iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL
FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 Issue: V Month of publication: May 2017

DOI:

www.ijraset.com
IC Value: 45.98

Volume 5 Issue V, May 2017
ISSN: 2321-9653

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

# Lattice Points Of A Cubic Diophantine Equation 11(X+Y)^2= 4(Xy+11z^3)

Manju Somanath[1], J. Kannan[2] , K. Raja[3]
[1, 3]Assistant Professor, Department of Mathematics, National College, Trichy – 620 001, Tamil Nadu, India.
[2]Research Scholar, Department of Mathematics, National College, Trichy – 620 001, Tamil Nadu, India.

***Abstract* - The ternary cubic Diophantine equation $11(x+y)^2 = 4xy + 44z^3$ , is considered for determining its non-zero distinct integral solutions employing the linear transformations $x = u + v$, $y = u - v$ and employing the method of factorization in complex conjugates in different patterns.**

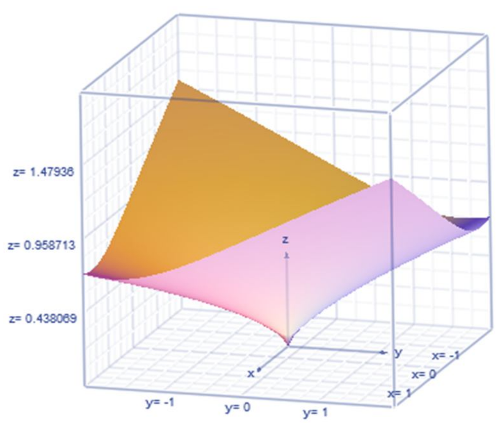
***Keywords* - Diophantine equations, integer solutions, lattice points, non - homogenous equation.**

## I. INTRODUCTION

Number theory, called the Queen of Mathematics, is a broad and diverse part of Mathematics that developed from the study of the integers. The foundations for Number theory as a discipline were laid by the Greek mathematician Pythagoras and his disciples (known as Pythagoreans). One of the oldest branches of mathematics itself, is the Diophantine equations since its origins can be found in texts of the ancient Babylonians, Chinese, Egyptians, Greeks and so on [7 - 8]. The theory of Diophantine equations is a treasure house in which the search for many hidden relations and properties among numbers form a treasure hunt. In fact, Diophantine problems dominated most of the unsolved mathematical problems. Certain Diophantine problems come from physics problems or from immediate Mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze [1 - 6]. In this context one may refer [9, 10].

In this communication, the non-homogenous ternary cubic Diophantine equation represented by $11(x+y)^2 = 4(xy + 11z^3)$ is considered for its non-zero distinct lattice points.

### A. Pictorial representation of the equation:




## II. METHOD OF ANALYSIS

The ternary cubic Diophantine equation under consideration is

$$11(x+y)^2 = 4xy + 44z^3 \tag{1}$$

Volume 5 Issue V, May 2017
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Introduction of the transformations

$$x = u + v,\ y = u - v \tag{2}$$

in (1) leads to

$$v^2 + 10u^2 = 11z^3 \tag{3}$$

Equation (3) is solved through different methods and thus, we obtain different patterns of solutions to (1)

*A. Pattern 1:*

Assume $z = a^2 + 10b^2$, where $a, b > 0$ (4)

Write 11 as $11 = (1 + i\sqrt{10})(1 - i\sqrt{10})$ (5)

Using (4) and (5) in (3) and employing the method of factorization,

define $(v + i\sqrt{10}\,u) = (1 + i\sqrt{10})(a + i\sqrt{10}\,b)^3$ (6)

Equating real and imaginary parts on both sides of (6), we get

$$v = a^3 - 30a^2b - 30ab^2 + 100b^3$$
$$u = a^3 + 3a^2b - 30ab^2 - 10b^3$$

Substituting the values of $u, v$ in (2), we obtain the solutions of (1) as

$$x = 2a^3 - 27a^2b - 60ab^2 + 90b^3$$
$$y = 33a^2b - 110b^3$$
$$z = a^2 + 10b^2$$

*1) Note 1:*

Write 11 as $11 = (-1 + i\sqrt{10})(-1 - i\sqrt{10})$

Proceeding as above, we obtain

$$x = -33a^2b + 110b^3$$
$$y = 2a^3 + 27a^2b - 60ab^2 - 90b^3$$
$$z = a^2 + 10b^2$$

*B. Pattern 2:*

Equation (3) can be written as

$$v^2 + 10u^2 = 11z^3 * 1 \tag{7}$$

Write 1 as $1 = \frac{(3+i2\sqrt{10})(3-i2\sqrt{10})}{7^2}$ (8)

Using (4), (5) and (8) in (7) and employing the method of factorization,

define, $v + i\sqrt{10}\,u = \frac{(1+i\sqrt{10})(a+i\sqrt{10}\,b)^3(3+i2\sqrt{10})}{7}$ (9)

Equating real and imaginary parts, we have

$$v = \frac{[-17(a^3-30ab^2)-50(3a^2b-10b^3)]}{7}$$
$$u = \frac{[5(a^3 - 30ab^2) - 17(3a^2b - 10b^3)]}{7}$$

Since our interest centers on finding integral solutions, replace $a$ by $7A$ and $b$ by $7B$ in the above equations. Thus the corresponding solutions to (1) are given by

$$x = 7^2[-12A^3 + 360AB^2 - 201A^2B + 670B^3]$$
$$y = 7^2[22A^3 + 99A^2B - 660AB^2 - 330B^3]$$
$$z = 7^2[A^2 + 10B^2]$$

*1) Note 2:*

Write 11 as $11 = (-1 + i\sqrt{10})(-1 - i\sqrt{10})$

Proceeding as above, we obtain

$$x = 7^2[-22A^3 + 660AB^2 - 99A^2B + 330B^3]$$
$$y = 7^2[24A^3 - 720AB^2 - 39A^2B + 130B^3]$$
$$z = 7^2[A^2 + 10B^2]$$

*C. Pattern 3:*

Instead of (8) we can also write 1 as

$$1 = \frac{(3+i4\sqrt{10})(3-i4\sqrt{10})}{13^2}$$

Proceeding as above, we obtain the solution as

$$x = 13^2[-30A^3 + 900AB^2 - 321A^2B + 1070B^3]$$
$$y = 13^2[44A^3 + 99A^2B - 1320AB^2 - 330B^3]$$
$$z = 13^2[A^2 + 10B^2]$$

*2) Note 3:*

Write 11 as $11 = (-1 + i\sqrt{10})(-1 - i\sqrt{10})$

Proceeding as above, we obtain

$$x = 13^2[-44A^3 - 99A^2B + 1320AB^2 + 330B^3]$$
$$y = 13^2[42A^3 - 1260AB^2 - 159A^2B + 530B^3]$$
$$z = 13^2[A^2 + 10B^2]$$

*D. Pattern 4*

Yet another representation of 1 is

$$1 = \frac{(1+i6\sqrt{10})(1-i6\sqrt{10})}{19^2}$$

Proceeding as above, we obtain the solutions of (1) as

$$x = 19^2[-52A^3 + 1560AB^2 - 387A^2B + 1290B^3]$$
$$y = 19^2[66A^3 + 33A^2B - 1980AB^2 - 110B^3]$$
$$z = 19^2[A^2 + 10B^2]$$

*1) Note 4:*

Write 11 as $11 = (-1 + i\sqrt{10})(-1 - i\sqrt{10})$

Proceeding as above, we obtain

$$x = 19^2[-66A^3 - 33A^2B + 1980AB^2 + 110B^3]$$
$$y = 19^2[56A^3 - 1680AB^2 - 333A^2B + 1110B^3]$$
$$z = 19^2[A^2 + 10B^2]$$

## III. CONCLUSION

In this paper, we have made an attempt to obtain a complete set of non-trivial distinct integral solutions for the non-homogeneous ternary cubic equation. To conclude, one may search for other choices of solutions to the considered cubic equation and further cubic equations with multi variables.

## REFERENCES

[1] Andre weil, Number Theory : An Approach through History, From Hammurapito to Legendre, Bikahsuser, Boston, 1987.
[2] Bibhotibhusan Batta and Avadhesh Narayanan Singh, History of Hindu Mathematics, Asia Publishing House, 1983.
[3] Boyer. C. B., History of mathematics, John Wiley & sons lnc., New York, 1968.
[4] L.E. Dickson, History of Theory of Numbers, Vol.2, Chelsea Publishing Company, New York, 1952.
[5] Davenport, Harold (1999), The higher Arithmetic: An introduction to the Theory of Numbers (7th ed.) Cambridge University Press.
[6] John Stilwell, Mathematics and its History, Springer Verlag, New York, 2004.
[7] James Matteson, M.D. "A Collection of Diophantine problems with solutions" Washington, ArtemasMartin, 1888.
[8] Tituandreescu, DorinAndrica, "An introduction to Diophantine equations" Springer Publishing House, 2002.
[9] M. A .Gopalan, Manju Somanath, K.Geetha, On Ternary Quadratic Diophantine Equation $z^2 = 50x^2 + y^2$, International Journal for research in emerging Science and technology, Vol.3, Issue3, Pp.644-648,February, 2016.
[10] M. A .Gopalan, Manju Somanath, K.Geetha, On Ternary Quadratic Diophantine Equation $z^2 = 7x^2 + 9y^2$, Bulletin of Mathematics and Statistics

research,Vol.2,issue1,Pp.1-8,2014.

[11] Manju Somanath, J. Kannan, K.Raja, "Gaussian integer solutions to space Pythagorean Equation $x^2 + y^2 + z^2 = w^2$", International Journal of Modern Trends in Engineering and Research, Volume 3, Issue 4, April 2016, pp . 287 - 289.

[12] Manju Somanath, J. Kannan, K.Raja , "Gaussian Pythagorean Triples", International Journal of Engineering Research and Management (IJERM), Volume 03, Issue 04, April 2016, pp . 131 - 132.

[13] Manju Somanath, J. Kannan, "Congruum Problem", International Journal of Pure and Applied Mathematical Sciences (IJPAMS), Volume 9, Number 2 (2016), pp. 123-131.

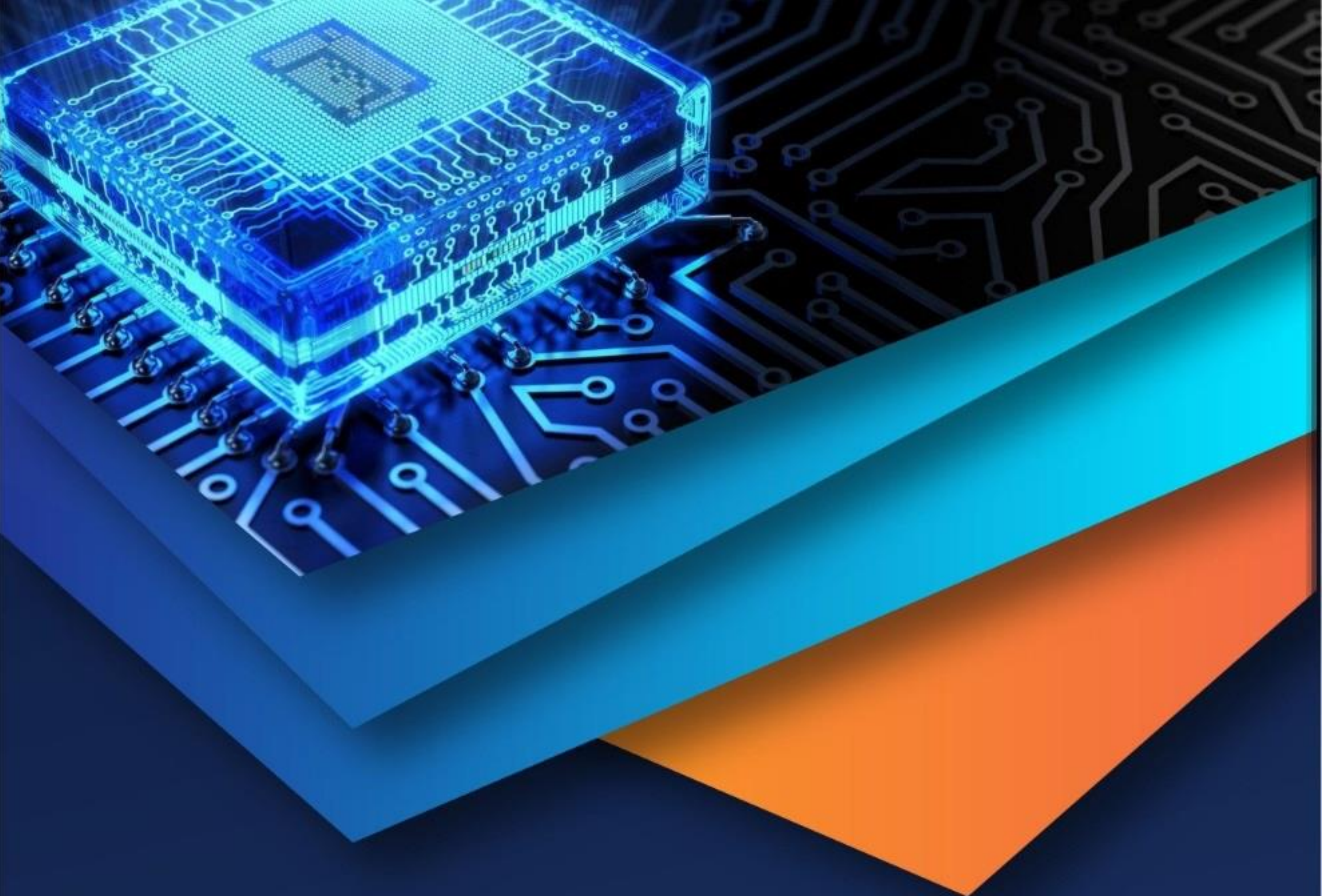
[14] M. A. Gopalan, J. Kannan, Manju Somanath, K.Raja, "Integral Solutions of an Infinite Elliptic Cone $x^2 = 4y^2 + 5z^2$", IJIRSET, Volume 5, Issue10, October 2016, pp .17551 - 17557.

[15] Manju Somanath, J. Kannan, K.Raja, "Lattice Points of an Infinite Cone $x^2 + y^2 = 85z^2$", International Journal of Recent Innovation in Engineering and Research, Vol. 1 Issue. 5, September 2016, pp. 14 -17.

[16] Manju Somanath, J. Kannan, K.Raja, "Integral Solutions of an Infinite Cone $\alpha(x^2 + y^2) = (2a - 1)xy + (4\alpha - 1)z^2$ ", International Journal for Research in Applied Science and Engineering Technology , Vol. 4 Issue X, October 2016, pp(504 - 507).

[17] Manju Somanath, J. Kannan, K.Raja, "Lattice Points of an Infinite Cone $x^2 + y^2 = (\alpha^{2n} + \beta^{2n})z^2$", International Journal of Mathematical Trends and Technology, Vol. 38 No. 2, October 2016, pp(95 - 98).

[18] Manju Somanath, J. Kannan, K.Raja, "Families of Solutions of a Cubic Diophantine Equation", International Journal for Research in Applied Science and Engineering Technology, Vol. 4 Issue XI, November 2016, pp(432 - 434).




10.22214/IJRASET

INDEX COPERNICUS
45.98

IMPACT FACTOR: 7.129

TOGETHER WE REACH THE GOAL
IMPACT FACTOR: 7.429

# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY