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Variational Techniques for Image Denoising: A Review

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Abstract: Image Restoration is an important and growing area in the field of Digital Image Processing. It is the process which tries to recover an image which has been degraded by some degradation function. In doing so, a prior knowledge of this degradation function may come handy. Traditional denoising methods are not very efficient in case of mix noises. In the recent time, there has been tremendous work on Image recovery by various approaches. Variational approaches which are based on classic ROF model are very popular in image recovery. In this paper, we have described variational technique in detail with mathematical preliminaries. A few notable researches are also discussed as review.

Keywords: Image restoration, gradient descent, Euler- Lagrange eqn., Denoising, deblurring, variational techniques, ROF model

I. INTRODUCTION

The aim of the Image Restoration is to recover the original image *f* from the degraded image [1]

$$g = H[f] + n \qquad (1)$$

where g is the observed degraded image, f is the original undegraded image, H is the degradation function such as blur or turbulence and n is the additive noise. The approach is to obtain an estimate of the original image given by $\hat{f}$

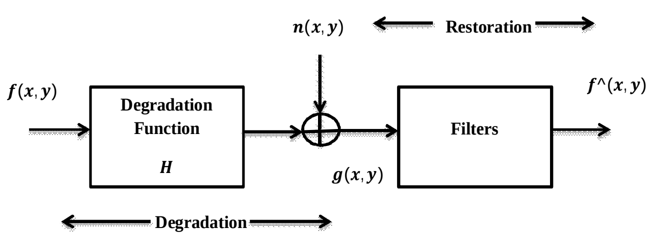

Figure 1: Image Degradation and Restoration Model

Degradation function is assumed to be Linear, space invariant

[2] which holds true for most cases. Prior information about the degradation function is very useful in Image restoration. This is where image recovery distinguishes from image enhancement [3]. In image enhancement there is no need to consider degradation function. Degradation function can be estimated by methods [4-6] like observation, experimentation or mathematical modeling as per suitability. If prior information is not available, the process is called Blind Deconvolution.

(a) *(b)* *(c)*

Figure 2: (a) Original image, (b) Blurred Image, (c) Restored Image

Simplest ways to recover original image using knowledge of degradation operator is to use the Inverse of this operator. This may work if noise does not exist, but in general, inverse filtering [6] enhances the noise and restored image is very bad. There are other approaches like constrained least square filtering [7] which is a deterministic approach and Wiener filtering [8-9] which is stochastic method of image recovery. But these techniques have drawbacks as they cannot work in Blur and Mix noise environment. Also the methods based on filtering techniques [10-12] like wavelet, FIR, IIR and DFT etc. are not very efficient in case of mix noises.
ROF model is a famous image denoising model proposed by L. Rudin, S. Osher and E. Fatemi in 1992 [13]. This model tries to minimize energy functional. Two terms are considered in functional: one for fidelity and other for smoothness in image. Regularization is provided by total variation penalty. Several other models have been developed by some variations in ROF model which are briefly described in the coming sections of this paper.

II. MATHEMATICS PRELIMINARIES

A. Euler-Lagrange Differential Equation

The Euler-Lagrange differential equation is used to solve variations problems and is of the form

$$J = \int f(t, y, \dot{y})dt \tag{2}$$

where $\dot{y} = \dfrac{dy}{dt}$

The stationary value of J is defined as

$$\frac{\partial f}{\partial y} - \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0 \tag{3}$$

In case of space derivative above equation (3) changes to

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y_x}\right) = 0 \tag{4}$$

B. Gradient Descent

If a function $f(x)$ is defined a neighborhood of point a, then $f(x)$ decreases fast in the direction of negative gradient at point a i.e. $-\nabla f(a)$. So we have

$$a^{n+1} = a^n - \gamma \nabla f(a^n) \tag{5}$$

However, γ is small enough, such that $f(a^n) \geq f(a^{n+1})$. So in any problem we begin with some initial guess (x_0) for a local minimum of f and consider sequence $x_0, x_1, \ldots, x_n, \ldots$ such that

$$x_{n+1} = x_n - \gamma \nabla f(x_n), \quad n \geq 0 \tag{6}$$

We get

$f(x_0) \geq f(x_1) \geq f(x_2) \geq \ldots.$ and so on.

III. ROF MODEL

A constrained optimization type of numerical algorithm for removing noise from images is presented. The total variation of the image is minimized subject to constraints involving the statistics of the noise. The constraints are imposed using Lagrange multipliers. The solution is obtained using the gradient-projection method. This amounts to solving a time dependent partial differential equation on a manifold determined by the constraints. As $t \to \infty$ the solution converges to a steady state which is the denoised image. The numerical algorithm is simple and relatively fast. The results appear to be state-of-the-art for very noisy images. The method is non-invasive, yielding sharp edges in the image. The technique could be interpreted as a first step of moving each level set of the image normal to itself with velocity equal to the curvature of the level set divided by the magnitude of the gradient of the image, and a second step which projects the image back onto the constraint set.

For the image reconstruction f(x) from the degraded image $g(x)$ Rudin, Osher and E. Fatemi proposes a model [13] based on variational principle as given below:

In ROF model they solved the following constrained minimization problem:

$$\text{Let } u_0(x,y) = u(x,y) + n(x,y) \tag{7}$$

Where, $u_0(x,y)$ is noisy image and $u(x,y)$ is desired clean image.

Minimize $\int_{\Omega} \sqrt{u_x^2 + u_y^2}\, dxdy$

Under the condition of mean $\int_{\Omega} udxdy = \int_{\Omega} u_0 dxdy$ and variance as $\int_{\Omega} (u-u_0)^2 dxdy = \sigma^2$

To convert this constrained problem to unconstrained problem, the Lagrange multiplier (λ) is used. Finally our energy function is the below unconstrained function:

$$J(u) = \int_{\Omega} \sqrt{u_x^2 + u_y^2}\, dxdy + \lambda_1 \int_{\Omega} udxdy + \lambda_2 \int_{\Omega} (u-u_0)^2 dxdy - \sigma^2 = 0$$

(8)

To minimize this energy functional we take its Euler - Langrage and we get:

$$0 = \frac{\partial}{\partial x} \frac{u_x}{\sqrt{u_x^2 + u_y^2}} + \frac{\partial}{\partial y} \frac{u_y}{\sqrt{u_x^2 + u_y^2}} + \lambda_1 - 2\lambda_2 (u - u_0)$$

Or we can write

$$0 = \frac{\partial}{\partial x} \frac{u_x}{\sqrt{u_x^2 + u_y^2}} + \frac{\partial}{\partial y} \frac{u_y}{\sqrt{u_x^2 + u_y^2}} - \lambda (u - u_0)$$

In general, when noise is completely eliminated, the above equation holds true. But in iterative algorithms, the above expression will approach to zero after some definite number of iterations. So for a particular iteration 't', we define above relation as

$$u_t = div\left(\frac{\nabla u}{|\nabla u|}\right) - \lambda (u - u_0) \tag{9}$$

where u_t is the step size of the gradient descent.

Considering $\Delta(t)$ as the primal step size, the final gradient descent equation is:

$$u^{n+1} = u^n + \Delta(t)\left(div\left(\frac{\nabla u}{|\nabla u|}\right) - \lambda (u - u_0)\right) \tag{10}$$

As n (number of time steps) increases, we approach a denoised version of the image.

IV. NOTABLE CONTRIBUTIONS

A. Lin He, Antonio Marquina, Stanley J. Osher (2005) [14]

In this paper author formulate a new time dependent model for blind deconvolution based on a constrained variational model. This model uses the sum of the total variation norms of the signal and the kernel as a regularizing functional. They also present an analytical study of the model discussing uniqueness of the solution, convergence to steady state and a priori parameter estimation. However, the quality of recovered image is not of good quality.

B. Gilles Aubert and Jean-François Aujol (2008) [15]

This paper proposes a method for multiplicative noise removal. In this work the modelling of speckle noise is done. By using a MAP estimator, we can derive a functional whose minimizer corresponds to the denoised image we want to recover.

C. Y. Wang, W. Chen, S. Zhou, T. Yu and Y. Zhang (2011) [16]

The total variation (TV) model is a second-order partial differential equation (PDE) based model for image noise removal. However, the second-order PDEs suffer from the so-called staircase effect. This paper, proposed is a modified TV (MTV) model, which is staircase-free by minimising the variation of the image along the direction tangential to the a line where the intensity of light is the same. Its effectiveness in preserving edges, avoiding staircases and removing noises is shown by experiment and comparison.

D. Paul Rodríguez (2013) [17]

This paper focuses on review of the most relevant TV numerical algorithms for solving the noise removal problem for grayscale/color images corrupted with several noise models, that is, Gaussian, Salt & Pepper, Poisson, and Speckle (Gamma) noise and for the mixed noise scenarios, such the mixed Gaussian and impulse model. They also include the description of the maximum a posteriori (MAP) estimator for each model as well as a summary of general optimization procedures that are typically used to solve the TV problem.

E. Liang-Jian Deng, Huiqing Guo, Ting-Zhu Huang (2015) [18]

In this paper explain a two level algorithm, i.e., deblurring step and denoising step. In the deblurring step, Fourier transform is employed for. In the denoising step, they use a simple and fast method, called fast iterative shrinkage/thresholding algorithm (FISTA), to reduce image noise.

F. Jianlou Xu , Aifen Feng , Yan Hao , Xuande Zhang , Yu Han (2016) [19]

Total variation method has been widely used in image processing. However, it produces undesirable staircase effect. To solve the staircase effect, fourth order variational models were studied, which lead to the restored images smoothing and some important details lost. To compute the new model effectively, they employ an alternating iterative method for recovering images from the blurry and noisy observations. The iterative algorithm decouples deblurring and denoising steps in the restoration process. In the deblurring step, fast transforms is employed. In the denoising step, the primal–dual method is adopted. The numerical experiments show that the new model can obtain better results than those by some recent methods.

V. CONCLUSIONS

This paper describes the basics of ROF model and gives the mathematical background required to understand the concept. Then various approaches for image denoising are discussed. These approaches are obtained by certain modifications in ROF model. However, the above described methods fail in most of the cases when blurring and noise is significant. Further work can be done to develop a method that gives satisfactory results in blur and mixed noise environment.

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