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International Journal For Research in  
Applied Science and Engineering Technology



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

**Volume: 5**

**Issue: V**

**Month of publication: May 2017**

**DOI:**

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# Application of Homotopy Analysis Method for Solution of Instability Phenomenon Arising in Fluid Flow through Porous Media with Inclination and Gravitational Effect

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**Abstract:** In this paper, instability phenomenon in homogeneous porous media with inclination effect is discussed. Homotopy analysis method is used here to study the saturation rate for homogeneous porous media. A detailed discussion of saturation rate at various inclination angle has been done with dimensionless time and a comparison study with its physical interpretation has been done by using mathematican software

**Keywords:** Instability, imbibition, gravitation, homotopy analysis method, porous media.

## I. INTRODUCTION

Imbibitions phenomenon occurs due to difference in wetting abilities of the two immiscible fluids flowing in the medium. Imbibitions means that a wetting fluid displaces the non-wetting one, while the opposite case is called drainage. The instability phenomenon has much significance in secondary oil recovery process of petroleum technology. The instability or fingering phenomenon in porous media may arise when two phases of different viscosities and densities are separated by an initially sharp interface that becomes unstable under certain conditions. Figure-1 represents the diagram of instabilities in cylindrical piece of porous media. When native phase (oil) is filled in pores of the porous media and another phase (water) is injected, then instead of regular displacement of whole front at common interface protuberances take place which shoot through the porous media at relatively very high speed.

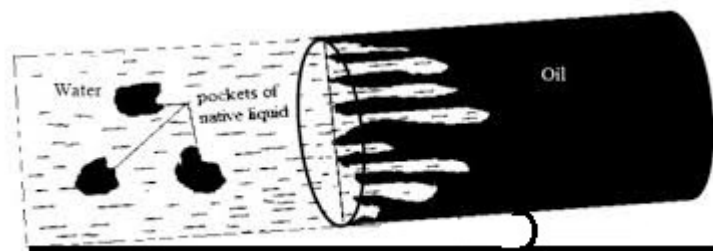


Figure-1 The Instability Phenomenon in Vertical Cross Section of Cylindrical Porous Media

## II. MATHEMATICAL FORMATION

From Darcy's law the seepage velocity of water  $V_w$  and oil  $V_o$  can be written

$$V_w = -\frac{K_w}{\mu_w} K \left( \frac{\partial P_w}{\partial x} + \rho_w g \sin \alpha \right) \quad (1)$$

$$V_o = -\frac{K_o}{\mu_o} K \left( \frac{\partial P_o}{\partial x} + \rho_w g \sin \alpha \right) \quad (2)$$

where "K" is permeability of homogeneous medium,  $K_o$  and  $K_w$  are relative permeability of oil and water which are function

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of  $S_o$  and  $S_w \cdot \rho_w$  and  $\rho_w$  are constant densities of water and oil respectively, while  $\mu_w$  and  $\mu_o$  are constant kinematic viscosity of the phases in homogeneous porous media,  $\alpha$  is the inclination of the bed,  $g$  is acceleration due to gravity.

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \tag{3}$$

$$\phi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \tag{4}$$

Where " $\phi$ " is the porosity of medium. From the definition of phase saturation, we have

$$S_w + S_o = 1 \tag{5}$$

The capillary pressure, which is defined as the pressure discontinuity of the following phases across the common interface is written as

$$P_c = P_o - P_w \tag{6}$$

Implies

$$\frac{\partial P_w}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} \tag{7}$$

The equation of motion for saturation can be obtained by substituting of the values of  $V_w$  and  $V_o$  from equation (1) and (2) in equation (3) and (4) respectively. Thus, we have

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left( \left( \frac{K_w}{\mu_w} K \right) \left( \frac{\partial P_w}{\partial x} + \rho_w g \sin \alpha \right) \right) \tag{8}$$

$$\phi \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x} \left( \left( \frac{K_o}{\mu_o} K \right) \left( \frac{\partial P_o}{\partial x} + \rho_o g \sin \alpha \right) \right) \tag{9}$$

Equation (7) and Equation (8) together

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left( \left( \frac{K_w}{\mu_w} K \right) \left( \frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} + \rho_w g \sin \alpha \right) \right) \tag{10}$$

Now by considering Equation (5), (9) and (10), it becomes

$$\left[ \frac{\partial P_o}{\partial x} K \left( \frac{K_w}{\mu_w} + \frac{K_o}{\mu_o} \right) - \frac{K_w}{\mu_w} k \frac{\partial P_c}{\partial x} + g \sin \alpha K \left( \frac{K_o}{\mu_o} \rho_o + \frac{K_w}{\mu_w} \rho_w \right) \right] = -q \tag{11}$$

Where  $q$  is constant of integration.

Equation (11) Implies

$$\frac{\partial P_o}{\partial x} = \left[ \frac{-q + \frac{K_w}{\mu_w} k \frac{\partial P_c}{\partial x} - g \sin \alpha K \left( \frac{K_o}{\mu_o} \rho_o + \frac{K_w}{\mu_w} \rho_w \right)}{K \left( \frac{K_w}{\mu_w} + \frac{K_o}{\mu_o} \right)} \right] \tag{12}$$

Equation (10) and (12) together gives

$$\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left[ \frac{\frac{K \cdot K_o}{\mu_o} \left( \frac{\partial P_c}{\partial x} + g \sin \alpha (\rho_o - \rho_w) \right)}{\left( 1 + \frac{K_o \cdot \mu_w}{K_w \cdot \mu_o} \right)} + \frac{q}{\left( 1 + \frac{K_o \cdot \mu_w}{K_w \cdot \mu_o} \right)} \right] = 0 \tag{13}$$

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The value of the pressure of oil ( $P_o$ ) can be written as

$$P_o = \bar{P} + \frac{1}{2} P_c \Rightarrow \frac{\partial P_o}{\partial x} = \frac{1}{2} \frac{\partial P_c}{\partial x} \quad (14)$$

where  $\bar{P}$  is the mean pressure.

Equation (14), (11) together with equation (13) gives,

$$\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left( \left( \frac{K_w}{\mu_w} K \right) \left( \frac{1}{2} \frac{\partial P_c}{\partial x} - g \rho_w \sin \alpha \right) \right) = 0 \quad (15)$$

on using equation (10) in equation (15), it reduces to

$$\frac{\partial S_w}{\partial t} - \frac{\partial}{\partial x} \left( \left( \frac{S_w}{\mu_w} K \right) \left( \frac{\beta}{2} \frac{\partial S_w}{\partial x} + g \rho_w \sin \alpha \right) \right) = 0 \quad (16)$$

Equation (16) describe the equation of motion for saturation of wetting phase in fingering imbibition in fluid flow through fracture porous media with inclination and gravitational effect.

Using the dimensionless variables, Equation (16) reduces to,  $X = \frac{x}{L_c}$ ,  $T = \left( \frac{g K \rho_w}{\mu_w \phi L_c} \right) t$

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X} \left( S_w \frac{\partial S_w}{\partial X} \right) + \sin \alpha \frac{\partial S_w}{\partial X} = 0 \quad (17)$$

We choose appropriate initial and Dirichlets boundary condition due to the behaviour of saturation of displaced water at the interface in instability phenomena; that is, instability of oil and water zone at the interface is high, and it becomes stable as it becomes away from the interface and by [11] as,

$$\begin{aligned} S_w(X, 0) &= e^{-X}, \\ S_w(0, T) &= f_1(T), \\ S_w(1, T) &= f_2(T) \end{aligned} \quad (18)$$

Where  $f_1$  and  $f_2$  are the saturation of water at common interface  $X = 0$  and saturation of water at end of matrix of length  $X = 1$  (i.e.  $x = L$ ). It is necessary to discuss the behaviour of saturation of displace water by solving (17) together with (18). Equation (17) is the desired nonlinear partial differential equation with suitable initial and boundary conditions which describes the saturation of displaced water in instability phenomena arising during oil recovery process.

### III. RESULTS AND DISCUSSION

After applying homotpy analysis method on the governing equation (17), we get analytical solution  $S_w(X, T)$ . Using Mathematica software; all numerical values have been derived.

From figure-2 to 4 and table-I to III, it is apparent that Saturation of water increases as time increases at a fixed point  $X = 0.2$ ,  $X = 0.5$  and  $X = 0.8$ . For inclination at  $\alpha = 0^\circ$ ,  $\alpha = 5^\circ$ ,  $\alpha = 10^\circ$ ,  $\alpha = 15^\circ$  checked the value of saturation rate.

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Table –I Saturation Vs Time with inclination effect [ $X = 0.2$ ]

$T$	$\alpha = 0^0$	$\alpha = 5^0$	$\alpha = 10^0$	$\alpha = 15^0$
0.2	0.824093	0.823246	0.822456	0.821778
0.4	0.829456	0.827761	0.826181	0.824824
0.6	0.834818	0.832276	0.829906	0.827871
0.8	0.840181	0.836791	0.833631	0.830918

Table –II Saturation Vs Time with inclination effect [ $X = 0.5$ ]

$T$	$\alpha = 0^0$	$\alpha = 5^0$	$\alpha = 10^0$	$\alpha = 15^0$
0.2	0.609474	0.608846	0.608261	0.607758
0.4	0.612417	0.611161	0.609991	0.608986
0.6	0.615360	0.613476	0.611721	0.610213
0.8	0.618363	0.615791	0.613451	0.611441

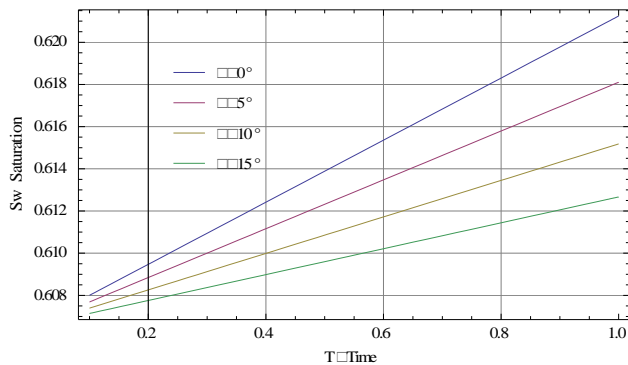


Figure -2 Saturation Vs Time (X=0.5)

Table –III Saturation Vs Time with inclination effect [ $X = 0.8$ ]

$T$	$\alpha = 0^0$	$\alpha = 5^0$	$\alpha = 10^0$	$\alpha = 15^0$
0.2	0.450944	0.450479	0.450045	0.449673
0.4	0.452559	0.451629	0.450762	0.450018
0.6	0.454174	0.452779	0.451479	0.450362
0.8	0.455790	0.453929	0.452195	0.450705

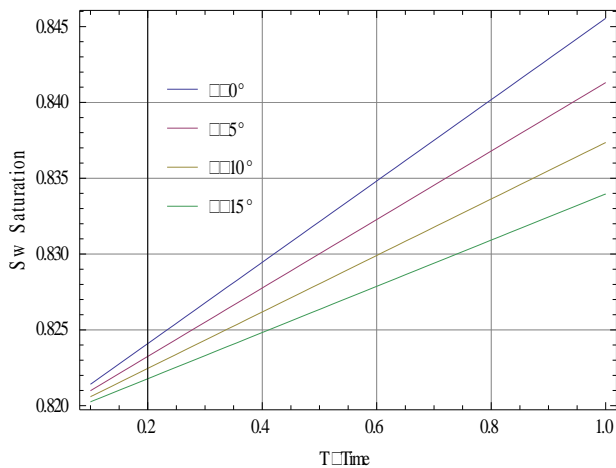


Figure -3 Saturation Vs Time (X=0.2)

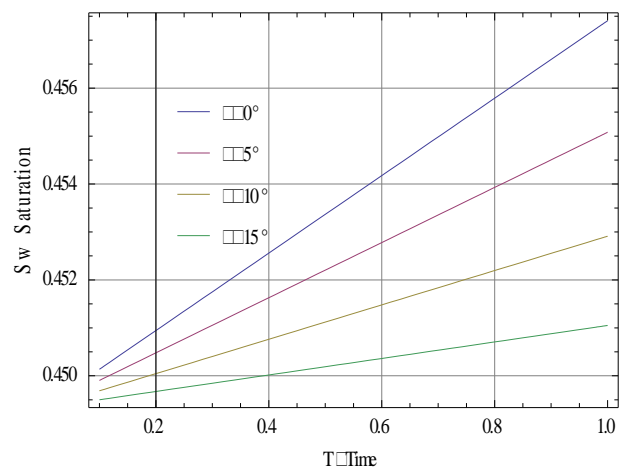


Figure- 4 Saturation Vs Time (X=0.8)

### IV. CONCLUSION

In the instability phenomenon, Homotopy analysis method has been applied successfully using freedom of choosing parameter. Equation (17) represents the saturation of wetting phase for instability phenomena with inclination and dimensionless time and distance. Studied the variation of saturation of water in and direction for particular parametric values of the parameter. Tables and graphs shows that the saturation of wetting phase is maximum for zero inclination which leads increase the rate of saturation.

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