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A Study on Some Important Aspects of Fuzzy Logic

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Abstract: The present paper deals with the problems involving human factors, such as decision making, utility theory and behavioural models of different factors that could not be formulated properly by mathematical model based on two valued logic or traditional set theory. In view of fuzzy human feelings and concepts, fuzzy human feelings cannot be described by black and white models only. Another purpose of this paper is to inspect the basic definitions in this field. Definitions, theorems and proofs on fuzzy set theory always hold for non fuzzy sets. Due to this generalization, theory of fuzzy sets has a wider scope of applicability than that of an abstract set theory in solving problems which involves, to some extent, subjective evaluation. This theory of fuzzy sets and fuzzy logic has been studied throughout the paper.
Keywords: Fuzzy sets, fuzzy logic, mathematical model, decision making, traditional set theory

I.   FUZZY SETS: AN INTRODUCTION

Every individual who are from any discipline requires a structure for existence on globe. Every structure desires a theory, which is useful on a collection of well defined objects, which is called 'set'. The worth of any structure depends upon the worth of theory of sets .It means that higher the quality of set is superior the quality of structure people for such structure, are encountered with mans' problems in their lives. To represent these problems, several mathematicians have introduced the concept of sets in their own ways. These ways of representing problems are more rigid. The solutions using this concept are not so meaningful in many circumstances. This difficulty was overcome by the fuzzy concept, which was first introduced by an eminent American Cyberneticist *Prof. L.A.Zadeh in 1965*. Since then it has occupied almost all domains of human lives. The Concept of almost all branches of human knowledge has been redefined using fuzzy sets.

In the Classical set /Traditional set the characteristic function assigns only two values 0 and 1, that means there are only two possibilities a member belongs to or not belongs to in that set.

But in the case of fuzzy set , the characteristic function is defined in such a way that the values assigned to the elements of the universal set drop within a specified range of real numbers in the interval [0,1].It also contains all those members which are partially belongs to in that set.

II.   FUZZY SETS: THE PAST

Fuzzy Set Theory was introduced by Lofti A Zadeh Professor of electrical engineering with the University of California at Berkeley in 1965.He published the first paper on his new approach of fuzzy sets and systems. Since 1980s, this mathematical theory of "unsharp amounts" has been applied with great success in many different fields. Thanks not least of all too extensive advertising campaigns for fuzzy-controlled household appliances and to their prominent presence in the media, first in Japan and then in other countries, the word "fuzzy" has also become very well-known among non-scientists. On the other hand, the story of how Fuzzy Set Theory and its earliest applications originated has remained largely unknown. In this project, the history of the theory of Fuzzy Sets and Systems and the ways it was first used will be incorporated into the history of 20th century science and technology. Influences from system theory and cybernetics stemming from the earliest part of the 20th century are considered alongside those communication and control theory from mid-century.

III.   SOME IMPORTANT DEFINITIONS

A.   Definition 1 - Classical Sets
Either an element belongs to the set or it does not. For example, either you are in the Chhattisgarh or you are not .Another example is for good students, one cannot say either he/she good or not good. Classical sets are also called "Crisp sets".

B.   Definition 2 - Membership Functions

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Fuzziness in a fuzzy set is characterized by its membership functions. A fuzzy set has a graphical description that expresses how the transition takes place to another this graphical description is called "membership functions".

C. Definition 3 - Fuzzy Sets
A set whose elements have degree of membership is called "Fuzzy sets". Fuzzy means "vagueness". Fuzziness occurs when set boundaries are not clear- cut.

IV.   MATHEMATICAL DEFINITION OF FUZZY SET

Let X be the universe of discourse and [0, 1] be the closed interval of real numbers. Then a mapping A: X $\square$ [0, 1] is called a fuzzy set A or fuzzy subset A of X for which

$$A(x) = \square,$$

Where x is a member of X and $\square$ is a real number belonging to [0,1].
 Example IV(i)
Let   X =   {1, 2, 3, 4, 5, 6} be the universal set and
                 A : X $\square$ [0,1] such that
A (1) = .3   i.e 1 $\square$.3 A means 1 $\square$ A with gradation .3
A (2) = .1   i.e 2 $\square$.1 A means 2 $\square$ A with gradation .1
A (3) = .7   i.e 3 $\square$.7 A means 3 $\square$ A with gradation .7
A (4) = .8   i.e 4 $\square$.8 A means 4 $\square$ A with gradation .8
A (5) = .9   i.e 5 $\square$ .9 A means 5 $\square$ A with gradation .9
A(6)=1   i.e. 6$\square$ 1 A means 6 $\square$ A with gradation 1.
Then the fuzzy set A is written as
A = {(1, .3), (2, .1), (3, .7), (4, .8), (5,.9),(6,1)}

V.   THE CRISP SET VS THE FUZZY SET

The classical set is defined by crisp boundaries, i.e., there is no uncertainty in the place of the boundaries of the set.
In a fuzzy set, its set boundaries are ambiguously particular.

A. Example
To differentiate classical set and fuzzy sets, we take an example-
Suppose Poverty is the main problem of our state Chhattisgarh. The government wants to solve this problem by making a definite rule that "a person is poor if he earns Rs. 15,000 or less per annum and he will get concessions from the government time to time.
Suppose Mr. A earns Rs. 15,000 per annum and Mr. B earns Rs. 15,100 per annum. Then according to this given rule, A is poor and B is not. A is eligible for the concessions while B is not, this is meaningless conclusion. Though a clear line of demarcation defining the jurisdiction of the collection of the poor people does not exist, but it is obvious that some vague demarcation must exist. A person who earns Rs. 10,000 p.a is surely poor while a person earning Rs.1,00000 p.a is certainly not poor. So while moving from the financial status of the former person to later one, this line of demarcation must have been crossed somewhere. Such jurisdiction of the collection of poor people may start gradually.
The above example is now represented in both classical and fuzzy way as follows.

B. Classical Representation of the Problem
To solve the main problem of example IV (i), non fuzzy mathematicians have represented the collection of poor people in the following manner:
Let I be the set of citizens of Chhattisgarh and A be the collection of poor people of Chhattisgarh. Also let E (x) denotes the annual earning of each people of Chhattisgarh x.Then the mathematical representation of the set A of poor people of Chhattisgarh is viewed as a characteristic function.

$$A: I \rightarrow [0, 1]$$

Such that if A(x) be the grade of membership of a person of Chhattisgarh, then
$$A(x) = 1 \text{ if } E(x) \ \square \ 15,000$$

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= 0 if E(x) > 15,000

Here if E(Roshan)= 14,000 then Roshan is poor.

if E(Rohit) =15,000 then Rohit is poor.

if E (Manoj) = 15,100 then Manoj is not poor.

if E(Sunil) = 15,500 then Sunil is not poor.

if E(John) = 16,000 then John is not poor.

In this representation, only those people of Chhattisgarh are poor i.e. those people of Chhattisgarh are members of A, whose membership grade is 1, i.e. whose annual earning is Rs. 15,000 or less. But people of Chhattisgarh with annual earnings Rs. 15,100; 15,500; 16,000; 40,000; ……., 1, 00,000; and more are not poor. They all have membership grade 0 and they are not members of the set A.

C.   Fuzzy Representation of the Problem

The rigidity of the above classical representation of the problem can be viewed with the fact that one can easily understand about a person who earns Rs. 15,000 is poor but at the same time how a person is not poor if he earns a slightly more amount than Rs. 15,000. No government can succeed in solving this problem having such a rigid representation. A group of some people having earnings like 15,100; 15,500; 16,000 even 17000, 18000 and so on will not be benefitted from the financial concession given by the government.

Fuzzy Mathematicians have represented the fuzzy set A of poor people of Chhattisgarh as the function.

$$A: I \square [0, 1]$$

Where,

A(x) = 1 if E(x) $\square$ 15,000 means person of Chhattisgarh x is poor by all means.

= .99 if E (x) = 21,000 means person of Chhattisgarh x is almost surely poor.

= .70 if E(x) = 50,000 means person of Chhattisgarh x is more or less poor.

= .50 if E(x) = 60,000 means person of Chhattisgarh x may or may not be poor.

= .30 ii F(x) = 90,000 means person of Chhattisgarh x is definitely not rich hut it will be odd to call him poor.

= .01 if E(x) = 1, 00,000 means person of Chhattisgarh x is almost surely not poor.

= 0 if E(x) = 5, 00,000 means person of Chhattisgarh x is definitely not poor.

As earlier, we have seen that a member of a traditional set A has only two grades 0 & 1. If x $\square$ A then A(x) = 1 and if x $\square$ A then A(x) = 0. But it does not happen in the case of fuzzy set. It has a whole range of grades (1, .99,.70, .50, .30, .01, ………0) between 0 & 1. This fuzzy set of poor people of Chhattisgarh has indefinite boundaries. Every member of I i.e. every person of Chhattisgarh is a member of the fuzzy set A i.e. every person of Chhattisgarh is poor with different grade of membership.

Thus we see that the problem of example (4.1) can be solved in a better way by fuzzy set than a way by classical set. Each and every person of Chhattisgarh will be benefitted from the above rule of example (4.1).

They will get appropriate aid according to their grade of poorness.

If A (x) = .5

Then

Chhattisgarhrian x will get 50% of the aid,

If A(y) = .1

Then

Chhattisgarhrian y will get 15% and

If A(z) = 1

Then,

Chhattisgarhrian z will get 100% of the aid announced by the government.

D. Few more Examples of Fuzzy Sets are:

1)   The set of shades of grey colors,

2)   The set of delicious dishes

3)   The set of good cricketers

4) The set of very good students
5) The set of numbers much greater than one,
6) The set of beautiful girls etc.

In all these sets, the adjectives 'poor', 'red', delicious', 'good', 'very good', 'much greater', beautiful bring a situation of vagueness in the listener.

All the above cited examples with adjectives are not sets in classical sense. The transition from membership to non membership is unexpected in the classes. We know these sets have indefinite limitations that make possible regular transition from membership to non-membership.

VI.   FUZZY SETS: BASIC OPERATIONS

There are three basic operations in Fuzzy Sets:

A.   *Union-* $\mu_{A \cup B}(x) = \mu_A(x) \lor \mu_B(x)$
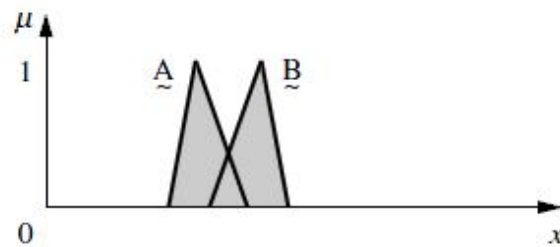
Here $\lor$ is the symbol for maximum.

Fig. 1 Shaded portion shows union of the fuzzy set

B.   *Intersection-* $\mu_{A \cap B}(x) = \mu_A(x) \land \mu_B(x)$
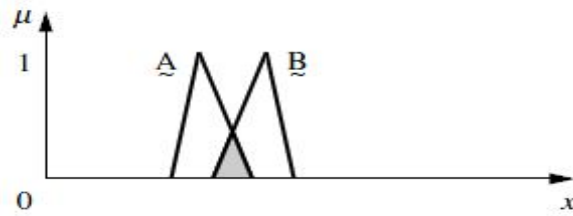
Here $\land$ is the symbol for maximum.

Fig. 2 Shaded portion shows intersection of the fuzzy set

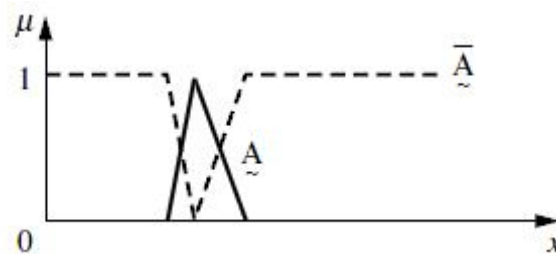C.   *Complement-* $\mu_{\overline{A}}(x) = 1 - \mu_A(x)$

Fig. 3 Doted line shows complement of the fuzzy set

VII.   FUZZY LOGIC

Fuzzy logic is an extension of fuzzy set theory which deals with reasoning. *L.A. Zadeh* was the father of *Fuzzy logic*. It allows some linguistic form like "slightly", "quite", and "very". Fuzzy logic includes 0 and 1 as extreme cases of truth (or "the state of matters" or "fact") but also includes the various states of truth in between so that, for example, the result of a comparison between two things

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could be not "beautiful" or "ugly" but .21 of beauteous.

Fuzzy logic is extremely useful for many people involved in research and development including engineers (electrical, mechanical, civil, chemical, aerospace, agricultural, biomedical, computer, environmental, geological, industrial, and mechatronics), mathematicians, computer software developers and researchers, natural scientists (biology, chemistry, earth science, and physics), medical researchers, social scientists (economics, management, political science, and psychology), public policy analysts, business analysts, and jurists.

VIII.   CONCLUSION

So, we came to the conclusion that the set boundaries of Fuzzy sets are broader than Crisp set. Prof. Zadeh developed the concept of modified set which is called *fuzzy set* and it is used as a mathematical tool to handle different types of uncertainties. *Fuzzy logic* is an extension of fuzzy set theory as it suggests making the membership functions (or the value F & T) apart over the range of real numbers [0, 1]. We also discussed basic operations about fuzzy set theory. Therefore, we can say that by applying fuzzy logic, you will feel much less fuzzy.

IX.   ACKNOWLEDGEMENT

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