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# Certain Transformations Involving q-Series

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**Abstract :** In this paper, certain transformations for basic hypergeometric series have been established by making use of a known identity.

**Keywords:** Hypergeometric functions, Summation, Transformation, Polybasic, Converges

## I. INTRODUCTION

$$\text{In the following identity } \sum_{m=0}^n \delta_m \sum_{r=0}^m \alpha_r = \sum_{r=0}^n \alpha_r \sum_{m=0}^n \delta_m - \sum_{r=0}^{n-1} \alpha_{r+1} \sum_{m=0}^r \delta_m \tag{1.1}$$

If we take  $\alpha_j = z^j$ , (1.1) takes the following form :

$$\sum_{m=0}^n \delta_m z^m = \sum_{m=0}^n \delta_m + (1-z) \sum_{r=0}^{n-1} z^r \sum_{m=0}^r \delta_m \tag{1.2}$$

## II. DEFINITIONS AND NOTATIONS

For real or complex  $q$  ( $|q| < 1$ ), we get

$$(\lambda; q)_\infty = \prod_{j=0}^{\infty} (1 - \lambda q^j) \tag{2.1}$$

and let  $(\lambda; q)_\mu$  be defined by

$$(\lambda; q)_\mu = \frac{(\lambda; q)_\infty}{(\lambda q^\mu; q)_\infty} \tag{2.2}$$

For arbitrary parameters  $\lambda$  and  $\mu$ , so that

$$(\lambda; q)_n = \begin{cases} 1, & n = 0 \\ (1 - \lambda)(1 - \lambda q) \dots (1 - \lambda q^{n-1}), & n \in (1, 2, 3, \dots) \end{cases} \tag{2.3}$$

A truncated basic hypergeometric series is defined as

$${}_r\Phi_s \left[ \begin{matrix} a_1, a_2, \dots, a_r; q; Z \\ b_1, b_2, \dots, b_s \end{matrix} \right]_N = \sum_{n=0}^N \frac{\prod_{i=1}^r [a_i; q]_n z^n}{\prod_{j=1}^s [b_j; q]_n (q; q)_n} \tag{2.4}$$

Where  $|q| < 1, |z| < 1$  and no zero appear in the denominator.

The truncated polybasic hypergeometric series of one variable is defined as

$${}_r\Phi_s \left[ \begin{matrix} a_1, a_2, \dots, a_r; c_{1,1}, \dots, c_{m,1}; \dots, c_{m,r_m}; q_1, q_2, \dots, q_m; Z \\ b_1, b_2, \dots, b_s; d_{1,1}, \dots, d_{1,s_1}; \dots, d_{m,s_m}; \dots, d_{m,s_m} \end{matrix} \right]_N = \sum_{n=0}^N \frac{[a_1, a_2, \dots, a_r; q]_n z^n}{[q, b_1, b_2, \dots, b_s; q]_n} \prod_{j=1}^m \frac{[c_{j,1}, \dots, a_r; q_j]_n}{[d_{j,1}, \dots, d_{j,s_j}; q_j]_n} \tag{2.5}$$

The series (2.5) converges for  $|q||q_1|, \dots, |q_m| < 1, |z| < 1$ .

The other notations appearing in this paper shall stand for their usual meaning.

We shall use the following summations of truncated series in our analysis

$${}_2\Phi_1 \left[ \begin{matrix} a, y; q \\ ayq \end{matrix} \right]_n = \frac{[aq, yq; q]_n}{[q, ayq; q]_n} \tag{2.6}$$

$${}_4\Phi_3 \left[ \begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, e; q; \frac{1}{e} \\ \sqrt{a}, -\sqrt{a}, \frac{aq}{e} \end{matrix} \right]_n = \frac{[aq, eq; q]_n}{[q, \frac{aq}{e}; q]_n} e^n \tag{2.7}$$

$${}_6\Phi_5 \left[ \begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, d; q; q \\ \sqrt{a}, -\sqrt{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d} \end{matrix} \right]_n = \frac{[aq, bq, cq, dq; q]_n}{[q, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d}; q]_n} \tag{2.8}$$

Provided  $a = bcd$ .

$$\sum_{k=0}^n \frac{(1 - ap^k q^k) [a; p]_k [c; q]_k c^{-k}}{(1 - a) [q; q]_k \left[ \frac{ap}{c}; p \right]_k} = \frac{[ap; p]_n [cq; q]_n c^{-n}}{[q; q]_n [ap/c; p]_n} \tag{2.9}$$

$$\sum_{k=0}^n \frac{(1 - ap^k q^k)(1 - bp^k q^{-k}) [a, b; p]_k \left[ c, \frac{a}{bc}; q \right]_k q^k}{(1 - a)(1 - b) \left[ q, \frac{aq}{b}; q \right]_k \left[ \frac{ap}{c}, bcp; p \right]_k} = \frac{[ap, bp; p]_n \left[ cq, \frac{aq}{bc}; q \right]_n}{\left[ q, \frac{aq}{b}; q \right]_n \left[ \frac{ap}{c}, bcp; p \right]_n} \tag{2.10}$$

$$\begin{aligned} & \sum_{k=0}^n \frac{(1 - adp^k q^k)(1 - \frac{b}{d} p^k q^{-k}) [a, b; p]_k \left[ c, \frac{ad^2}{bc}; q \right]_k q^k}{(1 - ad)(1 - \frac{b}{d}) \left[ dq, \frac{adq}{b}; q \right]_k \left[ \frac{adp}{c}, \frac{bcp}{d}; p \right]_k} \\ &= \frac{(1 - a)(1 - b)(1 - c) \left( 1 - \frac{ad^2}{bc} \right)}{d(1 - ad) \left( 1 - \frac{b}{d} \right) \left( 1 - \frac{c}{d} \right) \left( 1 - \frac{ad}{bc} \right)} \times \left[ \frac{[ap, bp; p]_n \left[ cq, \frac{ad^2 q}{bc}; q \right]_n}{\left[ dq, \frac{adq}{b}; q \right]_n \left[ \frac{adp}{c}, \frac{bcp}{d}; p \right]_n} - \frac{(b - ad)(c - ad)(d - bc)(1 - d)}{d(1 - a)(1 - b)(1 - c)(bc - ad^2)} \right] \tag{2.11} \end{aligned}$$

Which is  $m=0$  case of [Gasper and Rahman 3; App. II (II.36)]

$$\begin{aligned} & \sum_{k=0}^n \frac{(1 - adp^k q^k P^k Q^k) \left( 1 - \frac{d}{c} \frac{P^k Q^k}{p^k q^{-k}} \right) \left( 1 - \frac{b}{d} \frac{p^k P^k}{q^k Q^k} \right) \left( 1 - \frac{ad}{bc} \frac{p^k Q^k}{q^k P^k} \right)}{(1 - ad) \left( 1 - \frac{d}{c} \right) \left( 1 - \frac{b}{d} \right) \left( 1 - \frac{ad}{bc} \right)} \\ & \times \frac{[a, p^2]_k [c, q^2]_k [b, P^2]_k \left[ \frac{ad}{bc}; Q^2 \right]_k q^{2k}}{\left[ \frac{d}{p} \frac{qPQ}{P}; \frac{qPQ}{P} \right]_k \left[ \frac{ad}{c} \frac{pPQ}{q}; \frac{pPQ}{q} \right]_k \left[ \frac{ad}{b} \frac{pqQ}{P}; \frac{pqQ}{P} \right]_k \left[ \frac{bc}{d} \frac{pqP}{Q}; \frac{pqP}{Q} \right]_k} \\ &= \frac{(1 - a)(1 - b)(1 - c) \left( 1 - \frac{ad^2}{bc} \right)}{(1 - ad)(c - d) \left( 1 - \frac{b}{d} \right) \left( 1 - \frac{ad}{bc} \right)} \\ & \times \left[ \frac{[ap^2; p^2]_n [cq^2; q^2]_n [bP^2; P^2]_n \left[ \frac{ad^2}{bc} Q^2; Q^2 \right]_n}{\left[ \frac{d}{p} \frac{qPQ}{P}; \frac{qPQ}{P} \right]_n \left[ \frac{ad}{c} \frac{pPQ}{q}; \frac{pPQ}{q} \right]_n \left[ \frac{ad}{b} \frac{pqQ}{P}; \frac{pqQ}{P} \right]_n \left[ \frac{bc}{d} \frac{pqP}{Q}; \frac{pqP}{Q} \right]_n} - \frac{(b - ad)(c - ad)(d - bc)(1 - d)}{d(1 - a)(1 - b)(1 - c)(bc - ad^2)} \right] \tag{2.12} \end{aligned}$$

Which is  $m = 0$  case of [Agarwal et. al.<sup>4</sup> ; (18) p. (89)]

### III.MAIN RESULTS

If we take  $\delta_m = \frac{[a, y; q]_m q^m}{[q, ayq; q]_m q^m}$  in (1.2) and make use of (2.6) we get :

$${}_2\Phi_1 \left[ \begin{matrix} a, y; q; qz \\ ayq \end{matrix} \right]_n = \frac{z^n [aq, yq; q]_n}{[q, ayq; q]_n} + (1-z) {}_2\Phi_1 \left[ \begin{matrix} aq, yq; q; z \\ ayq \end{matrix} \right]_{n-1}, \quad |z| < 1 \quad (3.1)$$

As  $n \rightarrow \infty$  (3.1) yields the following transformation

$$\backslash \quad {}_2\Phi_1 \left[ \begin{matrix} a, y; q; qz \\ ayq \end{matrix} \right] = (1-z) {}_2\Phi_1 \left[ \begin{matrix} aq, yq; q; z \\ ayq \end{matrix} \right], \quad |z| < 1 \quad (3.2)$$

(i) Taking  $\delta_m = \frac{[a, q\sqrt{a}, -q\sqrt{a}; e; q]_m}{[q, \sqrt{a}, -\sqrt{a}, aq/e; q]_m e^m}$  in (1.2) and making use of (2.7) we get

$${}_4\Phi_3 \left[ \begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, e; z/e \\ \sqrt{a}, -\sqrt{a}, aq/e \end{matrix} \right]_n = \frac{z^n [aq, eq; q]_n}{e^n [e, aq/e; q]_n} + (1-z) {}_2\Phi_1 \left[ \begin{matrix} aq, eq, q; q; z/e \\ e, aq/e \end{matrix} \right]_{n-1} \quad (3.3)$$

For  $|z| < 1$  and  $|e| > 1$  (1.1) yields the following transformation, when  $n \rightarrow \infty$ ,

$${}_4\Phi_3 \left[ \begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, e; z/e \\ \sqrt{a}, -\sqrt{a}, aq/e \end{matrix} \right] = (1-z) {}_2\Phi_1 \left[ \begin{matrix} aq, eq, q; q; z/e \\ e, aq/e \end{matrix} \right] \quad (3.4)$$

(ii) Taking  $\delta_m = \frac{[a, q\sqrt{a}, -q\sqrt{a}, b, c, a/bc; q]_m}{[q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bcq; q]_m}$  in (1.2) and making use of (2.8) we get

$${}_6\Phi_5 \left[ \begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, a/bc; q; zq \\ q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bcq \end{matrix} \right]_n = \frac{[aq, bq, cq, aq/bc; q; z]_n z^n}{[q, aq/b, aq/c, bcq; q]_n} + (1-z) {}_2\Phi_1 \left[ \begin{matrix} aq, bq, cq, aq/bc; q; z \\ aq/b, aq/c, bcq \end{matrix} \right]_{n-1}, \quad |z| < 1 \quad (3.5)$$

As  $n \rightarrow \infty$ , (3.5) yield the following transformation

$${}_6\Phi_5 \left[ \begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, a/bc; q; zq \\ q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bcq \end{matrix} \right] = (1-z) {}_4\Phi_3 \left[ \begin{matrix} aq, bq, cq, aq/bc; q; z \\ aq/b, aq/c, bcq \end{matrix} \right] \quad (3.6)$$

(iii) Taking  $\delta_m = \frac{[a, pq; pq]_m [a; p]_m [c; q]_m c^{-m}}{[a; pq]_m [a; q]_m [ap/c; p]_m}$  in (1.2) and making use of (2.9) we get

$${}_3\Phi_2 \left[ \begin{matrix} c; a; apq; q, p, pq, z/c \\ -; ap/c; a; \end{matrix} \right]_n = \frac{z^n [ap; p]_n [cq; q]_n}{c^n [q; q]_n [ap/c; p]_n} (1-z) {}_2\Phi_1 \left[ \begin{matrix} cq, ap; p; z/c \\ -; ap/c \end{matrix} \right]_{n-1} \quad (3.7)$$

For  $|z| < 1$  and  $|c| > 1$  and  $n \rightarrow \infty$ , (3.7) yields

$${}_3\Phi_2 \left[ \begin{matrix} c; a; apq; q, p, pq, z/c \\ -; ap/c; a; \end{matrix} \right] = (1-z) {}_2\Phi_1 \left[ \begin{matrix} cq, ap; p; z/c \\ -; ap/c \end{matrix} \right] \quad (3.8)$$

(iv) Taking  $\delta_m = \frac{[apq; pq]_m [bp/q; p/q]_m [a, b; p]_m [c, a/bc; q]_m q^m}{[a; pq]_m [b; p/q]_m [q, aq/b; q]_m [ap/c, bcp; p]_m}$  in (1.2) and making use (2.10), we get

$${}_6\Phi_5 \left[ \begin{matrix} c; aq/bc; ap, bp; q, p, pq, z \\ aq/b; ap/c, bcp; \end{matrix} \right]_n = \frac{[ap, bp; p]_n [cq, aq/bc; q]_n z^n}{[q, aq/b; q]_n [ap/c, bcp; p]_n}$$

$$+ (1-z) {}_4\Phi_3 \left[ \begin{matrix} cq, aq / bc; ap, bp, q, p; z \\ ap / c; ap / c, bcp \end{matrix} \right]_{n-1}, |z| < 1 \tag{3.9}$$

As  $n \rightarrow \infty$ , (3.9) yields the following transformations

$${}_6\Phi_5 \left[ \begin{matrix} c; aq / bc; ap, bp, q, p, pq, z \\ aq / b; ap / c, bcp \end{matrix} \right] = (1-z) {}_4\Phi_3 \left[ \begin{matrix} cq, aq / bc; ap, bp, q, p; z \\ ap / c; ap / c, bcp \end{matrix} \right] \tag{3.10}$$

(v) Taking  $\delta_m = \frac{[adpq; pq]_m \left[ \frac{b}{d}, \frac{p}{q} \right]_m [a, b; p]_m \left[ c, \frac{ad^2}{bc}; q \right]_m q^m}{[a; pq]_m \left[ \frac{b}{d}, \frac{p}{q} \right]_m [q, aq / b; q; q]_m \left[ \frac{adp}{c}, \frac{bcp}{d}; p \right]_m}$

In (1.2) and making use of (2.11)

$$\begin{aligned} & {}_7\Phi_6 \left[ \begin{matrix} c; ad^2 / bc, q; a, b, adpq; b / dq; q, p, pq, p / q; zq \\ dq, adq / b; adp / c, bcp / d; ad; b / d \end{matrix} \right]_n \\ &= \frac{z^n (1-a)(1-b)(1-c)(1-ad^2 / bc)[ap, bp; p]_n [cq, ad^2q; q]_n}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)[dq, adq / b; q]_n [adp / c; bcp / d; p]_n} \\ &\quad - \frac{(1-ad)(c-ad)(d-bc)(1-d)}{bcd^2(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \\ &+ (1-z) \frac{(1-a)(1-b)(1-c)(1-ad^2 / bc)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times {}_5\Phi_4 \left[ \begin{matrix} cq; ad^2q / bc; q; ap, bp, q, p; z \\ dq, adq / b, adp / c, bcp / d \end{matrix} \right]_{n-1}, |z| < 1 \end{aligned} \tag{3.11}$$

As  $n \rightarrow \infty$ , (3.11) yields

$$\begin{aligned} & {}_7\Phi_6 \left[ \begin{matrix} c; ad^2 / bc, q; a, b, adpq; b / dq; q, p, pq, p / q; zq \\ dq, adq / b; adp / c, bcp / d; ad; b / d \end{matrix} \right] \\ &= (1-z) \frac{(1-a)(1-b)(1-c)(1-ad^2 / bc)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times {}_5\Phi_4 \left[ \begin{matrix} cq; ad^2q / bc; q; ap, bp, q, p; z \\ dq, adq / b, adp / c, bcp / d \end{matrix} \right] \\ &\quad - \frac{(1-ad)(c-ad)(d-bc)(1-d)}{bcd^2(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \end{aligned} \tag{3.12}$$

#### IV. CONCLUSION

In this paper, an attempt has been made to establish six certain transformation formulae for basic hypergeometric series by making use of the identity (1.2)

#### V. ACKNOWLEDGEMENT

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