & \text{if } v \in V_2 - V_1 \end{cases} \\
 (\beta_1 \cup \beta_1')(v) &= \begin{cases} \beta_1(v) & \text{if } v \in V_1 - V_2 \\ \beta_1'(v) & \text{if } v \in V_2 - V_1 \end{cases} \\
 \text{And } (\mu_2 \cup \mu_2')(v_i v_j) &= \begin{cases} \mu_{2ij} & \text{if } e_{ij} \in E_1 - E_2 \\ \mu_{2ij}' & \text{if } e_{ij} \in E_2 - E_1 \end{cases}
 \end{aligned}$$

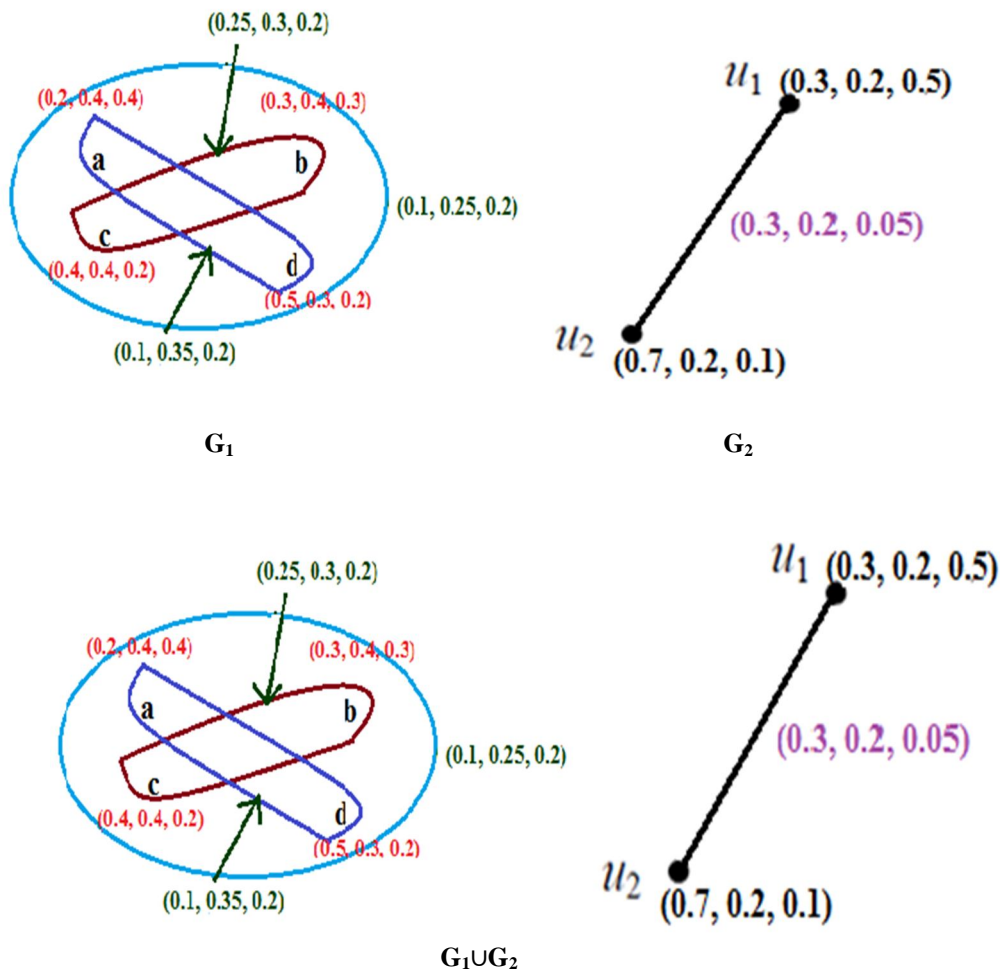
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$$(\gamma_2 \cup \gamma_2')(v_i v_j) = \begin{cases} \gamma_{2ij} & \text{if } e_{ij} \in E_1 - E_2 \\ \gamma_{2ij}' & \text{if } e_{ij} \in E_2 - E_1 \end{cases}$$

$$(\beta_2 \cup \beta_2')(v_i v_j) = \begin{cases} \beta_{2ij} & \text{if } e_{ij} \in E_1 - E_2 \\ \beta_{2ij}' & \text{if } e_{ij} \in E_2 - E_1 \end{cases}$$

where  $(\mu_1, \gamma_1, \beta_1)$  and  $(\mu_1', \gamma_1', \beta_1')$  refer the vertex membership, non-membership and hesitant element of  $G_1$  and  $G_2$  respectively;  $(\mu_2, \gamma_2, \beta_2)$  and  $(\mu_2', \gamma_2', \beta_2')$  refer the edge membership, non-membership and hesitant element of  $G_1$  and  $G_2$  respectively.

1) *Example 2:* Consider the Hesitant Fuzzy Hypergraphs  $G_1 = (V_1, E_1)$  where  $V_1 = \{a, b, c, d\}$  and  $G_2 = (V_2, E_2)$  where  $V_2 = \{u_1, u_2\}$ .



### B. Definition 2.2

Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two HFHG. Then the **Join** of  $G_1$  and  $G_2$  is an HFHG  $G = G_1 + G_2 = (V_1 \cup V_2, E_1 \cup E_2)$  defined by

$$(\mu_1 + \mu_1')(v) = (\mu_1 \cup \mu_1')(v) \quad \text{if } v \in V_1 \cup V_2$$

$$(\gamma_1 + \gamma_1')(v) = (\gamma_1 \cup \gamma_1')(v) \quad \text{if } v \in V_1 \cup V_2$$

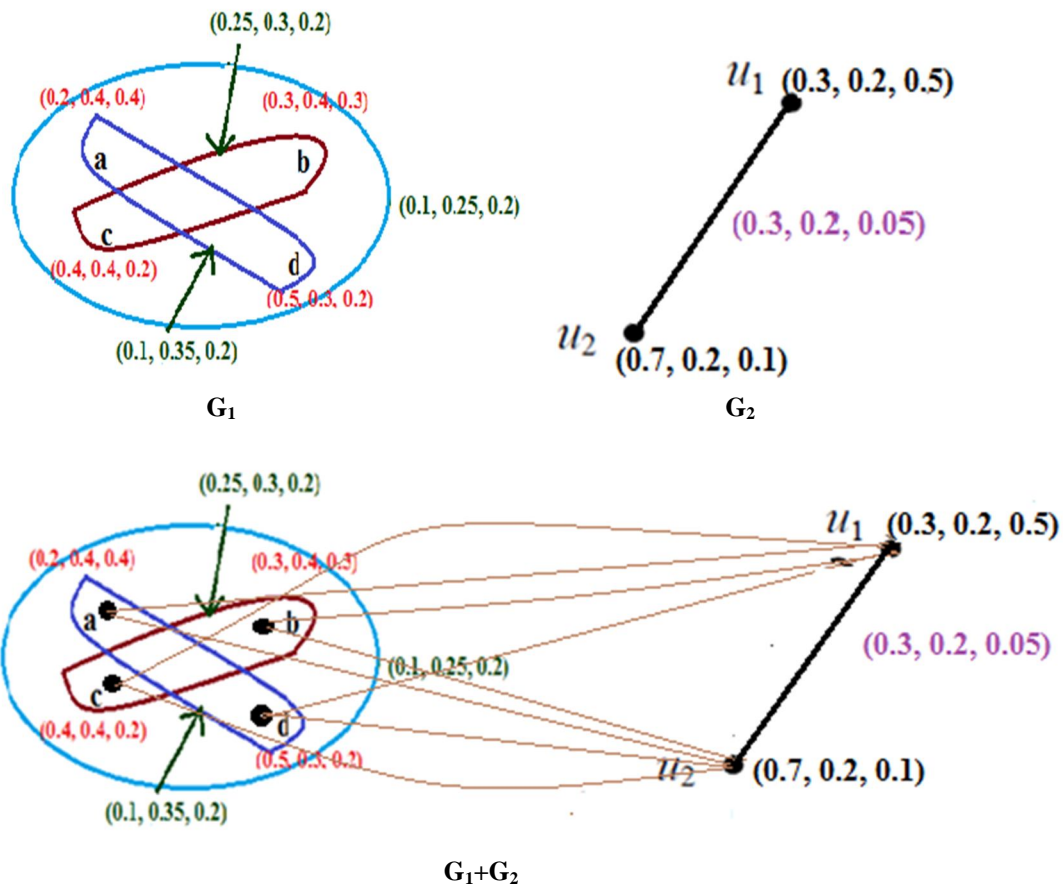
$$(\beta_1 + \beta_1')(v) = (\beta_1 \cup \beta_1')(v) \quad \text{if } v \in V_1 \cup V_2$$

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$$(\mu_1 + \mu'_1)(v_i v_j) = \begin{cases} (\mu_1 + \mu'_1)(v_i v_j) & \text{if } v_i v_j \in E_1 \cup E_2 \\ \mu_1(v_i) \mu'_1(v_j) & \text{if } v_i v_j \in E' \end{cases}$$

$$(\gamma_1 + \gamma'_1)(v_i v_j) = \begin{cases} (\gamma_1 + \gamma'_1)(v_i v_j) & \text{if } v_i v_j \in E_1 \cup E_2 \\ \gamma_1(v_i) \gamma'_1(v_j) & \text{if } v_i v_j \in E' \end{cases} \quad (\beta_1 + \beta'_1)(v_i v_j) = \begin{cases} (\beta_1 + \beta'_1)(v_i v_j) & \text{if } v_i v_j \in E_1 \cup E_2 \\ \beta_1(v_i) \beta'_1(v_j) & \text{if } v_i v_j \in E' \end{cases}$$

1) *Example 3:* Consider the Hesitant Fuzzy Hypergraphs  $G_1 = (V_1, E_1)$  where  $V_1 = \{a, b, c, d\}$  and  $G_2 = (V_2, E_2)$  where  $V_2 = \{u_1, u_2\}$ .



### C. Definition 2.3

Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two HFHGs. Then the **Intersection** of HFHGs  $G_1$  and  $G_2$ , denoted by  $G_1 \cap G_2$ , is an HFHG defined by

$$(\mu_1 \cap \mu'_1)(v) = \begin{cases} \mu_1(v) & \text{if } v \in V_1 \text{ and } V_2 \\ \mu'_1(v) & \text{if } v \in V_2 \text{ and } V_1 \end{cases}$$

$$(\gamma_1 \cap \gamma'_1)(v) = \begin{cases} \gamma_1(v) & \text{if } v \in V_1 \text{ and } V_2 \\ \gamma'_1(v) & \text{if } v \in V_2 \text{ and } V_1 \end{cases}$$

$$(\beta_1 \cap \beta'_1)(v) = \begin{cases} \beta_1(v) & \text{if } v \in V_1 \text{ and } V_2 \\ \beta'_1(v) & \text{if } v \in V_2 \text{ and } V_1 \end{cases}$$

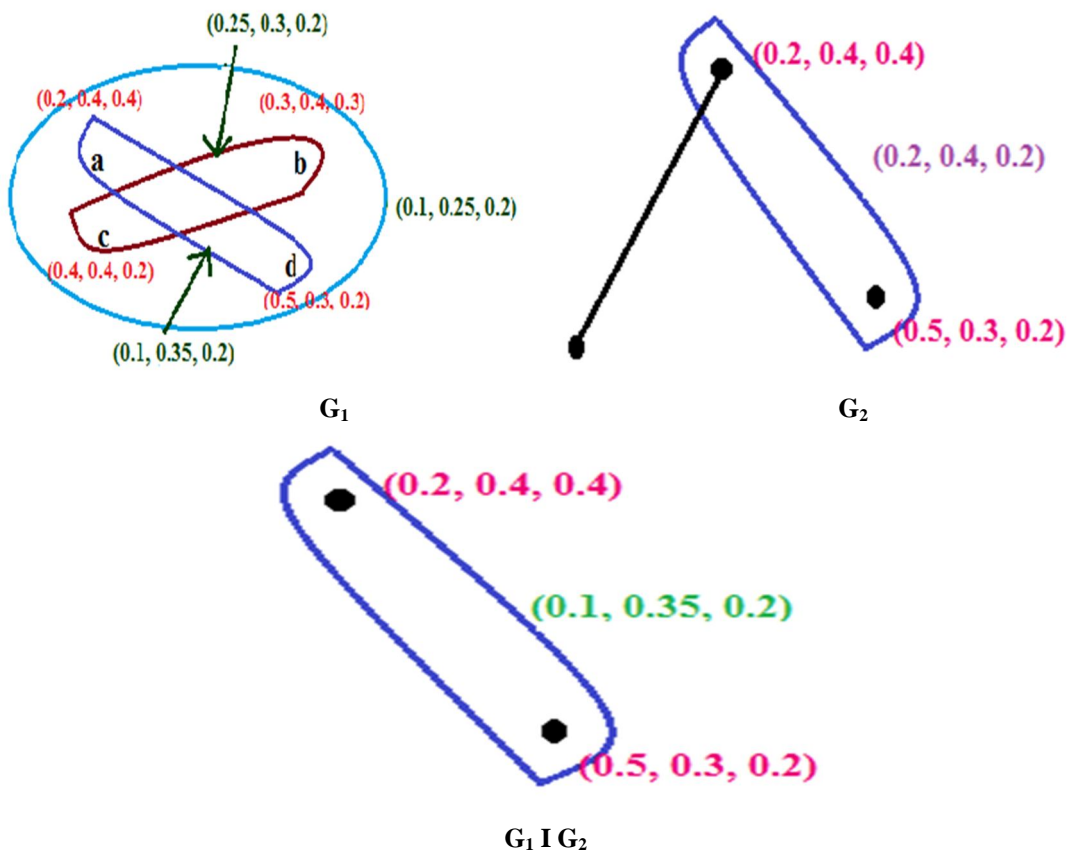
$$(\mu_2 \cap \mu'_2)(v_i v_j) = \begin{cases} \mu_{2ij} & \text{if } e_{ij} \in E_1 \text{ and } E_2 \\ \mu'_{2ij} & \text{if } e_{ij} \in E_2 \text{ and } E_1 \end{cases}$$

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$$(\gamma_2 \text{ I } \gamma_2')(v_i v_j) = \begin{cases} \gamma_{2ij} & \text{if } e_{ij} \in E_1 \text{ and } E_2 \\ \gamma'_{2ij} & \text{if } e_{ij} \in E_2 \text{ and } E_1 \end{cases}$$

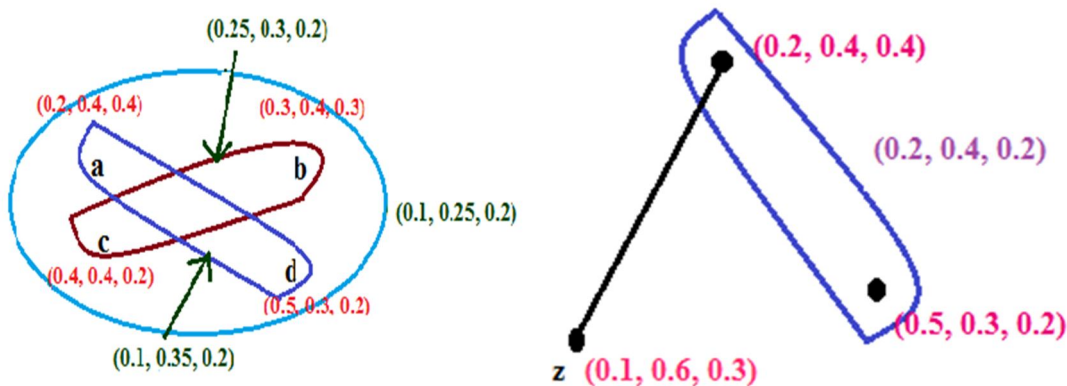
$$(\beta_2 \text{ I } \beta_2')(v_i v_j) = \begin{cases} \beta_{2ij} & \text{if } e_{ij} \in E_1 \text{ and } E_2 \\ \beta'_{2ij} & \text{if } e_{ij} \in E_2 \text{ and } E_1 \end{cases}$$

1) Example 4:

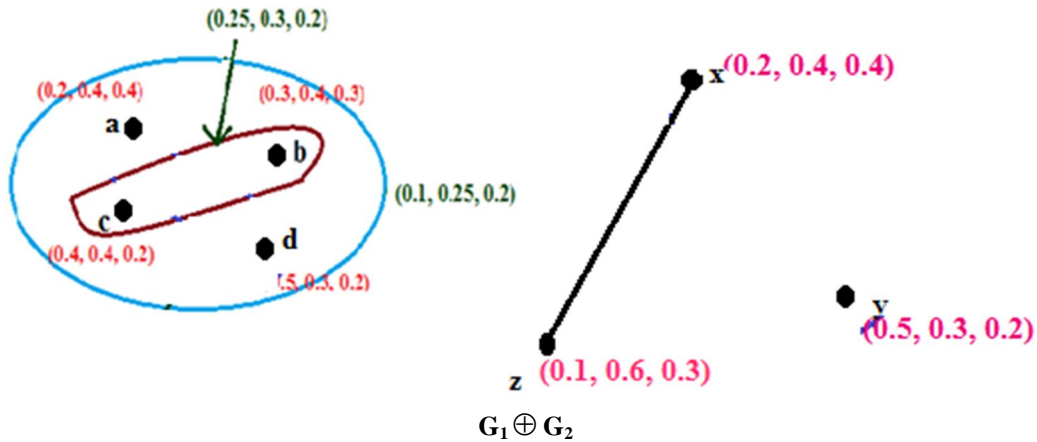


**D. Definition 2.4**

Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two HFHGs. Then the ring sum of HFHGs  $G_1$  and  $G_2$  is an HFHG defined by  $G_1 \oplus G_2 = (G_1 \cup G_2) - (G_1 \text{ I } G_2)$ .



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**E. Definition 2.5**

Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two HFHGs. Then the Cartesian product of HFHGs  $G_1$  and  $G_2$  is an HFHG, denoted by  $G = G_1 \times G_2$ , defined by  $G = (V, E)$   $V = V_1 \times V_2$  and

$$E = \{(u, u_2)(u, v_2) : u \in V_1, u_2 v_2 \in E_2\} \cup \{(u_1, w)(v_1, w) : w \in V_2, u_1 v_1 \in E_1\}$$

where

(i)

$$(\mu_1 \times \mu_1')(u_1, u_2) = \mu_1(u_1) \cdot \mu_1'(u_2) \text{ for every } (u_1, u_2) \in V$$

$$(\gamma_1 \times \gamma_1')(u_1, u_2) = \gamma_1(u_1) \cdot \gamma_1'(u_2) \text{ for every } (u_1, u_2) \in V \text{ and}$$

$$(\beta_1 \times \beta_1')(u_1, u_2) = \beta_1(u_1) \cdot \beta_1'(u_2) \text{ for every } (u_1, u_2) \in V$$

(ii)

$$(\mu_2 \times \mu_2')(u, u_2)(u, v_2) = \mu_1(u) \cdot \mu_2(u_2 v_2) \text{ for every } u \in V_1, u_2 v_2 \in E_2$$

$$(\gamma_2 \times \gamma_2')(u, u_2)(u, v_2) = \gamma_1(u) \cdot \gamma_2(u_2 v_2) \text{ for every } u \in V_1, u_2 v_2 \in E_2$$

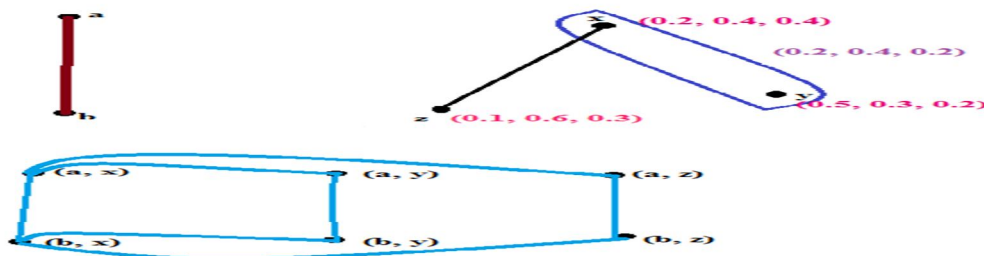
$$(\beta_2 \times \beta_2')(u, u_2)(u, v_2) = \beta_1(u) \cdot \beta_2(u_2 v_2) \text{ for every } u \in V_1, u_2 v_2 \in E_2 \text{ and}$$

$$(\mu_2 \times \mu_2')(u_1, w)(v_1, w) = \mu_1(w) \cdot \mu_2(u_1 v_1) \text{ for every } w \in V_2, u_1 v_1 \in E_1$$

$$(\gamma_2 \times \gamma_2')(u_1, w)(v_1, w) = \gamma_1(w) \cdot \gamma_2(u_1 v_1) \text{ for every } w \in V_2, u_1 v_1 \in E_1$$

$$(\beta_2 \times \beta_2')(u_1, w)(v_1, w) = \beta_1(w) \cdot \beta_2(u_1 v_1) \text{ for every } w \in V_2, u_1 v_1 \in E_1$$

**1) Example 5:**



**F. Definition 2.6**

Let  $G = G_1 \circ G_2 = (V_1 \times V_2, E)$  be the Composition of two graphs  $G_1$  and  $G_2$  where  $V = V_1 \times V_2$  and

$$E = \{(u, u_2)(u, v_2) : u \in V_1, u_2 v_2 \in E_2\} \cup \{(u_1, w)(v_1, w) : w \in V_2, u_1 v_1 \in E_1\}$$

$$\cup \{(u_1, u_2)(v_1, v_2) : u_1 v_1 \in E_1, u_2 \neq v_2\}$$

Then, the ‘‘Composition of HFHGs  $G_1$  and  $G_2$ ’’, denoted by  $G = G_1 \circ G_2$ , is an HFHG defined by

(i)

$$(\mu_1 \circ \mu_1')(u_1, u_2) = \mu_1(u_1) \cdot \mu_1'(u_2) \text{ for every } (u_1, u_2) \in V_1 \times V_2$$

$$(\gamma_1 \circ \gamma_1')(u_1, u_2) = \gamma_1(u_1) \cdot \gamma_1'(u_2) \text{ for every } (u_1, u_2) \in V_1 \times V_2 \text{ and}$$

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$$(\beta_1 \circ \beta_1')(u_1, u_2) = \beta_1(u_1) \cdot \beta_1'(u_2) \text{ for every } (u_1, u_2) \in V_1 \times V_2$$

(ii)

$$(\mu_2 \circ \mu_2')(u, u_2)(u, v_2) = \mu_1(u) \cdot \mu_2(u_2 v_2) \text{ for every } u \in V_1, u_2 v_2 \in E_2$$

$$(\gamma_2 \circ \gamma_2')(u, u_2)(u, v_2) = \gamma_1(u) \cdot \gamma_2(u_2 v_2) \text{ for every } u \in V_1, u_2 v_2 \in E_2$$

$$(\beta_2 \circ \beta_2')(u, u_2)(u, v_2) = \beta_1(u) \cdot \beta_2(u_2 v_2) \text{ for every } u \in V_1, u_2 v_2 \in E_2$$

$$(\mu_2 \circ \mu_2')(u_1, w)(v_1, w) = \mu_1(w) \cdot \mu_2(u_1 v_1) \text{ for every } w \in V_2, u_1 v_1 \in E_1$$

$$(\gamma_2 \circ \gamma_2')(u_1, w)(v_1, w) = \gamma_1(w) \cdot \gamma_2(u_1 v_1) \text{ for every } w \in V_2, u_1 v_1 \in E_1$$

$$(\beta_2 \circ \beta_2')(u_1, w)(v_1, w) = \beta_1(w) \cdot \beta_2(u_1 v_1) \text{ for every } w \in V_2, u_1 v_1 \in E_1 \text{ and}$$

$$(\mu_2 \circ \mu_2')(u_1, u_2)(v_1, v_2) = \mu_1'(u_2) \cdot \mu_1'(v_2) \cdot \mu_2(u_1 v_1) \text{ for every } (u_1, u_2), (v_1, v_2) \in E - E''$$

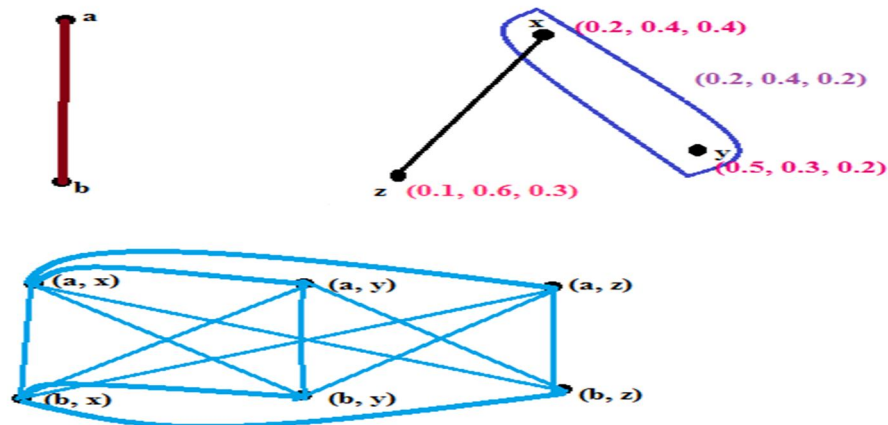
$$(\gamma_2 \circ \gamma_2')(u_1, u_2)(v_1, v_2) = \gamma_1'(u_2) \cdot \gamma_1'(v_2) \cdot \gamma_2(u_1 v_1) \text{ for every } (u_1, u_2), (v_1, v_2) \in E - E''$$

$$(\beta_2 \circ \beta_2')(u_1, u_2)(v_1, v_2) = \beta_1'(u_2) \cdot \beta_1'(v_2) \cdot \beta_2(u_1 v_1) \text{ for every } (u_1, u_2), (v_1, v_2) \in E - E''$$

where

$$E'' = \{(u, u_2)(u, v_2) : u \in V_1, u_2 v_2 \in E_2\} \cup \{(u_1, w)(v_1, w) : w \in V_2, u_1 v_1 \in E_1\}$$

1) Example 6:



### IV. CONCLUSION

In this paper, order of the HFHG, size of the HFHG, Operations like complement, join, union, intersection, ringsum, Cartesian product, composition on HFHGs are defined. Currently, the authors are working on existing clustering techniques. Further, it is proposed to apply the properties of HFHGs to develop a new clustering algorithm and the same may be checked with a numerical dataset.

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