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Thermo-mechanical Analysis of Functionally Graded Rotating Disc

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Abstract: The study investigates the thermo-mechanical response for a variable thickness rotating disc made of functionally graded materials (FGM). The thickness of FGM disc is varying non-linearly along the radius. The distribution of the SiC_p content in the Al matrix is non-linearly decreasing from inner to the outer radius according to the power law. The effect of different thickness profiles have been investigated on the stresses and strains in the FG disc. It is observed that the FGM disc with radially decreasing thickness shows the lowest stresses and strains. The FGM disc, operating under radially increasing temperature profile exhibits lower radial and tangential strains over a larger portion of the disc.
Key words: Rotating disc, Stress, Variable thickness, Thermal gradient

I. INTRODUCTION

Rotating disc is a very important component in engineering applications like flywheels, automobiles brakes and computer disc drives. In these the disc is operating under high mechanical and thermal loadings, therefore deformations will produce [1-3]. Functionally Graded Material (FGM) may be characterized by the variation in material composition over volume, resulting in changes in the material properties [4, 5].

Timoshenko and Goodier [2] was the first to obtain a closed form solution for homogeneous rotating disc operating at constant temperature. Reddy and Srinath [6] studied the effect of material density on the stresses and displacements in rotating disc made of orthotropic material. Horgan and Chan [7] investigated the effects of material inhomogeneity on the stresses in rotating disc of constant thickness. They observed that the stresses in inhomogeneous disc are significantly different as compared to homogeneous disc. Yeh and Han [8] proposed a symmetric formulation to predict elastic stresses in an inhomogeneous rotating disc with arbitrary thickness, operating under thermal loading. Zenkour [9] presented a solution procedure to estimate the distribution of elastic stresses in rotating FGM disc of constant thickness. The study indicates that the stresses and deformations in the FGM disc are lower when the material of the disc is tailored in such a way that the elastic modulus and density are more near the inner radius than those noticed towards the outer radius. Afsar [10] carried out finite element analysis of rotating FGM disc subjected to thermal load. It is observed that thermo-elastic characteristics of the FGM disc are significantly influenced by temperature and thickness profile of the disc, apart from angular speed of the disc. Callioglu et al [11] analyzed elastic stresses in constant thickness rotating FGM disc subjected to internal pressure and radially varying temperature distribution. It is revealed that the radial displacement in FGM disc subjected to uniform temperature is higher than the FGM disc subjected to temperature gradient. Peng and Li [12] performed elastic analysis for a constant thickness rotating sandwich solid disc made of FGM. The numerical results obtained are observed to be in excellent agreement with the exact solution, for some specific power-law gradients.

It is revealed from the literature that many attempts have been made to study elastic stresses in FGM disc having constant thickness. The variable thickness disc performs better than a constant thickness disc [13]. Further, the analyses carried out, mostly assume the disc to operate at constant temperature. However, in numerous applications of rotating disc, viz. turbine rotor and disc brake, the disc is subjected to radial thermal gradient [5, 14]. Keeping this in view, it has been decided to analyze elastic stresses and strains in a variable thickness rotating FG disc subjected to different temperature profiles.

A. Disc Profile and It's Material

Consider a variable thickness FG rotating disc with the inner radius a (=0.04 m) and the outer radius b (=0.1 m). The thickness h(r) of the disc is assumed to vary according to the power law,

$$h(r) = h_b\left(\frac{r}{b}\right)^k \tag{1}$$

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where h_b is thickness of disc at the outer radius and k is the disc thickness gradation index.

In this study, we have compared the elastic stresses in variable thickness FGM disc with a similar FGM disc of uniform thickness (t = 5 mm). For the purpose of comparison, the volume of both the FGM discs are kept equal i.e.,

$$\int_a^b 2\pi\, rh(r)dr = \pi(b^2 - a^2)t \tag{2}$$

Substituting h(r) from Equation (1) into above equation and simplifying, we get,

$$h_b = \frac{(b^2 - a^2)t(2+k)b^k}{2(b^{2+k} - a^{2+k})} \tag{3}$$

Assuming different value's of k, we can find h_b from the eq. (3).

The FGM disc is assumed to be made of Al-SiC_p composite. The SiC_p content in the FGM disc is assumed to vary along the radial distance (r) according to the following relation,

$$V(r) = V_o\left(\frac{r}{b}\right)^n \tag{4}$$

Where V_o is the SiC_p content at the outer radius and $n(=-0.5)$ is SiC_p gradation index.

If V_{av} is average SiC_p content in the FGM disc then on equating total SiC_p content in variable thickness FGM disc with that in uniform thickness FGM disc, we get,

$$\int_a^b 2\pi r h(r)V(r)dr = V_{av}\left[\pi(b^2 - a^2)t\right] \tag{5}$$

Substituting the values of h(r) and V(r), respectively from equations (1) and (4), into equation (5), the minimum SiC_p content (V_o) at the outer radius of FGM disc may be estimated as,

$$V_o = \frac{V_{av}\,(2+k+n)\,(b^{2+k} - a^{2+k})b^n}{(2+k)\,(b^{2+k+n} - a^{2+k+n})} \tag{6}$$

The properties of disc material, like density (ρ), Young's modulus (E) and coefficient of thermal expansion (α) are assumed to vary radially as given by,

$$\rho(r) = \rho_0\left(\frac{r}{b}\right)^{n_1},\ E(r) = E_0\left(\frac{r}{b}\right)^{n_2},\ \alpha(r) = \alpha_0\left(\frac{r}{b}\right)^{n_3} \tag{7}$$

where ρ_o, E_o and α_o are the values at the outer radius of FGM disc and n_1, n_2 and n_3 denotes gradation indices. In this study, we have assumed that the FGM disc is subject to linearly varying thermal gradient along the radial directions, as given by,

$$T(r) = T_a + (T_b - T_a)\frac{(r-a)}{(b-a)} \tag{8}$$

where T_a and T_b respectively denote temperatures, at the inner and outer disc radii.

B. *Mathematical Formulation*

The radial strain (ε_r) and tangential strain (ε_θ) are related according to compatibility equation,

$$\varepsilon_r = \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr} \tag{9}$$

For elastic deformations, the constitutive equations between elastic stresses and strains for an isotropic FGM disc under plane stress condition are given by [11],

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$$\varepsilon_r = \frac{1}{E(r)}(\sigma_r - \nu\sigma_\theta) + \alpha(r)T(r) \tag{10}$$

$$\varepsilon_\theta = \frac{1}{E(r)}(\sigma_\theta - \nu\sigma_r) + \alpha(r)T(r) \tag{11}$$

where σ_r and σ_θ are the radial and tangential stresses in the disc.

The force equilibrium equation for a variable thickness rotating FGM disc is given as [2],

$$\frac{d}{dr}\left[rh(r)\sigma_r\right] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0 \tag{12}$$

where $\rho(r)$ is the density and ω is angular velocity of the disc. The angular velocity (ω) is taken 1570 rad/s in this study.

The equilibrium equation (12) is satisfied by the stress function F, defined as,

$$\sigma_r = \frac{F}{r\,h(r)}, \quad \sigma_\theta = \frac{1}{h(r)}\frac{dF}{dr} + \rho(r)\omega^2 r^2 \tag{13}$$

Substituting the values of ε_r and ε_θ from equations (10) and (11) respectively and using equation (13) into the compatibility equation (9), we get,

$$r^2\frac{d^2F}{dr^2} + r\frac{dF}{dr}\left[1 - \frac{r}{h(r)}\frac{dh}{dr} - \frac{r}{E(r)}\frac{dE}{dr}\right] + F\left[\frac{\nu r}{h(r)}\frac{dh}{dr} + \frac{\nu r}{E(r)}\frac{dE}{dr} - 1\right] = -\rho(r)\omega^2 r^3 h(r)\left[3 + \upsilon - \frac{r}{E(r)}\frac{dE}{dr}\right]$$
$$-\omega^2 r^4 h(r)\frac{d\rho}{dr} - r^2 h(r)E(r)\alpha(r)\frac{dT}{dr} - r^2 h(r)E(r)T(r)\frac{d\alpha}{dr} \tag{14}$$

Substituting the values of properties from equation (7) into equation (14), we get,

$$r^2\frac{d^2F}{dr^2} + r\frac{dF}{dr}\left[1 - k - n_2\right] + F\left[\nu k + \nu n_2 - 1\right] = -\frac{\rho_o\omega^2 h_b r}{b^{n_1+k}}(3 + \upsilon + n_1 - n_2)r^{3+n_1+k}$$
$$-\frac{E_o\alpha_o h_b(1+n_3)}{b^{n_2+n_3+k}}\left[\frac{(T_b - T_a)}{(b-a)}\right]r^{2+k+n_2+n_3} - \frac{E_o\alpha_o h_b n_3}{b^{n_2+n_3+k}}\left[T_a - \frac{(T_b - T_a)}{(b-a)}\right]r^{1+k+n_2+n_3} \tag{15}$$

Solving the above this differential equation, the stress function (F) can be obtained as,

$$F = C_1 r^{\frac{n_2+k+m}{2}} + C_2 r^{\frac{n_2+k-m}{2}} + Ar^{3+n_1+k} + Br^{1+k+n_2+n_3} + Cr^{2+k+n_2+n_3} \tag{16}$$

where C_1 and C_2 are the constants of integration and A, B, C and m are given by,

$$A = \frac{-\rho_o\omega^2 h_b(3 + v + n_1 - n_2)}{b^{n_1+k}(8 + n_1{}^2 + n_1k + 3k + 6n_1 - n_1n_2 - n_2k - 3n_2 + vn_2 + vk)} \quad ; \; m = \sqrt{(n_2+k)^2 - 4(vn_2 + vk - 1)}$$

$$B = \frac{E_o\alpha_o h_b n_3}{b^{n_2+n_3+k}(n_3{}^2 + n_2n_3 + n_3k + k + n_2 + 2n_3 + vn_2 + vk)}\left[T_a - \frac{(T_b - T_a)}{(b-a)}\right]$$

$$C = \frac{-E_o\alpha_o h_b(1+n_3)}{b^{n_2+n_3+k}(n_3{}^2 + 3 + n_2n_3 + n_3k + 2k + 2n_2 + 4n_3 + vn_2 + vk)}\left[\frac{(T_b - T_a)}{(b-a)}\right]$$

The radial and tangential stresses can be obtained by substituting the values of stress function (F) in equation (13) as,

$$\sigma_r = \frac{1}{h(r)}\left[C_1 r^{\frac{n_2+k+m-2}{2}} + C_2 r^{\frac{n_2+k-m-2}{2}} + Ar^{2+n_1+k} + Br^{n_2+n_3+k} + Cr^{n_2+n_3+1+k}\right] \tag{17}$$

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$$\sigma_\theta = \frac{1}{h(r)}\left[\begin{array}{l}\left(\frac{n_2+k+m}{2}\right)C_1 r^{\frac{n_2+k+m-2}{2}} + \left(\frac{n_2+k-m}{2}\right)C_2 r^{\frac{n_2+k-m-2}{2}} + (3+n_1+k)Ar^{2+n_1+k} \\ +(1+n_2+n_3+k)Br^{n_2+n_3+k} + (2+k+n_2+n_3+)Cr^{1+k+n_2+n_3+}\end{array}\right] + \rho(r)r^2\omega^2 \quad (18)$$

It is assumed that the FGM disc under investigation is fitted on a splined shaft where small axial movement is permitted. Thus, we may use the following free-free boundary conditions [5],

$$\sigma_r = 0 \text{ at } r = a \text{ and } \sigma_r = 0 \text{ at } r = b \quad (19)$$

Using the above boundary condition in equation (17), we get,

$$C_1 = \frac{D_2 b^{\frac{2+m-n_2-k}{2}} - D_1 a^{\frac{2+m-n_2-k}{2}}}{(b^m - a^m)} \text{ and } C_2 = \frac{D_1 b^m a^{\frac{2+m-n_2-k}{2}} - D_2 b^{\frac{2+m-n_2-k}{2}} a^m}{(b^m - a^m)} \quad (20)$$

where,

$$D_1 = -Aa^{2+n_1+k} - Ba^{n_2+n_3+k} - Ca^{n_2+n_3+1+k} \;;\; D_2 = -Ab^{2+n_1+k} - Bb^{n_2+n_3+k} - Ca^{1+k+n_2+n_3} \quad (21)$$

II. RESULTS AND DISCUSSION

On the basis of analysis studied in the previous section, a code has been generated to calculate the stresses and strains in disc. For the validation, we have calculated the radial stress for a constant thickness rotating composite disc (Refer Table 1). The results obtained are compared with earlier published work [15] for the same kind of disc. A good co-relation is observed between the present results and earlier work (refer Figure 1).

Table 1 Parameters and operating conditions used for validation

a (m)	b (m)	E (GPa)	v	ρ (kg/m^3)	ω (rad/s)	n	T_a (K)	T_b (K)
0.04	0.1	72	0.33	2800	1570	0	0	0

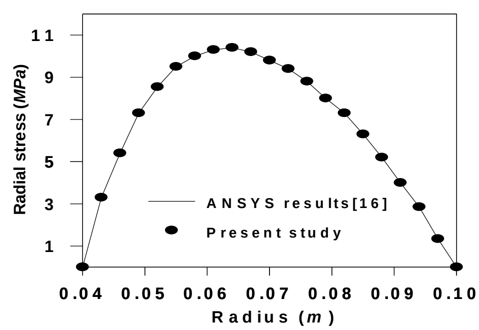

Fig. 1: Comparison of radial stress in disc: Present study and ANSYS results

The results are estimated for discs with different thickness profile (refer Table 2). The values of various constants and properties used during the computation process are summarized in Table 3. The values of gradation indices n_1, n_2 and n_3 have calculated using regression analysis.

Table 2 Description of FGM discs with varying thickness gradient (TG)

Disc Notation	*k*	h_a (*mm*)	h_b (*mm*)	TG (h_a- h_b) (*mm*)	SiC$_p$ Content (*vol %*)			SiC$_p$ Gradient (*vol %*)
					$V_i(r=a)$	V_{av}	$V_o(r=b)$	$(V_i - V_0)$
Disc (D1)	0.5	3.69	5.80	-2.11	27.07	20	17.12	9.95
Disc (D2)	0	5.00	5.00	0	26.66	20	16.86	9.80
Disc (D3)	-0.50	6.66	4.21	2.45	26.25	20	16.60	9.65

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Table 3 Values of material properties and constants

Disc Notation	ρ_o	n_1	E_o	n_2	α_o (x10^{-6})	n_3
Disc (D1)	2784.95	-0.01958	134.51	-0.2696	1.9890	0.1045
Disc (D2)	2783.64	-0.01929	133.53	-0.2677	1.9940	0.1026
Disc (D3)	2782.33	-0.01900	132.55	-0.2658	1.9990	0.1007

A. Effect of varying disc thickness profile on elastic stresses and strains

The purpose of this section is to compare elastic stresses and strains in FGM disc having increasing thickness with k= 0.5 (Disc D1), constant thickness disc k= 0 (Disc D2) and radially decreasing thickness with k = -0.5 (Disc D3), as mention in Table 2. The variation of disc thickness with radius (r) for the three discs is depicted in Fig. 2.

The radial stress in all the FGM discs, Fig. 3a, increases from zero at the inner radius, reaches a maximum, before decreasing to zero again at the outer radius, under the imposed boundary conditions given in equation (19). As compared to uniform thickness disc (D2), the radial stress in FGM disc D1 is relatively higher, but in the FGM disc D3, it is relatively lower. The variation noticed in value of radial stress in various FGM discs is more towards the inner radius than noticed near the outer radius. Similar to radial stress, the tangential stress (Fig. 3b) in FGM disc D3, having radially decreasing thickness profile, is significantly lower than the constant thickness FGM disc D2 and FGM disc D1 with radially increasing thickness.

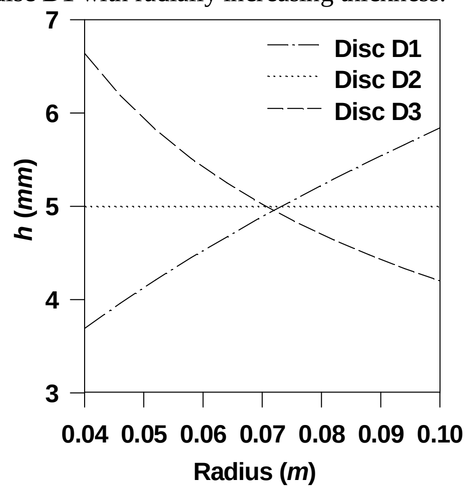

Fig. 2: Variation of disc thickness

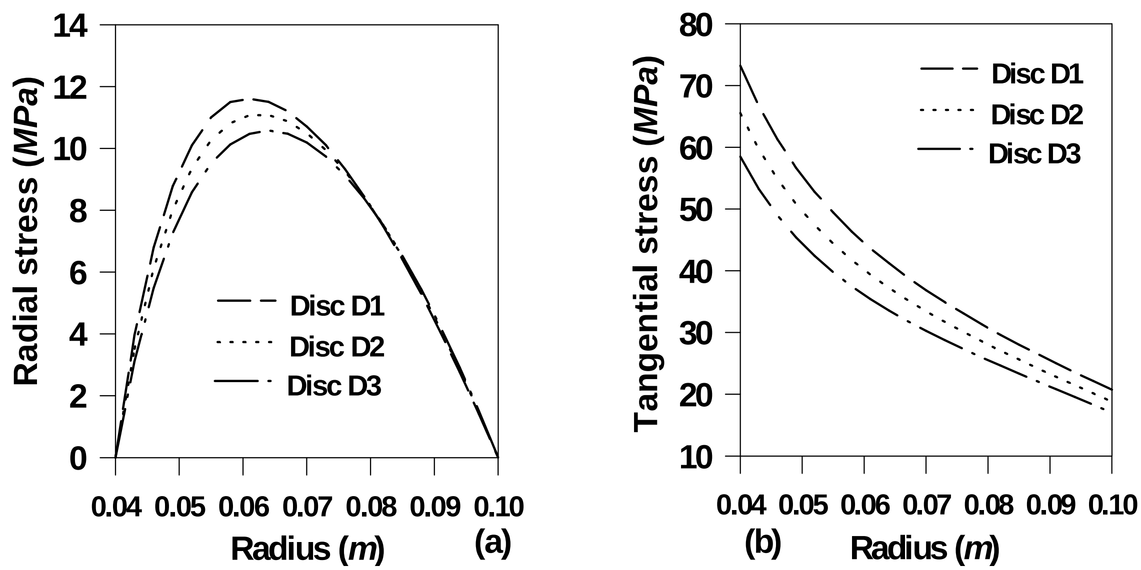

Fig. 3: Variation of: (a) Radial and (b) Tangential stresses in FGM discs.

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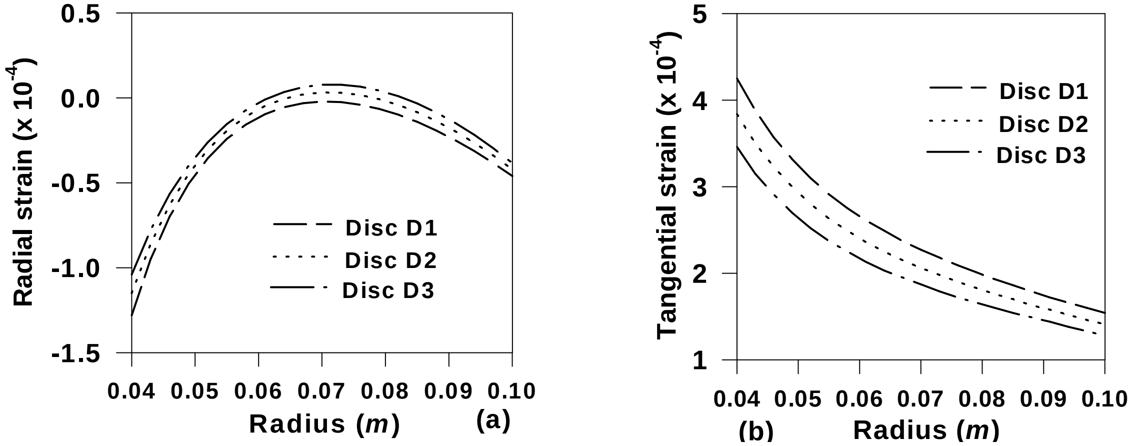

Fig. 4: Variation of: (a) Radial and (b) Tangential strains in FGM discs.

The radial strain in the FGM discs remains compressive, except for some tensile value in the middle of FGM disc D3 (Fig. 4a). The radial strain in FGM disc D3 is relatively lower than the FGM discs D1 and D2. Similar to radial strain, the tangential strain in FGM disc D3 is lower than the FGM discs D1 and D2 (Fig. 4b). Thus, it is evident that the elastic response of the FGM disc D4, having radially decreasing thickness, is superior to any other FGM discs.

B. *Effect of different temperature profiles on elastic stresses and strain*

The effect of different thermal profiles (refer Table 4) is investigated on the elastic deformations in rotating FG disc. In all the three cases, the average temperature has been kept same as 12.5 *K*. The FGM disc-T1, operating under a radially decreasing temperature, exhibits the lowest radial stress (Fig. 8a) whereas the disc operating radial increasing radial temperature gradient shows the highest radial stress. The tangential stress is the lowest in FGM disc T1, near the inner radius, but towards the outer radius it is the lowest in FGM disc T2 (Fig. 8b). The maximum value of tangential stress in FGM disc T1 is significantly reduced as compared to FGM discs T2 and T3 and in addition the distribution of tangential stress is more uniform in disc T1. Upon imposing a radially decreasing thermal gradient in FGM disc (*i.e.* FGM disc T1), the radial strain (Fig. 8c) increases near the inner radius but decreases towards the outer radius, when compared to radial strain in constant temperature FGM disc T3. In some portion, towards the outer radius of the FGM disc T1, the nature of radial strain become compressive. The imposition of radially increasing thermal gradient, as in FGM disc T2, reduces the radial strain near the inner radius but increases its value towards the outer radius. Except for some portion near the inner radius, the radial strain in FGM disc T2 is always tensile. The tangential strain (Fig., 8d) is the highest in FGM disc T1 and the lowest in FGM disc T2, except for some portion near the outer radius where the trend is just opposite.

Table 4 Description of FGM disc under thermal distribution

Disc notation	T_a (K)	T_b (K)	T_{avg} (K)
Disc (T1)	25	0	12.5
Disc (T2)	0	25	12.5
Disc (T3)	12.5	12.5	12.5

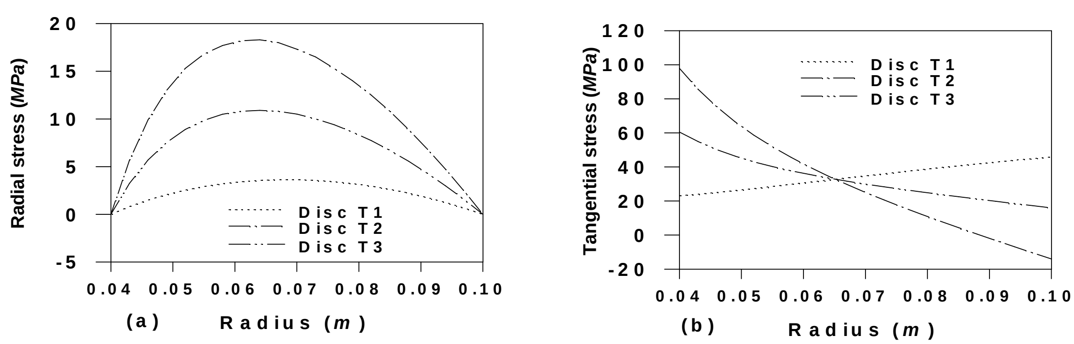

FIG 5

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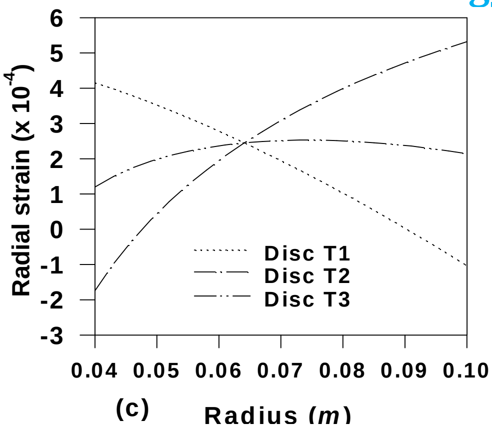

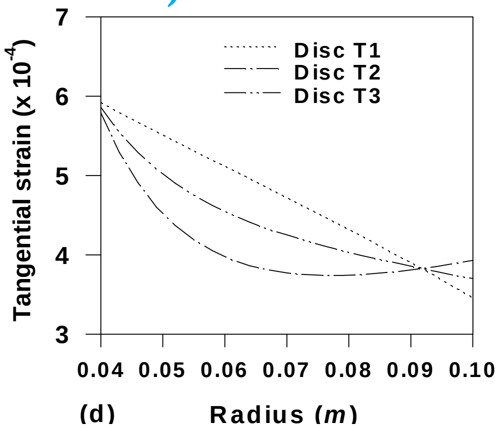

Fig. 6: Effect of temperature profiles on Stresses and Strain in FGM disc

III. CONCLUSIONS

It is concluded from the study that the elastic stresses and strains in rotating FGM disc could be reduced by employing disc having radially decreasing thickness profile. The radial stress (over the entire disc) and tangential stress (near the inner radius) are lower in FGM disc operating under a linearly decreasing radial thermal gradient as compared to FGM disc operating under linearly increasing radial thermal gradient or constant temperature. However, the FGM disc operating under a linearly increasing radial thermal gradient shows the lower tangential stress towards the outer radius.

The radial strain near the inner radius is the lowest in FGM disc operating under a radially increasing temperature profile but towards the outer radius it is lowest in FGM disc operating under a linearly decreasing temperature. The tangential strain in FGM disc operating under a radially increasing temperature is the lowest, except for some portion towards the outer radius where it is the lowest in FGM disc subjected to radially decreasing temperature.

REFERENCES

[1] Gupta, V.K, Singh, S B, Chandrawat, H N and Ray, S (2004) Steady state creep and material parameters in a rotating disc of Al-SiC_p composite, European Journal of Mechanics A/Solids, 23: 335-344.
[2] Timoshenko S P and Goodier J N, Theory of Elasticity, McGraw Hill, California (1970).
[3] Ugural A C and Fenster S K, Advanced Strength and Applied Elasticity, Elsevier, New York (1981).
[4] Groves, J.F. and Wadley, H.N.G. (1997) Functionally graded materials synthesis via low vacuum directed vapor deposition, Composites part B: Engineering, 28: 57–69.
[5] Gupta, V.K., Singh, S.B., Chandrawat, H.N. and Ray, S. (2005) Modeling of creep behavior of a rotating disc in the presence of both composition and thermal gradients, Journal of Engineering Materials and Technology, 127: 97-105.
[6] Reddy, T.Y. and Srinath, H. (1974) Elastic stresses in a rotating anisotropic annular disc of variable thickness and variable density, International Journal of Mechanical Sciences, 16: 85-89.\
[7] Horgan, C O and Chan, A M (1999) The stress response of functionally graded isotropic linearly elastic rotating disks, Journal of Elasticity, 55: 219-230.
[8] Yeh, K Y and Han, R P S (1994) Analysis of high-speed rotating disks with variable thickness and inhomogeneity, *Journal* of A*pplied* Mechanics, 61: 186-191.
[9] Zenkour, A M (2009) Stress distribution in rotating composite structures of functionally graded solid disks, Journal of Materials Processing Technology, 209: 3511-3517.
[10] Afsar, A M and Go, J (2010) Finite element analysis of thermoelastic field in a rotating FGM circular disk, Applied Mathematical Modelling, 34: 3309-3320
[11] Callioglu, H, Sayer, M and Demir, E (2011) Stress analysis of functionally graded rotating discs under mechanical and thermal loads, Indian Journal of Engineering and Material Sciences, 18: 111-118.
[12] Peng, X L and Li, X F (2012) Effects of gradient on stress distribution in rotating functionally graded solid disks, Journal of Mechanical Science and Technology, 26: 1483-1492.
[13] Garg M, Salaria B S and Gupta V K (2012), Effect of disc geometry on the steady state creep in a rotating disc made of functionally graded materials, Material Science Forum, 736: 183-190.
[14] Garg M, Salaria B S and Gupta V K (2012) Effect of thermal gradient on steady state creep in a rotating disc of variable thickness, Procedia Engineering, 55: 542–547.
[15] Callioglu, H, Bektas, N B and Sayer, M (2011), Stress analysis of functionally graded rotating discs analytical and numerical solutions, Acta Mechanica Sinica, 27: 950-955.