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Fixed Point Theorem on Dislocated B-Metric Spaces

A.Thanga Pandi¹, J. Maria Joseph²

^{1,2}Department of Mathematics, St.Joseph's college, Tiruchirappalli, TamilNadu, India

Abstract: In this paper, we prove Fixed point theorem for cyclic contractions in dislocated b-metric spaces. This paper generalized many result in the current literature. **Mathematics Subject Classification:** 47H05; 47H10; 47J25

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I. INTRODUCTION AND PRELIMINARIES

Fixed point theory plays one of the important roles in Mathematical Analysis. Many authors [1-6] presented fixed point theorem in different ways. In Banach contraction principle was introduced in 1922 by Banach [7] as follows:

Let (X, d) be a metric space and $T : X \rightarrow X$. Then T is called a Banach contraction mapping if there exists $k \in [0, 1)$ such that $d(Tx, Ty) \leq kd(x, y)$ for all $x, y \in X$.

The concept of Kannan mapping was introduced in 1969 by Kannan [8] as follows: (ii) T is called a Kannan mapping if there exists $r \in [0, 1)$ such that

$$d(Tx, Ty) \leq rd(x, Tx) + rd(y, Ty) \text{ for all } x, y \in X$$

Now, we recall the definition of cyclic map. Let A and B be non-empty subsets of a metric space (X, d) and $T : A \cup B \rightarrow A \cup B$. T is called a cyclic map iff $T(A) \subseteq B$ and $T(B) \subseteq A$.

In 2003, Kirk et al. [9] introduced cyclic contraction as follows:

(iii) A cyclic map $T : A \cup B \rightarrow A \cup B$ is said to be cyclic contraction if there exists $\alpha \in [0, 1)$ such that

$$d(Tx, Ty) \leq \alpha d(x, y) \text{ for all } x \in A \text{ and } y \in B.$$

In 2010, Karapinar and Erhan [10] introduced Kannan type cyclic contraction as follows:

(iv) A cyclic map $T : A \cup B \rightarrow A \cup B$ is called a Kannan type cyclic contraction if there exists $b \in [0, 1)$ such that

$$d(Tx, Ty) \leq bd(x, Tx) + bd(y, Ty) \text{ for all } x \in A \text{ and } y \in B.$$

If (X, d) is a complete metric space, at least one of (i), (ii), (iii) and (iv) holds, then it has a unique fixed point [7-10]. Next, we discuss the development of space.

Definition 1.1. [7] Let X be a nonempty set. Suppose that the mapping

$d : X \times X \rightarrow [0, \infty)$ satisfies the following conditions:

(d1) $d(x, x) = 0$ for all $x \in X$

(d2) $d(x, y) = d(y, x) = 0$ implies $x = y$ for all $x, y \in X$

(d3) $d(x, y) = d(y, x)$ implies for all $x, y \in X$

(d4) $d(x, y) \leq [d(x, z) + d(z, y)]$ for all $x, y, z \in X$

If d satisfies conditions (d1), (d2) and (d4), then d is called a quasi-metric on X . If

d satisfies conditions (d2), (d3) and (d4), then d is called a dislocated metric on X . If

d satisfies conditions (d1), (d3) and (d4), then d is called a metric on X .

In 2005 the concept of dislocated quasi-metric, which is a new generalization of quasi-b-metric spaces and dislocated b-metric space, was introduced. By Definition

1.1, if setting conditions (d2), and (d4) holds true, then d is called a dislocated quasi-metric on X .

A. Remark

It is obvious that metric spaces are quasi-metric spaces and dislocated metric spaces, but the converse is not true. **Definition 1.2.** [11] Let X be a non-empty set. Suppose that the mapping

$b : X \times X \rightarrow [0, \infty)$ such that the constant $s \geq 1$ satisfies the following conditions:

(b1) $b(x, y) = b(y, x) = 0 \Leftrightarrow x = y$ for all $x, y \in X$

(b2) $b(x, y) = b(y, x) = 0$ for all $x, y \in X$

(b3) $b(x, y) \leq s[b(x, z) + b(z, y)]$ for all $x, y, z \in X$

The pair (X, d) is then called a b -metric space.

II. MAIN RESULTS

A. The Following Theorem Generalizes Theorem

Theorem 2.1. Let (X, d) be a complete dislocated b -metric with $s \geq 1$. Let A and B be a non-empty closed subset of X . Let $T : A \cup B \rightarrow A \cup B$ be a self map such that $d(Tx, Ty) \leq a_1 d(x, y) + a_2 d(Tx, x) + a_3 d(Ty, y) + a_4 d(Ty, x) + a_5 d(y, Tx)$ where $a_i \geq 0, i = 1, 2, 3, 4, 5$ and $a_1 + a_2 + a_3 + 2sa_4 + 2sa_5 < 1$. Then T has a unique fixed point in $A \cap B$.

Proof. Let $T^{2n} \subseteq X, \{T^{2n}\} \subseteq A$ and $\{T^{2n+1}\} \subseteq B$. Fix $x \in A$.

$$\begin{aligned} d(T^{2n}x, Tx) &= d(T(Tx), Tx) \\ &\leq a_1 d(Tx, x) + a_2 d(T^{2n}x, Tx) + a_3 d(Tx, x) + a_4 d(Tx, Tx) + a_5 d(x, T^{2n}x) \\ &\leq a_1 d(Tx, x) + a_2 d(T^{2n}x, Tx) + a_3 d(Tx, x) + sa_4 [d(Tx, x) + d(x, Tx)] \\ &\quad + sa_5 [d(x, Tx) + d(Tx, T^{2n}x)] \\ d(T^{2n}x, Tx) &\leq a_1 d(Tx, x) + a_2 d(T^{2n}x, Tx) + a_3 d(Tx, x) + 2sa_4 d(Tx, x) \\ &\quad + sa_5 d(x, Tx) + sa_5 d(Tx, T^{2n}x) \\ &= (a_1 + a_3 + 2sa_4 + sa_5)d(Tx, x) + (a_2 + sa_5)d(T^{2n}x, Tx) \\ &\leq \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)} d(Tx, x) \end{aligned}$$

$$d(T^{2n}x, Tx) \leq kd(Tx, x), \text{ where } k = \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)}$$

Now,

$$\begin{aligned} d(T^3x, T^2x) &= d(T(T^2x), T(Tx)) \\ &\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) + a_4 d(T^2x, T^2x) \\ &\quad + a_5 d(Tx, T^3x) \\ &\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) + sa_4 [d(T^2x, Tx) + d(Tx, T^2x)] \\ &\quad + sa_5 [d(Tx, T^2x) + d(T^2x, T^3x)] \\ &\leq a_1 d(T^2x, Tx) + a_2 d(T^3x, T^2x) + a_3 d(T^2x, Tx) \\ &\quad + 2sa_4 d(T^2x, Tx) + sa_5 d(Tx, T^2x) + a_5 d(T^2x, T^3x) \end{aligned}$$

$$\begin{aligned} d(T^3x, T^2x) &\leq (a_1 + a_3 + 2sa_4 + sa_5)d(T^2x, Tx) + (a_2 + sa_5)d(T^2x, T^3x) \\ &\quad + \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)} d(T^2x, Tx) \\ &\leq \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)} d(T^2x, Tx) \end{aligned}$$

$$d(T^3x, T^2x) \leq kd(T^2x, Tx), \text{ where } k = \frac{(a_1 + a_3 + 2sa_4 + sa_5)}{1 - (a_2 + sa_5)}$$

B. Fixed Point Theorem on Dislocated b-Metric Spaces

$$d(T^3x, T^2x) \leq k[kd(Tx, x)]$$

$$d(T^3x, T^2x) \leq k^2 d(Tx, x) \text{ induction,}$$

$$d(T^{n+1}x, T^nx) \leq k^n d(Tx, x)$$

In general n, m ∈ N where m > n

$$\begin{aligned} d(T^nx, T^{n+m}x) &\leq sd(T^nx, T^{n+1}x) + s^2 d(T^{n+1}x, T^{n+2}x) + \dots + s^m d(T^{n+m-1}x, T^{n+m}x) \\ &\leq sk^n d(x, Tx) + s^2 k^{n+1} d(x, Tx) + \dots + s^m k^{n+m-1} d(x, Tx) \\ &\leq sk^n [1 + sk + \dots + (sk)^{m-1}] d(x, Tx) \end{aligned}$$

$$d(T^nx, T^{n+m}x) \leq sk^n \frac{1 - (sk)^m}{1 - sk} d(x, Tx)$$

$$d(T^nx, T^{n+m}x) \rightarrow 0 \text{ as } n \rightarrow \infty$$

Hence {Tⁿ} is a Cauchy sequence.

Since (X, d) is complete, Then {Tⁿ} coverages to some point x ∈ X. Since {T²ⁿ} ⊆ A and {T²ⁿ⁺¹} ⊆ B.

Thus x ∈ A ∩ B

We show that Tx = x. Now,

$$\begin{aligned} d(Tx, x) &= d(Tx, T^{2n}x) \\ &\leq a_1 d(x, x) + a_2 d(Tx, x) + a_3 d(T^{2n}x, x) + a_4 d(T^{2n}x, x) + a_5 d(x, Tx) \text{ Letting } n \rightarrow \infty \\ d(Tx, x) &\leq a_1 d(x, x) + a_2 d(Tx, x) + a_3 d(x, x) + a_4 d(x, x) + a_5 d(x, Tx) \\ &\leq sa_1 [d(x, Tx) + d(Tx, x)] + a_2 d(Tx, x) + sa_3 [d(x, Tx) + d(Tx, x)] \\ &\quad + a_4 [d(x, Tx) + d(Tx, x)] + a_5 d(x, Tx) \\ &\leq 2sa_1 d(x, Tx) + a_2 d(Tx, x) + 2sa_3 d(x, Tx) + 2sa_4 d(x, Tx) + a_5 d(x, Tx) \\ &\leq (2sa_1 + a_2 + 2sa_3 + 2sa_4 + a_5)d(x, Tx) \end{aligned}$$

$$[1 - (2sa_1 + a_2 + 2sa_3 + 2sa_4 + a_5)]d(x, Tx) \leq 0$$

Hence d(Tx, x) = 0.

This implies Tx = x. Therefore, T has a fixed point.

Uniqueness: Let x and y be two fixed points of T, that is Tx = x and Ty = y

$$\begin{aligned} d(x, y) &= d(Tx, Ty) \\ &\leq a_1 d(x, y) + a_2 d(Tx, x) + a_3 d(Ty, y) + a_4 d(Ty, x) + a_5 d(y, Tx) \\ &\leq a_1 d(x, y) + a_2 d(x, x) + a_3 d(y, y) + a_4 d(y, x) + a_5 d(y, x) \\ &\leq a_1 d(x, y) + a_2 s[d(x, y) + d(y, x)] + sa_3 [d(y, x) + d(x, y)] + a_4 d(y, x) + a_5 d(y, x) \\ d(x, y) &\leq (a_1 + 2sa_2 + 2sa_3 + a_4 + a_5)d(y, x) \end{aligned}$$

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$$[1 - (a_1 + 2sa_2 + 2sa_3 + a_4 + a_5)]d(x, y) \leq 0$$

Hence d(x, y) = 0.

Therefore, x = y.

Hence, T has a unique fixed point.

Example 2.2. Let X = R, A = [-2, 0]; B = [0, 2]. Define $d(a, b) = \frac{1}{2} [|a - b| + |a|]$

+ |b|]

Then d is the dislocated metric us define $T : A \cup B \rightarrow A \cup B$ by $Ta = -a$

Then T is a cyclic mapping. Here 0^0 is the unique fixed point.

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