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Some Elementary Operations on Intuitionistic Fuzzy Graph Structures

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Abstract: In this paper, some elementary operations like union, join, cartesian product, composition on intuitionistic fuzzy graph structure (IFGS) $\tilde{G}$ are defined and their properties are discussed. We also obtain the Phi-complement of these elementary operations on intuitionistic fuzzy graph structures.
Keywords: Intuitionistic fuzzy graph structure, union, join, cartesian product, composition, phi-complement of union and join of IFGS. 2010 Mathematics Subject Classification: 05C72, 05C76, 05C38, 03F55.

I. INTRODUCTION

The idea of fuzzy sets was introduced by Prof. Zadeh [8] in 1965. Then Rosenfeld [9] laid down the foundation of the concept of fuzziness in relations and graphs in 1975. Later on the thought of intuitionistic fuzzy sets was proposed by Atanassov [5]. Further the notion of graph structure was discussed by Sampathkumar in [1]. Dinesh and Ramakrishnan [2] gave the idea of fuzzy graph structure. The notion of intuitionistic fuzzy graph structure (IFGS) $\tilde{G} = (A, B_1, B_2, ...B_k)$ are defined and discussed by the authors in [6] and [7]. In this paper some elementary operations on intuitionistic fuzzy graph structure are defined and their phi-complements are obtained.

II. PRELIMINARIES

In this section, we review some definitions and results that are necessary in this paper, which are mainly taken from [1], [2], [6], [7] and [10].

A. *Definition (2.1)*

$G = (V, R_1, R_2, ..., R_k)$ is a graph structure if V is a non empty set and $R_1, R_2, ..., R_k$ are relations on V which are mutually disjoint such that each R_i, i=1,2,3,...,k, is symmetric and irreflexive.

B. *Definition (2.2)*

An intuitionistic fuzzy graph is of the form G = (V, E) where

i) $V=\{v_1,v_2,...,v_n\}$ such that $\mu_1:V\to[0,1]$ and $\gamma_1:V\to[0,1]$ denote the degree of membership and non - membership of the element $v_i \in V$, respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$, for every $v_i \in V$, (i = 1,2,….,n),

ii) $E \subseteq V \times V$ where $\mu_2: V\times V \to [0,1]$ and $\gamma_2 : V\times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \le \min\{\mu_1(v_i), \mu_1(v_j)\} \quad \text{and} \quad \gamma_2(v_i, v_j) \le \max\{\gamma_1(v_i), \gamma_1(v_j)\}$$

and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$, for every $(v_i, v_j) \in E$, (i, j = 1,2,….,n).

C. *Definition (2.3)*

Let $G = (V, R_1, R_2, ..., R_k)$ be a graph structure and $A, B_1, B_2, …., B_k$ be intuitionistic fuzzy subsets of $V, R_1, R_2, ..., R_k$ respectively such that

$$\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v) \text{ and } \nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v) \quad \forall u, v \in V \text{ and } i = 1, 2, ..., k.$$

Then $\tilde{G} = (A, B_1, B_2, …., B_k)$ is an intuitionistic fuzzy graph structure (IFGS) of G.

D. *Example (2.4)*

Consider the graph structure $G = (V, R_1, R_2, R_3)$, where $V = \{u_0, u_1, u_2, u_3, u_4\}$ and $R_1 = \{(u_0, u_1), (u_0, u_2), (u_3, u_4)\}$, $R_2 = \{(u_1, u_2), (u_2, u_4)\}$, $R_3 = \{(u_2, u_3), (u_0, u_4)\}$ are the relations on V. Let $A = \{< u_0, 0.5, 0.4 >, < u_1, 0.6, 0.3>, < u_2, 0.2, 0.6 >, <u_3, 0.1, 0.8>, <u_4,$

0.4,0.3> } be an IFS on V and B_1 = { (u_0, u_1), 0.5,0.3 > , < (u_0, u_2), 0.1,0.3 >, < (u_3, u_4), 0.1,0.2 >}, B_2 = {< (u_1, u_2), 0.2,0.1 >,< (u_2, u_4),0.1,0.2 >}, B_3 ={< (u_2, u_3), 0.1,0.5 >, ,< (u_0, u_4),0.3,0.2 >} are IFRs on V. Here $\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v)$ $\forall\, u, v \in V$ and i =1,2,3. $\therefore\ \tilde{G}$ is an IFGS.

III.ELEMENTARY OPERATIONS ON INTUITIONISTIC FUZZY GRAPH STRUCTURES

A. *Definition (3.1)*

Let $\tilde{G}_1 = (A_1, B_{11}, B_{12}, \ldots, B_{1k})$ and $\tilde{G}_2 = (A_2, B_{21}, B_{22}, \ldots, B_{2k})$ be two IFGSs of graph structures $G_1 = (V, R_{11}, R_{12}, \ldots, R_{1k})$ and $G_2 = (V', R_{21}, R_{22}, \ldots, R_{2k})$ respectively then the union $\tilde{G}_1 \cup \tilde{G}_2$ of $\tilde{G}_1$ and $\tilde{G}_2$ is given by $(A_1 \cup A_2, B_{11} \cup B_{21}, B_{12} \cup B_{22}, \ldots, B_{1k} \cup B_{2k})$ where $A_1 \cup A_2$ and $B_{1i} \cup B_{2i}$ are given by

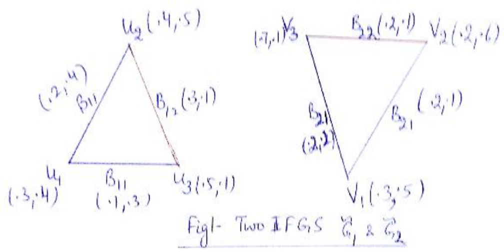
$$\mu_{A_1\cup A_2}(u) = \begin{cases} \mu_{A_1}(u) & ; \text{if } u \in V \text{ and } u \notin V' \\ \mu_{A_2}(u) & ; \text{if } u \in V' \text{ and } u \notin V \\ \mu_{A_1}(u) \vee \mu_{A_2}(u) & ; \text{if } u \in V \cap V' \end{cases} ; \quad \nu_{A_1\cup A_2}(u) = \begin{cases} \nu_{A_1}(u) & ; \text{if } u \in V \text{ and } u \notin V' \\ \nu_{A_2}(u) & ; \text{if } u \in V' \text{ and } u \notin V \\ \nu_{A_1}(u) \wedge \nu_{A_2}(u) & ; \text{if } u \in V \cap V' \end{cases}$$

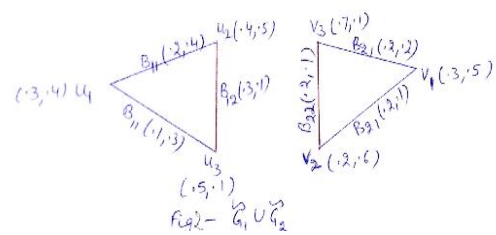
$$(\mu_{B_{1i}\cup B_{2i}})(uv) = \begin{cases} \mu_{B_{1i}}(uv) & \text{if } uv \in E_1 \text{ and } uv \notin E_2 \\ \mu_{B_{2i}}(uv) & \text{if } uv \in E_2 \text{ and } uv \notin E_1 \\ \mu_{B_{1i}}(uv) \vee \mu_{B_{2i}}(uv) & \text{if } uv \in E_1 \cap E_2 \end{cases} ; \quad (\nu_{B_{1i}\cup B_{2i}})(uv) = \begin{cases} \nu_{B_{1i}}(uv) & \text{if } uv \in E_1 \text{ and } uv \notin E_2 \\ \nu_{B_{2i}}(uv) & \text{if } uv \in E_2 \text{ and } uv \notin E_1 \\ \nu_{B_{1i}}(uv) \wedge \mu_{B_{2i}}(uv) & \text{if } uv \in E_1 \cap E_2 \end{cases}$$

for all i = 1, 2, 3 ,.......,k, where $E_1 = \bigcup_{i=1}^{k} R_{1i}$ and $E_2 = \bigcup_{i=1}^{k} R_{2i}$.

B. *Example (3.2*

Consider $\tilde{G}_1 = (A_1, B_{11}, B_{12})$ and $\tilde{G}_2 = (A_2, B_{21}, B_{22})$ be two IFGSs of graph structures $G_1 = (V, R_{11}, R_{12})$ and $G_2 = (V', R_{21}, R_{22})$ respectively where V= {u_1, u_2, u_3} and V′ ={v_1, v_2, v_3}. Let A_1 = {< u_1, 0.3,0.4 >, < u_2 , 0.4,0.5 >, <u_3,0.5,0.1>}and A_2 = {< v_1, 0.3,0.5 >, < v_2 , 0.2,0.6 >, <v_3,0.7,0.1>} be an IFS on V and B_{11}= {< (u_1, u_2), 0.2,0.4 > , < (u_1, u_3), 0.1,0.3 >}, B_{12} = { < (u_2, u_3),0.3,0.1 >} and B_{21} = { (v_1, v_2), 0.2,0.1> ,< (v_1, v_3), 0.2,0.2 >}, B_{22} = { < (v_2,v_3),0.2,0.1 >} are IFRs on V as shown in Fig 1. then the union $\tilde{G}_1 \cup \tilde{G}_2$ of $\tilde{G}_1$ and $\tilde{G}_2$ is as shown in Fig.2.





C. *Theorem (3.3)*

Let G = $G_1 \cup G_2$ be the union of two graph structures $G_1 = (V, R_{11}, R_{12}, \ldots, R_{1k})$ and $G_2 = (V', R_{21}, R_{22}, \ldots, R_{2k})$. Let $\tilde{G}_1 = (A_1, B_{11}, B_{12}, \ldots, B_{1k})$ and $\tilde{G}_2 = (A_2, B_{21}, B_{22}, \ldots, B_{2k})$ be IFGSs corresponding to the graph structures G_1 and G_2 respectively, then $\tilde{G} = \tilde{G}_1 \cup \tilde{G}_2 = (A_1 \cup A_2, B_{11} \cup B_{21}, B_{12} \cup B_{22}, \ldots, B_{1k} \cup B_{2k})$ is an IFGS of G.

1) *Proof*: For i = 1, 2, 3,….,k .

Case (i) : If $u \in V$ and $u \notin V'$ and if $uv \in E_1$ and $uv \notin E_2$ then

$$(\mu_{B_{1i}\cup B_{2i}})(uv) = \mu_{B_{1i}}(uv) = \mu_{A_1}(u) \wedge \mu_{A_1}(v) = \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v)$$

and $(\nu_{B_{1i}\cup B_{2i}})(uv) = \nu_{B_{1i}}(uv) = \nu_{A_1}(u) \wedge \nu_{A_1}(v) = \nu_{A_1\cup A_2}(u) \wedge \nu_{A_1\cup A_2}(v)$

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Case (ii) : - *If* $u \in V'$ and $u \notin V$ and if $uv \in E_2$ and $uv \notin E_1$ then

$$(\mu_{B_{1i}\cup B_{2i}})(uv) = \mu_{B_{2i}}(uv) = \mu_{A_2}(u) \wedge \mu_{A_2}(v) = \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v)$$

$$and\ (\nu_{B_{1i}\cup B_{2i}})(uv) = \nu_{B_{2i}}(uv) = \nu_{A_2}(u) \wedge \nu_{A_2}(v) = \nu_{A_1\cup A_2}(u) \wedge \nu_{A_1\cup A_2}(v)$$

Case (iii) : - *If* $u \in V \cap V'$ and if $uv \in E_1 \cap E_2$ then

$$(\mu_{B_{1i}\cup B_{2i}})(uv) = \mu_{B_{1i}}(uv) \vee \mu_{B_{2i}}(uv) \le [\mu_{A_1}(u) \wedge \mu_{A_1}(v)] \vee [\mu_{A_2}(u) \wedge \mu_{A_2}(v)]$$
$$\le [\mu_{A_1}(u) \vee \mu_{A_2}(u)] \wedge [\mu_{A_1}(v) \vee \mu_{A_2}(v)] = \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v)$$

$$\therefore (\mu_{B_{1i}\cup B_{2i}})(uv) \le \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v)$$

$$\text{and } (\nu_{B_{1i}\cup B_{2i}})(uv) = \nu_{B_{1i}}(uv) \wedge \nu_{B_{2i}}(uv) \le [\nu_{A_1}(u) \vee \nu_{A_1}(v)] \wedge [\nu_{A_2}(u) \vee \nu_{A_2}(v)]$$
$$\le [\nu_{A_1}(u) \wedge \nu_{A_2}(u)] \vee [\nu_{A_1}(v) \wedge \nu_{A_2}(v)] = \nu_{A_1\cup A_2}(u) \vee \nu_{A_1\cup A_2}(v)$$

$$\therefore (\nu_{B_{1i}\cup B_{2i}})(uv) \le \nu_{A_1\cup A_2}(u) \vee \nu_{A_1\cup A_2}(v)$$

$\therefore \breve{G} = (A_1 \cup A_2, B_{11} \cup B_{21}, B_{12} \cup B_{22}, ..., B_{1k} \cup B_{2k})$ is an IFGS of G.

D. Theorem (3.4)

Let $\breve{G} = (A_1 \cup A_2, B_{11} \cup B_{21}, B_{12} \cup B_{22}, ..., B_{1k} \cup B_{2k})$ be the union of IFGSs $\breve{G}_1 = (A_1, B_{11}, B_{12}, ..., B_{1k})$ and $\breve{G}_2 = (A_2, B_{21}, B_{22}, ..., B_{2k})$, then every IFGS $(A, B_1, B_2, ..., B_k)$ of G is the union of IFGSs of G_1 and G_2.

1) *Proof:* Define μ_{A_1} *and* μ_{A_2} as $\mu_{A_1}(u) = \mu_A(u)$ *if* $u \in V$; $\mu_{A_2}(u) = \mu_A(u)$ *if* $u \in V'$

and $\nu_{A_1}(u) = \nu_A(u)$ *if* $u \in V$; $\nu_{A_2}(u) = \nu_A(u)$ *if* $u \in V'$.

Define $\mu_{B_{1i}}$ and $\mu_{B_{2i}}$ *as* $\mu_{B_{1i}}(uv) = \mu_{B_i}(uv)$ *if* $uv \in R_{1i}$; $\mu_{B_{2i}}(uv) = \mu_{B_i}(uv)$ *if* $uv \in R_{2i}$

and $\nu_{B_{1i}}(uv) = \nu_{B_i}(uv)$ *if* $uv \in R_{1i}$; $\nu_{B_{2i}}(uv) = \nu_{B_i}(uv)$ *if* $uv \in R_{2i}$ for i= 1,2,3,….,k.

For j=1,2, $\mu_{B_{ji}}(u^j v^j) = \mu_{B_i}(u^j v^j) \le \mu_A(u^j) \wedge \mu_A(v^j) = \mu_{A_j}(u^j) \wedge \mu_{A_j}(v^j)$

and $\nu_{B_{ji}}(u^j v^j) = \nu_{B_i}(u^j v^j) \le \nu_A(u^j) \vee \nu_A(v^j) = \nu_{A_j}(u^j) \vee \nu_{A_j}(v^j)$.

$\therefore \breve{G}_{ji} = (A_j, B_{j1}, B_{j2}, ..., B_{jk})$ is an IFGS of G_j, j = 1, 2.

Thus an IFGS $(A, B_1, B_2, ...B_k)$ of G is a union of an IFGSs of G_1 and G_2 where $A = A_1 \cup A_2$ and $B_i = B_{1i} \cup B_{2i}$ for $i = 1, 2, ..., k$.

E. Theorem (3.5)

Let $G_1 = (V, R_{11}, R_{12}, ..., R_{1k})$ and $G_2 = (V', R_{21}, R_{22}, ..., R_{2k})$ be two graph structures with $V \cap V' = \varnothing$. Let A_1 and A_2 be intuitionistic fuzzy subsets of V and V′ and B_{1i}, B_{2i} be intuitionistic fuzzy subsets of R_{1i}, R_{2i} respectively for i =1,2,3,….,k. If $(A_1 \cup A_2, B_{11} \cup B_{21}, B_{12} \cup B_{22} ..., B_{1k} \cup B_{2k})$ is an IFGS of $G_1 \cup G_2$, then $(A_1, B_{11}, B_{12}, ..., B_{1k})$ and $(A_2, B_{21}, B_{22}, ..., B_{2k})$ are IFGSs of graph structures G_1 and G_2 respectively.

1) *Proof:* If $(A_1 \cup A_2, B_{11} \cup B_{21}, B_{12} \cup B_{22}, ..., B_{1k} \cup B_{2k})$ is an IFGS of $G_1 \cup G_2$,

$$(\mu_{B_{1i}\cup B_{2i}})(uv) \le \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v)\ and\ (\nu_{B_{1i}\cup B_{2i}})(uv) \le \nu_{A_1\cup A_2}(u) \vee \nu_{A_1\cup A_2}(v)$$

$\therefore uv \notin R_{2i}$.

Let $uv \in R_{1i}$ then $u \in V, v \in V$ *and hence* $u \notin V', v \notin V'$.

$$\therefore \mu_{B_{1i}}(uv) = (\mu_{B_{1i}\cup B_{2i}})(uv) \le \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v)$$
$$= \mu_{A_1}(u) \wedge \mu_{A_1}(v) \quad (\because u \notin V', v \notin V')$$

$$and\ \nu_{B_{1i}}(uv) = (\nu_{B_{1i}\cup B_{2i}})(uv) \le \nu_{A_1\cup A_2}(u) \vee \nu_{A_1\cup A_2}(v)$$
$$= \nu_{A_1}(u) \vee \nu_{A_1}(v) \quad (\because u \notin V', v \notin V').$$

This is true for i = 1,2,3,….,k .

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$\therefore$ $(A_1,B_{11},B_{12},...,B_{1k})$ is an IFGS of G_1.

Similarly it can be proved that $(A_2,B_{21},B_{22},...,B_{2k})$ is an IFGS of G_2.

F. Definition (3.6)

Let $\tilde{G}_1 = (A_1,B_{11},B_{12},...,B_{1k})$ and $\tilde{G}_2 = (A_2,B_{21},B_{22},...,B_{2k})$ be two IFGSs of G_1 and G_2 respectively then the join $\tilde{G}_1+\tilde{G}_2$ of $\tilde{G}_1$ and $\tilde{G}_2$ is given by $(A_1+A_2, B_{11}+B_{21}, B_{12}+B_{22},...,B_{1k}+B_{2k})$ where A_1+A_2 and $B_{1i}+B_{2i}$ are given by

$(\mu_{A_1+A_2})(u) = \max\{\mu_{A_1}(u), \mu_{A_2}(u)\}$ and $(\nu_{A_1+A_2})(u) = \min\{\nu_{A_1}(u), \nu_{A_2}(u)\}$, if $u \in V \cup V'$

$(\mu_{B_{1i}+B_{2i}})(uv) = \mu_{B_{1i}\cup B_{2i}}(uv)$ and $(\nu_{B_{1i}+B_{2i}})(uv) = \nu_{B_{1i}\cup B_{2i}}(uv)$, if $uv \in E_1 \cup E_2$

$(\mu_{B_{1i}+B_{2i}})(uv) = \min\{\mu_{A_1}(u), \mu_{A_2}(v)\}$ and $(\nu_{B_{1i}+B_{2i}})(uv) = \max\{\nu_{A_1}(u), \nu_{A_2}(v)\}$, if $uv \in E'$ where E' is the set of all edges joining the vertices of V and V', for all i = 1, 2, 3 ,.......,k.

G. Example (3.7)

Consider $\tilde{G}_1$ and $\tilde{G}_2$ be two IFGSs as shown in Fig1 then join $\tilde{G}_1 \cup \tilde{G}_2$ of $\tilde{G}_1$ and $\tilde{G}_2$ is as shown in Fig3.

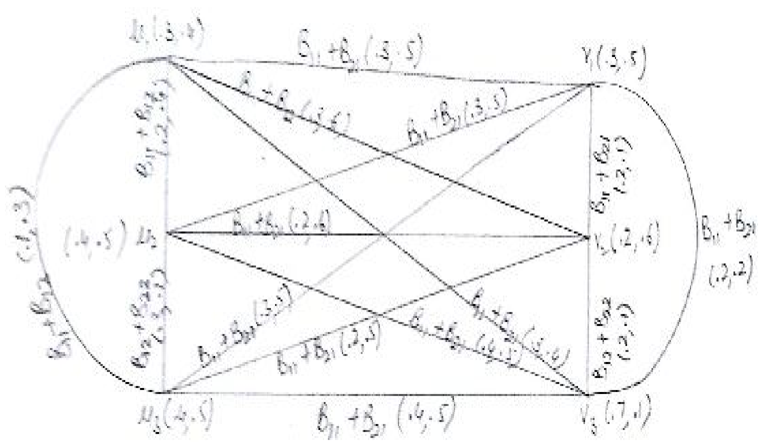

fig3:- $\tilde{G}_1+\tilde{G}_2$ - Join of $\tilde{G}_1$ & $\tilde{G}_2$.

H. Theorem (3.8)

Let G = $G_1 + G_2$ be the join of two graph structures G_1= (V, R_{11}, R_{12},...,R_{1k}) and G_2 = (V′, R_{21}, R_{22},...,R_{2k}). Let $\tilde{G}_1 = (A_1, B_{11}, B_{12},...,B_{1k})$ and $\tilde{G}_2 = (A_2, B_{21},B_{22},...,B_{2k})$ be IFGSs corresponding to graph structures G_1 and G_2 respectively then $(A_1+A_2, B_{11}+B_{21}, B_{12}+B_{22},...,B_{1k}+B_{2k})$ is an IFGS of G.

1) *Proof:* For i= 1,2,3,….,k

Case (i) : - If $uv \in E_1 \cup E_2$,

$$(\mu_{B_{1i}+B_{2i}})(uv) = (\mu_{B_{1i}\cup B_{2i}})(uv) = \mu_{B_{1i}}(uv) \vee \mu_{B_{2i}}(uv)$$
$$\le [\mu_{A_1}(u) \wedge \mu_{A_1}(v)] \vee [\mu_{A_2}(u) \wedge \mu_{A_2}(v)]$$
$$\le [\mu_{A_1}(u) \vee \mu_{A_2}(u)] \wedge [\mu_{A_1}(v) \vee \mu_{A_2}(v)]$$
$$= \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v) = \mu_{A_1+A_2}(u) \wedge \mu_{A_1+A_2}(v)$$

$$\therefore (\mu_{B_{1i}+B_{2i}})(uv) \le \mu_{A_1+A_2}(u) \wedge \mu_{A_1+A_2}(v)$$

and $(\nu_{B_{1i}+B_{2i}})(uv) = (\nu_{B_{1i}\cup B_{2i}})(uv) = \nu_{B_{1i}}(uv) \wedge \nu_{B_{2i}}(uv)$

$\leq [\nu_{A_1}(u) \vee \nu_{A_1}(v)] \wedge [\nu_{A_2}(u) \vee \nu_{A_2}(v)]$

$\leq [\nu_{A_1}(u) \wedge \nu_{A_2}(u)] \vee [\nu_{A_1}(v) \wedge \nu_{A_2}(v)]$

$= \nu_{A_1\cup A_2}(u) \vee \nu_{A_1\cup A_2}(v) = \nu_{A_1+A_2}(u) \vee \nu_{A_1+A_2}(v)$

$\therefore (\nu_{B_{1i}+B_{2i}})(uv) \leq \nu_{A_1+A_2}(u) \vee \nu_{A_1+A_2}(v)$

Case (ii) : - *If* $u \in V$ *and* $v \in V'$ *and* $uv \in E'$,

$(\mu_{B_{1i}+B_{2i}})(uv) = \mu_{A_1}(u) \wedge \mu_{A_2}(v) = \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v) \ \ (\because u \in V \text{ and } v \in V') = \mu_{A_1+A_2}(u) \wedge \mu_{A_1+A_2}(v)$

and $(\nu_{B_{1i}+B_{2i}})(uv) = \nu_{A_1}(u) \vee \nu_{A_2}(v) = \nu_{A_1\cup A_2}(u) \vee \nu_{A_1\cup A_2}(v) \ \ (\because u \in V \text{ and } v \in V') = \nu_{A_1+A_2}(u) \vee \nu_{A_1+A_2}(v)$

Case (iii) : - Similarly, *if* $u \in V'$ *and* $v \in V$ *and* $uv \in E'$,

$(\mu_{B_{1i}+B_{2i}})(uv) = \mu_{A_1}(u) \wedge \mu_{A_2}(v) = \mu_{A_1\cup A_2}(u) \wedge \mu_{A_1\cup A_2}(v) \ \ (\because u \in V' \text{ and } v \in V)$

$= \mu_{A_1+A_2}(u) \wedge \mu_{A_1+A_2}(v)$

and $(\nu_{B_{1i}+B_{2i}})(uv) = \nu_{A_1}(u) \vee \nu_{A_2}(v) = \nu_{A_1\cup A_2}(u) \vee \nu_{A_1\cup A_2}(v) \ \ (\because u \in V' \text{ and } v \in V)$

$= \nu_{A_1+A_2}(u) \vee \nu_{A_1+A_2}(v)$

These results hold for i= 1,2,3,….,k .

$\therefore$ $(A_1+A_2, B_{11}+B_{21}, B_{12}+B_{22},..., B_{1k}+B_{2k})$ is an IFGS of G.

I. Theorem (3.9)

Let G = $G_1 + G_2$ be the join of two graph structures G_1 and G_2 and $\breve{G}_1$ = (A_1, B_{11}, B_{12},...,B_{1k}) and $\breve{G}_2$ = (A_2, B_{21}, B_{22},...,B_{2k}) be IFGSs of G_1 and G_2 respectively. Let $\breve{G}$ = (A, B_1, B_2,…..,B_k) be an IFGS of G with the following conditions:

$\mu_{B_i}(uv) = \mu_A(u) \wedge \mu_A(v)$ *and* $\nu_{B_i}(uv) = \nu_A(u) \vee \nu_A(v) \quad \forall\ uv \in R_i, i = 1, 2, ..., k$ then $\breve{G}$ = (A, B_1, B_2,.., B_k) is the join of two IFGSs $\breve{G}_1$ and $\breve{G}_2$ such that

$\mu_{B_{1i}}(uv) = \mu_{A_1}(u) \wedge \mu_{A_1}(v)$ *if* $uv \in R_{1i}$, $\mu_{B_{2i}}(uv) = \mu_{A_2}(u) \wedge \mu_{A_2}(v)$ *if* $uv \in R_{2i}$

and $\nu_{B_{1i}}(uv) = \nu_{A_1}(u) \vee \nu_{A_1}(v)$ *if* $uv \in R_{1i}$, $\nu_{B_{2i}}(uv) = \nu_{A_2}(u) \vee \nu_{A_2}(v)$ *if* $uv \in R_{2i}$

1) Proof: Define μ_{A_1} *and* μ_{A_2} as $\mu_{A_1}(u) = \mu_A(u)$ *if* $u \in V$, $\mu_{A_2}(u) = \mu_A(u)$ *if* $u \in V'$

and $\nu_{A_1}(u) = \nu_A(u)$ *if* $u \in V$, $\nu_{A_2}(u) = \nu_A(u)$ *if* $u \in V'$ Define

$\mu_{B_{1i}}$ and $\mu_{B_{2i}}$ *as* $\mu_{B_{1i}}(uv) = \mu_{B_i}(u)$ *if* $uv \in R_{1i}$, $\mu_{B_{2i}}(uv) = \mu_{B_i}(u)$ *if* $uv \in R_{2i}$

and $\nu_{B_{1i}}(uv) = \nu_{B_i}(u)$ *if* $uv \in R_{1i}$, $\nu_{B_{2i}}(uv) = \nu_{B_i}(u)$ *if* $uv \in R_{2i}$ for i= 1,2,3,….,k ,

For j=1,2, $\mu_{B_{ji}}(u^j v^j) = \mu_{B_i}(u^j v^j) \leq \mu_A(u^j) \wedge \mu_A(v^j) = \mu_{A_j}(u^j) \wedge \mu_{A_j}(v^j)$

and $\nu_{B_{ji}}(u^j v^j) = \nu_{B_i}(u^j v^j) \leq \nu_A(u^j) \vee \nu_A(v^j) = \nu_{A_j}(u^j) \vee \nu_{A_j}(v^j)$

$\therefore$ $\breve{G}_1$ = (A_1, B_{11},B_{12},...,B_{1k}) and $\breve{G}_2$ = (A_2, B_{21},B_{22},...,B_{2k}) are IFGSs of G_1 and G_2 .

Case (i) : - If uv $\in R_{1i} \cup R_{2i}$,

$\mu_A(u) = \mu_{A_1\cup A_2}(u) = \mu_{A_1+A_2}(u)$ and $\nu_A(u) = \nu_{A_1\cup A_2}(u) = \nu_{A_1+A_2}(u)$

$\mu_{B_i}(uv) = (\mu_{B_{1i}\cup B_{2i}})(uv) = (\mu_{B_{1i}+B_{2i}})(uv)$ *and* $\nu_{B_i}(uv) = (\nu_{B_{1i}\cup B_{2i}})(uv) = (\nu_{B_{1i}+B_{2i}})(uv)$ by definition.

Case (ii) : - *If* $u \in V$ *and* $v \in V'$ *and* $uv \in E'$,

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$(\mu_{B_{1i}+B_{2i}})(uv) = \mu_{A_1}(u) \wedge \mu_{A_2}(v) = \mu_A(u) \wedge \mu_A(v) = \mu_{B_i}(uv)$ *by assumption*

and $(\nu_{B_{1i}+B_{2i}})(uv) = \nu_{A_1}(u) \vee \nu_{A_2}(v) = \nu_A(u) \vee \nu_A(v) = \nu_{B_i}(uv)$ by assumption

Case (iii) : - Similarly *if* $u \in V'$ and $v \in V$ and $uv \in E'$,

$(\mu_{B_{1i}+B_{2i}})(uv) = \mu_{A_1}(u) \wedge \mu_{A_2}(v) = \mu_A(u) \wedge \mu_A(v) = \mu_{B_i}(uv)$ *by assumption*

and $(\nu_{B_{1i}+B_{2i}})(uv) = \nu_{A_1}(u) \vee \nu_{A_2}(v) = \nu_A(u) \vee \nu_A(v) = \nu_{B_i}(uv)$ by assumption

These results are true for i= 1,2,3,….,k .

$\Rightarrow$ (A, $B_1, B_2, \ldots, B_k$) is a join of an IFGSs of G_1 and G_2 .

J. *Definition (3.10)*

Let $\breve{G}_1 = (A_1, B_{11}, B_{12}, \ldots, B_{1k})$ and $\breve{G}_2 = (A_2, B_{21}, B_{22}, \ldots, B_{2k})$ be two IFGSs of graph structures $G_1 = (V, R_{11}, R_{12}, \ldots, R_{1k})$ and $G_2 = (V', R_{21}, R_{22}, \ldots, R_{2k})$ respectively then the cartesian product $\breve{G}_1 \times \breve{G}_2$ of $\breve{G}_1$ and $\breve{G}_2$ is denoted by

$(A_1 \times A_2, B_{11} \times B_{21}, B_{12} \times B_{22} \ldots, B_{1k} \times B_{2k})$, where $A_1 \times A_2$ and $B_{1i} \times B_{2i}$ are defined by

$\mu_{A_1 \times A_2}(u^1, u^2) = \mu_{A_1}(u^1) \wedge \mu_{A_2}(u^2)$ *and* $\nu_{A_1 \times A_2}(u^1, u^2) = \nu_{A_1}(u^1) \vee \nu_{A_2}(u^2)$

$\mu_{B_{1i} \times B_{2i}}(uu^2, uv^2) = \mu_{A_1}(u) \wedge \mu_{B_{2i}}(u^2v^2)$ *and* $\nu_{B_{1i} \times B_{2i}}(uu^2, uv^2) = \nu_{A_1}(u) \vee \nu_{B_{2i}}(u^2v^2)$, $\forall u \in V$ *and* $u^2v^2 \in B_{2i}$

$(\mu_{B_{1i} \times B_{2i}})(u^1w, v^1w) = \mu_{A_2}(w) \wedge \mu_{B_{2i}}(u^1v^1)$ *and* $(\nu_{B_{1i} \times B_{2i}})(u^1w, v^1w) = \nu_{A_2}(w) \vee \nu_{B_{2i}}(u^1v^1)$, $\forall w \in V'$ *and* $u^1v^1 \in B_{1i}$

$(\mu_{B_{1i} \times B_{2i}})(u^1w, v^1w) = \mu_{A_2}(w) \wedge \mu_{B_{2i}}(u^1v^1)$ *and* $(\nu_{B_{1i} \times B_{2i}})(u^1w, v^1w) = \nu_{A_2}(w) \vee \nu_{B_{2i}}(u^1v^1)$, $\forall w \in V'$ *and* $u^1v^1 \in R_{1i}$.

K. *Example (3.11)*

Consider $\breve{G}_1 = (A_1, B_{11}, B_{12})$ and $\breve{G}_2 = (A_2, B_{21}, B_{22})$ be two IFGSs as shown in Fig1 then the cartesian product $\breve{G}_1 \times \breve{G}_2$ of $\breve{G}_1$ and $\breve{G}_2$ is as shown in Fig 4 below.

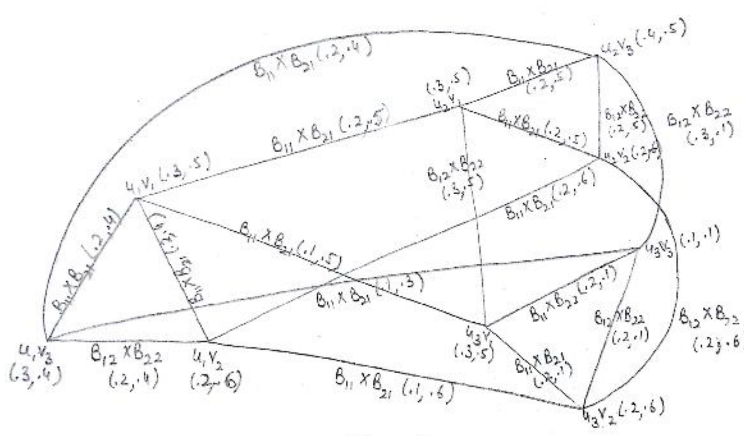

Fig4: $\breve{G}_1 \times \breve{G}_2$

L. *Theorem (3.12)*

Let G be cartesian product of two graph structures $G_1 = (V, R_{11}, R_{12}, \ldots, R_{1k})$ and $G_2 = (V', R_{21}, R_{22}, \ldots, R_{2k})$ and $\breve{G}_1 = (A_1, B_{11}, B_{12}, \ldots, B_{1k})$ and $\breve{G}_2 = (A_2, B_{21}, B_{22}, \ldots, B_{2k})$ be two IFGS of graph structures G_1 and G_2 respectively then $(A_1 \times A_2, B_{11} \times B_{21}, B_{12} \times B_{22} \ldots, B_{1k} \times B_{2k})$ is an IFGS of G.

1) *Proof:* Case (i) : - $\forall\ w \in V'$ *and* $u^1v^1 \in B_{1i}$

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$$(\mu_{B_{1i}\times B_{2i}})\left(u^1w, v^1w\right) = \mu_{A_2}(w)\wedge\mu_{B_{1i}}(u^1v^1) \le \mu_{A_2}(w)\wedge[\mu_{A_1}(u^1)\wedge\mu_{A_1}(v^1)]$$
$$= [\mu_{A_1}(u^1)\wedge \mu_{A_2}(w)]\wedge[\mu_{A_2}(w)\wedge\mu_{A_1}(v^1)] = [\mu_{A_1\times A_2}\left(u^1w\right)]\wedge[\mu_{A_1\times A_2}\left(v^1w\right)]$$
$$(\nu_{B_{1i}\times B_{2i}})\left(u^1w, v^1w\right) = \nu_{A_2}(w)\vee\nu_{B_{1i}}(u^1v^1) \le \nu_{A_2}(w)\vee[\nu_{A_1}(u^1)\vee\nu_{A_1}(v^1)]$$
$$= [\nu_{A_1}(u^1)\vee \nu_{A_2}(w)]\vee[\nu_{A_2}(w)\vee\nu_{A_1}(v^1)]\ [\nu_{A_1\times A_2}\left(u^1w\right)]\vee[\nu_{A_1\times A_2}\left(v^1w\right)]$$

Case (ii) : $\forall\, u\in V$ and $u^2v^2\in B_{2i}$,

$$(\mu_{B_{1i}\times B_{2i}})\left(uu^2, uv^2\right) = \mu_{A_1}(u)\wedge\mu_{B_{2i}}(u^2v^2) \le \mu_{A_1}(u)\wedge[\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)]$$
$$= [\mu_{A_1}(u)\wedge \mu_{A_2}(u^2)]\wedge[\mu_{A_2}(v^2)\wedge\mu_{A_1}(u)] = [\mu_{A_1\times A_2}\left(uu^2\right)]\wedge[\mu_{A_1\times A_2}\left(uv^2\right)]$$
$$(\nu_{B_{1i}\times B_{2i}})\left(uu^2, uv^2\right) = \nu_{A_1}(u)\vee\nu_{B_{2i}}(u^2v^2) \le \nu_{A_1}(u)\vee[\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)]$$
$$= [\nu_{A_1}(u)\vee \nu_{A_2}(u^2)]\vee[\nu_{A_2}(v^2)\vee\nu_{A_1}(u)] = [\nu_{A_1\times A_2}\left(uu^2\right)]\vee[\nu_{A_1\times A_2}\left(uv^2\right)]$$

These results hold for i= 1,2,3,….,k .

Therefore, $(A_1\times A_2,\ B_{11}\times B_{21},\ B_{12}\times B_{22}...,B_{1k}\times B_{2k})$ is an IFGS of G.

M. Definition (3.13)

Let $\tilde{G}_1 = (A_1, B_{11},B_{12},..,B_{1k})$ and $\tilde{G}_2 = (A_2, B_{21},B_{22},...,B_{2k})$ be two IFGSs of graph structures G_1 and G_2 respectively then the composition $\tilde{G}_1\circ\tilde{G}_2$ of G_1 and G_2 is given by $(A_1\circ A_2,\ B_{11}\circ B_{21},\ B_{12}\circ B_{22},...,B_{1k}\circ B_{2k})$ where $A_1\circ A_2$ and $B_{1i}\circ B_{2i}$ are given by

$$(\mu_{A_1\circ A_2})\left(u^1u^2\right) = \mu_{A_1}(u^1)\wedge\mu_{A_2}(u^2)\ \ and\ \ (\nu_{A_1\circ A_2})\left(u^1u^2\right) = \nu_{A_1}(u^1)\vee\nu_{A_2}(u^2),\ \forall\,(u^1,u^2)\in V\times V'\ and\ \text{for } i=1,2,...,k$$

and

$$(\mu_{B_{1i}\circ B_{2i}})\left(uu^2, uv^2\right) = \mu_{A_1}(u)\wedge\mu_{B_{2i}}(u^2v^2)\ and\ (\nu_{B_{1i}\circ B_{2i}})\left(uu^2, uv^2\right) = \nu_{A_1}(u)\vee\nu_{B_{2i}}(u^2v^2)\ \forall\,u\in V,\ u^2v^2\in B_{2i}$$
$$(\mu_{B_{1i}\circ B_{2i}})\left(u^1w, v^1w\right) = \mu_{A_2}(w)\wedge\mu_{B_{1i}}(u^1v^1)\ and\ (\nu_{B_{1i}\circ B_{2i}})\left(u^1w, v^1w\right) = \nu_{A_2}(w)\vee\nu_{B_{1i}}(u^1v^1),\ \forall\,w\in V',\ u^1v^1\in B_{1i}$$
$$(\mu_{B_{1i}\circ B_{2i}})\left(u^1u^2, v^1v^2\right) = \mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)\wedge\mu_{B_{1i}}(u^1v^1)\ and\ (\nu_{B_{1i}\circ B_{2i}})\left(u^1u^2, v^1v^2\right) = \nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)\vee\nu_{B_{1i}}(u^1v^1)$$
$$\forall\ (u^1u^2, v^1v^2)\in B_{1i}\circ B_{2i} - [\{(uu^2, uv^2):\ u\in V\ and\ u^2v^2\in B_{2i}\}\cup\{(u^1w, v^1w):\ w\in V'\ and\ u^1v^1\in B_{1i}\}]$$

N. Example (3.14)

Consider $\tilde{G}_1$ and $\tilde{G}_2$ be two IFGSs as shown in Fig1 then the composition $\tilde{G}_1\circ\tilde{G}_2$ of $\tilde{G}_1$ and $\tilde{G}_2$ is as shown in fig below.

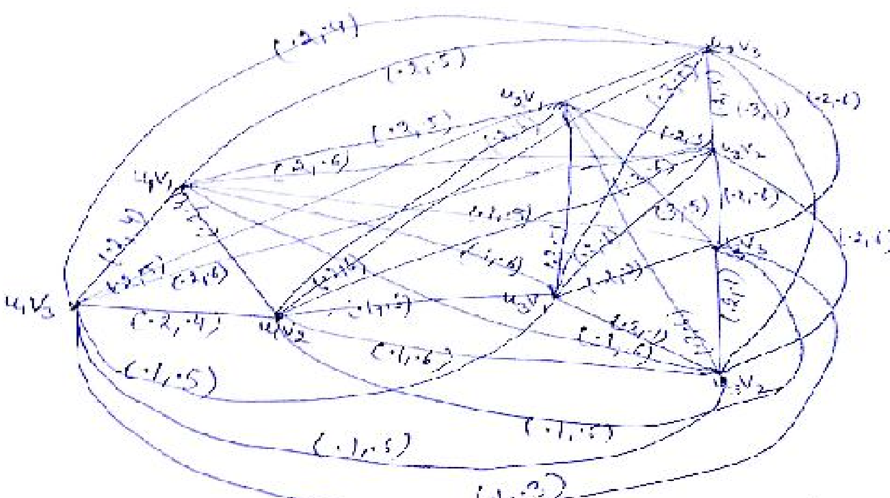

Fig . $\tilde{G}_1\circ\tilde{G}_2$ - Composition of $\tilde{G}_1$ & $\tilde{G}_2$

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O. Theorem (3.15)

Let G be composition of two graph structures G_1 and G_2 and $\tilde{G}_1 = (A_1, B_{11}, B_{12},...,B_{1k})$ and $\tilde{G}_2 = (A_2, B_{21},B_{22},...,B_{2k})$ be IFGSs of graph structures G_1 and G_2 respectively then $(A_1 \circ A_2, B_{11} \circ B_{21}, B_{12} \circ B_{22}..., B_{1k} \circ B_{2k})$ is an IFGS of G.

1) *Proof:* Case (i): $\forall\ w \in V'$ and $u^1v^1 \in B_{1i}$

$$(\mu_{B_{1i}\circ B_{2i}})(u^1w, v^1w) = \mu_{A_2}(w) \wedge \mu_{B_{1i}}(u^1v^1) \le \mu_{A_2}(w) \wedge [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1)]$$

$$= [\mu_{A_1}(u^1) \wedge \mu_{A_2}(w)] \wedge [\mu_{A_2}(w) \wedge \mu_{A_1}(v^1)] = [\mu_{A_1\circ A_2}(u^1w)] \wedge [\mu_{A_1\circ A_2}(v^1w)]$$

$$(\nu_{B_{1i}\circ B_{2i}})(u^1w, v^1w) = \nu_{A_2}(w) \vee \nu_{B_{1i}}(u^1v^1) \le \nu_{A_2}(w) \vee [\nu_{A_1}(u^1) \vee \nu_{A_1}(v^1)]$$

$$= [\nu_{A_1}(u^1) \vee \nu_{A_2}(w)] \vee [\nu_{A_2}(w) \vee \nu_{A_1}(v^1)] = [\nu_{A_1\circ A_2}(u^1w)] \vee [\nu_{A_1\circ A_2}(v^1w)]$$

Case (ii) : $\forall\ u \in V_1$ and $u^2v^2 \in B_{2i}$,

$$(\mu_{B_{1i}\circ B_{2i}})(uu^2, uv^2) = \mu_{A_1}(u) \wedge \mu_{B_{2i}}(u^2v^2) \le \mu_{A_1}(u) \wedge [\mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2)]$$

$$= [\mu_{A_1}(u) \wedge \mu_{A_2}(u^2)] \wedge [\mu_{A_2}(v^2) \wedge \mu_{A_1}(u)] = [(\mu_{A_1\circ A_2})(uu^2)] \wedge [(\mu_{A_1\circ A_2})(uv^2)]$$

$$(\nu_{B_{1i}\circ B_{2i}})(uu^2, uv^2) = \nu_{A_1}(u) \vee \nu_{B_{2i}}(u^2v^2) \le \nu_{A_1}(u) \vee [\nu_{A_2}(u^2) \vee \nu_{A_2}(v^2)]$$

$$= [\nu_{A_1}(u) \vee \nu_{A_2}(u^2)] \vee [\nu_{A_2}(v^2) \vee \nu_{A_1}(u)] = [(\nu_{A_1\circ A_2})(uu^2)] \vee [(\nu_{A_1\circ A_2})(uv^2)]$$

Case (iii) : $\forall\ u^1u^2, v^1v^2 \in B_{1i} \circ B_{2i} - [\{(uu^2, uv^2) : u \in V \text{ and } u^2v^2 \in B_{2i}\} \cup \{(u^1w, v^1w) : w \in V' \text{and } u^1v^1 \in B_{1i}\}]$

$$(\mu_{B_{1i}\circ B_{2i}})(u^1u^2, v^1v^2) = \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \mu_{B_{1i}}(u^1v^1) \le \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1)]$$

$$= [\mu_{A_1}(u^1) \wedge \mu_{A_2}(u^2)] \wedge [\mu_{A_1}(v^1) \wedge \mu_{A_1}(v^2)] = [\mu_{A_1\circ A_2}(u^1u^2)] \wedge [\mu_{A_1\circ A_2}(v^1v^2)]$$

$$(\nu_{B_{1i}\circ B_{2i}})(u^1u^2, v^1v^2) = \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \nu_{B_{1i}}(u^1v^1) \le \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee [\nu_{A_1}(u^1) \vee \nu_{A_1}(v^1)]$$

$$= [\nu_{A_1}(u^1) \vee \nu_{A_2}(u^2)] \vee [\nu_{A_1}(v^1) \wedge \nu_{A_1}(v^2)] = [\nu_{A_1\circ A_2}(u^1u^2)] \vee [\nu_{A_1\circ A_2}(v^1v^2)]$$

These results hold for i = 1, 2, 3,….,k.

Therefore, $(A_1 \circ A_2, B_{11} \circ B_{21}, B_{12} \circ B_{22}..., B_{1k} \circ B_{2k})$ is an IFGS of G.

IV. ϕ - COMPLEMENT OF OPERATIONS ON IFGSS

In this section, we obtained a relationship between phi-complements of union of two IFGSs with the sum of the phi-complements of the IFGSs.

Definition (4.1)[7] Let $\tilde{G} = (A,B_1,B_2,...,B_k)$ be an intuitionistic fuzzy graph structure of graph structure $G=(V,R_1,R_2,...,R_k)$. Let ϕ denotes the permutation on the set $\{R_1,R_2,....,R_k\}$ and also the corresponding permutation on $\{B_1, B_2,...,B_k\}$ i.e., $\phi(B_i) = B_i^{\phi} = B_j$ (i.e., $\phi\mu_{B_i} = \mu_{B_j}$ and $\phi\nu_{B_i} = \nu_{B_j}$) if and only if $\phi(R_i) = R_j$, then the ϕ - complement of $\tilde{G}$ is denoted $\tilde{G}^{\phi}$ and is given by $\tilde{G}^{\phi} = (A, B_1^{\phi}, B_2^{\phi},...,B_k^{\phi})$ where for each i = 1, 2, 3,…, k, we have

$$\mu_{B_i}{}^{\phi}(uv) = \mu_A(u) \wedge \mu_A(v) - \sum_{j \ne i}(\phi\mu_{B_j})(uv) \text{ and } \nu_{B_i}{}^{\phi}(uv) = \nu_A(u) \vee \nu_A(v) - \sum_{j \ne i}(\phi\nu_{B_j})(uv).$$

Definition (4.2): Let $G_1 = (V, R_{11}, R_{12},...,R_{1k})$ and $G_2 = (V', R_{21}, R_{22},...,R_{2k})$ be two graph structures such that V and V′ are disjoint. Let $\tilde{G}_1 = (A_1,B_{11},B_{12},...,B_{1k})$ and $\tilde{G}_2 = (A_2, B_{21}, B_{22},..., B_{2k})$ be IFGSs corresponding to the graph structures G_1 and G_2 respectively and ϕ_1 and ϕ_2 be permutations on the sets$\{B_{11},B_{12},...,B_{1k}\}$ and $\{B_{21},B_{22},...,B_{2k}\}$ respectively. Define ϕ on $(B_{11}+B_{21}, B_{12}+B_{22},...., B_{1k}+B_{2k})$ as follows

$$\phi(\mu_{B_{1i}+B_{2i}})(uv)=\begin{cases}\phi_1(\mu_{B_{1i}})(uv) & if\ \ uv\in B_{1i}\\ \phi_2(\mu_{B_{2i}})(uv) & if\ \ uv\in B_{2i}\\ \mu_A(u)\wedge\mu_A(v) & if\ \ u\in V, v\in V'\end{cases};\ \phi(\nu_{B_{1i}+B_{2i}})(uv)=\begin{cases}\phi_1(\nu_{B_{1i}})(uv) & if\ \ uv\in B_{1i}\\ \phi_2(\nu_{B_{2i}})(uv) & if\ \ uv\in B_{2i}\\ \nu_A(u)\vee\nu_A(v) & if\ \ u\in V, v\in V'\end{cases}.$$

Then clearly, ϕ is a permutation on the set $(B_{11}+B_{21}, B_{12}+B_{22},....,B_{1k}+B_{2k})$.

Theorem (4.3): Prove that ϕ - complement of join of G_1 and G_2 is the union of $G_1^{\phi_1}$ and $G_2^{\phi_2}$ i.e., $\left(G_1+G_2\right)^{\phi}=G_1^{\phi_1}\cup G_2^{\phi_2}$.

Proof: For i= 1, 2,3,….,k ,

$$(\mu_{B_{1i}+B_{2i}})^{\phi}(uv)=[\mu_{A_1+A_2}(u)\wedge\mu_{A_1+A_2}(v)]-\sum_{j\neq i}\phi(\mu_{B_{1j}+B_{2j}})(uv)$$

$$i.e.(\mu_{B_{1i}+B_{2i}})^{\phi}(uv)=[\mu_{A_1\cup A_2}(u)\wedge\mu_{A_1\cup A_2}(v)]-\sum_{j\neq i}\phi(\mu_{B_{1j}+B_{2j}})(uv)$$

$$if\ \ uv\in B_{1i}\cup B_{2i}\ ,\ i=1,2,3,\ldots,k$$

$$=[\mu_{A_1\cup A_2}(u)\wedge\mu_{A_1\cup A_2}(v)]-\sum_{j\neq i}[\mu_{A_1}(u)\wedge\mu_{A_2}(v)]$$

$$if\ \ uv\ joins\ a\ vertex\ u\ of\ V_1\ with\ a\ vertex\ v\ of\ V_2$$

$$=[\mu_{A_1\cup A_2}(u)\wedge\mu_{A_1\cup A_2}(v)]-\sum_{j\neq i}[\mu_{A_2}(u)\wedge\mu_{A_1}(v)]$$

$$if\ \ uv\ joins\ a\ vertex\ u\ of\ V_2\ with\ a\ vertex\ v\ of\ V_1$$

$$(\nu_{B_{1i}+B_{2i}})^{\phi}(uv)=[\nu_{A_1+A_2}(u)\vee\nu_{A_1+A_2}(v)]-\sum_{j\neq i}\phi(\nu_{B_{1j}+B_{2j}})(uv)$$

$$i.e.(\nu_{B_{1i}+B_{2i}})^{\phi}(uv)=[\nu_{A_1\cap A_2}(u)\vee\nu_{A_1\cap A_2}(v)]-\sum_{j\neq i}\phi(\nu_{B_{1j}+B_{2j}})(uv)$$

$$if\ \ uv\in B_{1i}\cup B_{2i}\ ,\ i=1,2,3,\ldots,k$$

$$=[\nu_{A_1\cap A_2}(u)\vee\nu_{A_1\cap A_2}(v)]-\sum_{j\neq i}[\nu_{A_1}(u)\vee\nu_{A_2}(v)]$$

$$\text{if } uv \text{ joins a vertex } u \text{ of } V_1 \text{ with a vertex } v \text{ of } V_2$$

$$=[\nu_{A_1\cap A_2}(u)\vee\nu_{A_1\cap A_2}(v)]-\sum_{j\neq i}[\nu_{A_2}(u)\vee\nu_{A_1}(v)]$$

$$\text{if } uv \text{ joins a vertex } u \text{ of } V_2 \text{ with a vertex } v \text{ of } V_1$$

Let i be fixed, three cases arises,

Case I: If $uv \in$ supp $(\phi_1 B_{1r})$ or $uv \in$ supp $(\phi_2 B_{2r})$, r ≠ i

If $uv \in$ supp $(\phi_1 B_{1r})$,

$$(\mu_{B_{1i}+B_{2i}})^{\phi}(uv)=[\mu_{A_1}(u)\wedge\mu_{A_1}(v)]-\phi(\mu_{B_{1r}})(uv)=[\mu_{A_1}(u)\wedge\mu_{A_1}(v)]-\phi_1(\mu_{B_{1r}})(uv)$$

$$(\mu_{B_{1i}})^{\phi_1}(uv)=[\mu_{A_1}(u)\wedge\mu_{A_1}(v)]-\sum_{j\neq i}\phi(\mu_{B_{1j}})(uv)=[\mu_{A_1}(u)\wedge\mu_{A_1}(v)]-\phi_1(\mu_{B_{1r}})(uv)$$

$$(\because uv \in \text{supp } (\phi_1 B_{1r}))$$

$$\therefore(\mu_{B_{1i}+B_{2i}})^{\phi}(uv)=\mu_{B_{1i}}{}^{\phi_1}(uv),$$

$$(\nu_{B_{1i}+B_{2i}})^{\phi}(uv)=[\nu_{A_1}(u)\vee\nu_{A_1}(v)]-\phi(\nu_{B_{1r}})(uv)=[\nu_{A_1}(u)\vee\nu_{A_1}(v)]-\phi_1(\mu_{B_{1r}})(uv)$$

$$(\nu_{B_{1i}})^{\phi}(uv)=[\nu_{A_1}(u)\vee\nu_{A_1}(v)]-\sum_{j\neq i}\phi(\nu_{B_{1j}})(uv)=[\nu_{A_1}(u)\vee\nu_{A_1}(v)]-\phi_1(\nu_{B_{1r}})(uv)$$

$$(\because uv \in \text{supp } (\phi_1 B_{1r}))$$

$$\therefore(\nu_{B_{1i}+B_{2i}})^{\phi}(uv)=\nu_{B_{1i}}{}^{\phi_1}(uv)$$

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Similarly if $uv \in$ supp $(\phi_2 B_{2r})$,

$(\mu_{B_{1i}+B_{2i}})^{\phi}(uv) = \mu_{B_{2i}}{}^{\phi_2}(uv)$ *and* $(\nu_{B_{1i}+B_{2i}})^{\phi}(uv) = \nu_{B_{2i}}{}^{\phi_2}(uv)$

Case II: If $uv \in$ supp $(\phi_1 B_{1i})$ or $uv \in$ supp $(\phi_2 B_{2i})$

If $uv \in$ supp $(\phi_1 B_{1i})$,

$(\mu_{B_{1i}+B_{2i}})^{\phi}(uv) = \mu_{A_1}(u) \wedge \mu_{A_1}(v) = \mu_{B_{1i}}{}^{\phi_1}(uv)$ and $(\nu_{B_{1i}+B_{2i}})^{\phi}(uv) = \nu_{A_1}(u) \vee \nu_{A_1}(v) = \nu_{B_{1i}}{}^{\phi_1}(uv)$

If $uv \in$ supp $(\phi_2 B_{2i})$,

$(\mu_{B_{1i}+B_{2i}})^{\phi}(uv) = \mu_{A_2}(u) \wedge \mu_{A_2}(v) = \mu_{B_{2i}}{}^{\phi_2}(uv)$ and $(\nu_{B_{1i}+B_{2i}})^{\phi}(uv) = \nu_{A_2}(u) \vee \nu_{A_2}(v) = \nu_{B_{2i}}{}^{\phi_2}(uv)$

Case III: If uv joins a vertex of V_1 with a vertex of V_2 or if uv joins a vertex of V_2 with a vertex of V_1

If uv joins a vertex of V_1 with a vertex of V_2, then

$(\mu_{B_{1i}+B_{2i}})^{\phi}(uv) = \mu_{A_1}(u) \wedge \mu_{A_2}(v) - [\mu_{A_1}(u) \wedge \mu_{A_2}(v)] = 0$ and

$(\nu_{B_{1i}+B_{2i}})^{\phi}(uv) = \nu_{A_1}(u) \vee \nu_{A_2}(v) - [\nu_{A_1}(u) \vee \nu_{A_2}(v)] = 0$

Similarly when uv joins a vertex of V_2 with a vertex of V_1, we get the results same as above.

Combining all the cases,

$(\mu_{B_{1i}+B_{2i}})^{\phi}(uv) = (\mu_{B_{1i}}{}^{\phi_1} \cup \mu_{B_{2i}}{}^{\phi_2})(uv)$ and $(\nu_{B_{1i}+B_{2i}})^{\phi}(uv) = (\nu_{B_{1i}}{}^{\phi_1} \cup \nu_{B_{2i}}{}^{\phi_2})(uv)$

$\therefore \left(G_1 + G_2\right)^{\phi} = G_1{}^{\phi_1} \cup G_2{}^{\phi_2}$.

Theorem (4.4): Let G_1 = (A_1, B_{11}, B_{12},..., B_{1k}) and G_2 = (A_2, B_{21}, B_{22},..., B_{2k}) be two IFGS of G_1 = (V, R_{11}, R_{12},..., R_{1k}) and G_2 = (V′, R_{21}, R_{22},..., R_{2k}) respectively such that V and V′ are disjoint and ϕ is a permutation on the set $(B_{11}+B_{21}, B_{12}+B_{22}, \ldots, B_{1k}+B_{2k})$, then prove that ϕ- complement of union of G_1 and G_2 is the join of $G_1{}^{\phi_1}$ and $G_2{}^{\phi_2}$ i.e. $\left(G_1 \cup G_2\right)^{\phi} = G_1{}^{\phi_1} + G_2{}^{\phi_2}$.

Proof : $(\mu_{B_{1i} \cup B_{2i}})^{\phi}(uv) = [\mu_{A_1 \cup A_2}(u) \wedge \mu_{A_1 \cup A_2}(v)] - \sum_{j \neq i} \phi(\mu_{B_{1j} \cup B_{2j}})(uv)$ for i = 1, 2, 3, …, k.

Fix an i, three cases arises,

Case I: If $uv \in$ supp $(\phi_1 B_{1r})$ or $uv \in$ supp $(\phi_2 B_{2r})$, r ≠ i

If $uv \in$ supp $(\phi_1 B_{1r})$,

$(\mu_{B_{1i} \cup B_{2i}})^{\phi}(uv) = [\mu_{A_1}(u) \wedge \mu_{A_1}(v)] - \phi(\mu_{B_{1r}})(uv) = [\mu_{A_1}(u) \wedge \mu_{A_1}(v)] - \phi_1(\mu_{B_{1r}})(uv)$

$(\mu_{B_{1i}})^{\phi_1}(uv) = [\mu_{A_1}(u) \wedge \mu_{A_1}(v)] - \sum_{j \neq i} \phi(\mu_{B_{1j}})(uv) = [\mu_{A_1}(u) \wedge \mu_{A_1}(v)] - \phi_1(\mu_{B_{1r}})(uv)$

$(\because uv \in \text{supp}(\phi_1 B_{1r}))$

$\therefore (\mu_{B_{1i} \cup B_{2i}})^{\phi}(uv) = \mu_{B_{1i}}{}^{\phi_1}(uv)$

$(\nu_{B_{1i} \cup B_{2i}})^{\phi}(uv) = [\nu_{A_1}(u) \vee \nu_{A_1}(v)] - \phi(\nu_{B_{1r}})(uv) = [\nu_{A_1}(u) \vee \nu_{A_1}(v)] - \phi_1(\mu_{B_{1r}})(uv)$

$(\nu_{B_{1i}})^{\phi_1}(uv) = [\nu_{A_1}(u) \vee \nu_{A_1}(v)] - \sum_{j \neq i} \phi(\nu_{B_{1j}})(uv) = [\nu_{A_1}(u) \vee \nu_{A_1}(v)] - \phi_1(\nu_{B_{1r}})(uv)$

$(\because uv \in \text{supp}(\phi_1 B_{1r}))$

$\therefore (\nu_{B_{1i} \cup B_{2i}})^{\phi}(uv) = \nu_{B_{1i}}{}^{\phi_1}(uv)$

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Similarly if $uv \in$ supp $(\phi_2 B_{2r})$,

$(\mu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = \mu_{B_{2i}}{}^{\phi_2}(uv)$ *and* $(\nu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = \nu_{B_{2i}}{}^{\phi_2}(uv)$

Case II: If $uv \in$ supp $(\phi_1 B_{1i})$ or $uv \in$ supp $(\phi_2 B_{2i})$

If $uv \in$ supp $(\phi_1 B_{1i})$,

$(\mu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = \mu_{A_1}(u) \wedge \mu_{A_1}(v) = \mu_{B_{1i}}{}^{\phi_1}(uv)$ and $(\nu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = \nu_{A_1}(u) \vee \nu_{A_1}(v) = \nu_{B_{1i}}{}^{\phi_1}(uv)$

If $uv \in$ supp $(\phi_2 B_{2i})$,

$(\mu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = \mu_{A_2}(u) \wedge \mu_{A_2}(v) = \mu_{B_{2i}}{}^{\phi_2}(uv)$ and $(\nu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = \nu_{A_2}(u) \vee \nu_{A_2}(v) = \nu_{B_{2i}}{}^{\phi_2}(uv)$

Case III: If uv joins a vertex of V_1 with a vertex of V_2 or if uv joins a vertex of V_2 with a vertex of V_1

When uv joins a vertex of V_1 with a vertex of V_2 ,

$(\mu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = \mu_{A_1}(u) \wedge \mu_{A_2}(v) - [\mu_{A_1}(u) \wedge \mu_{A_2}(v)] = 0$ and

$(\nu_{B_{1i}+B_{2i}})^{\phi}(uv) = \nu_{A_1}(u) \vee \nu_{A_2}(v) - [\nu_{A_1}(u) \vee \nu_{A_2}(v)] = 0.$

Similarly when uv joins a vertex of V_2 with a vertex of V_1, we get the results same as above.

Combining all the cases, we get

$(\mu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = (\mu_{B_{1i}}{}^{\phi_1} + \mu_{B_{2i}}{}^{\phi_2})(uv)$ and $(\nu_{B_{1i}\cup B_{2i}})^{\phi}(uv) = (\nu_{B_{1i}}{}^{\phi_1} + \nu_{B_{2i}}{}^{\phi_2})(uv)$

$\therefore \left(\tilde{G}_1 \cup \tilde{G}_2\right)^{\phi} = \tilde{G}_1{}^{\phi_1} + \tilde{G}_2{}^{\phi_2}$.

Result (4.5): Let ϕ_1 and ϕ_2 be permutations on the sets $\{B_{11}, B_{12}, \ldots, B_{1k}\}$ and $\{B_{21}, B_{22}, \ldots, B_{2k}\}$ respectively. Define ϕ on $\{B_{11}+B_{21}, B_{12}+B_{22}, \ldots, B_{1k}+B_{2k}\}$ as follows

$\phi(B_{1i}+B_{2i}) = \phi_1(B_{1i}) + \phi_2(B_{2i})$ then ϕ is a permutation on the set $\{B_{11}+B_{21}, B_{12}+B_{22}, \ldots, B_{1k}+B_{2k}\}$.

Definition (4.6): $\tilde{G} = (A, B_1, B_2, \ldots, B_k)$ of $G = (V, R_1, R_2, \ldots, R_k)$ is a B_i- strong IFGS if

$\mu_{B_i}(u,v) = \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) = \nu_A(u) \vee \nu_A(v)$ $\forall uv \in B_i$ and $i = 1, 2, \ldots, k$.

Theorem (4.7): Let $\tilde{G}_1 = (A_1, B_{11}, B_{12}, \ldots, B_{1k})$ be a B_{1i}- strong IFGS and $\tilde{G}_2 = (A_2, B_{21}, B_{22}, \ldots, B_{2k})$ is a B_{2i}- strong IFGS then $\tilde{G}_1 \circ \tilde{G}_2$ is a B_i- strong IFGS where $B_i = B_{1i} \circ B_{2i}$.

Proof : *Let* $A = A_1 \circ A_2$ and $B_i = B_{1i} \circ B_{2i}$.

$\therefore \mu_A(u) = \mu_{A_1 \circ A_2}(u)$ and $\nu_A(u) = \nu_{A_1 \circ A_2}(u)$, $\mu_{B_i}(uv) = \mu_{B_{1i}\circ B_{2i}}(uv)$ and $\nu_{B_i}(uv) = \nu_{B_{1i}\circ B_{2i}}(uv)$.

Case (i): $\forall w \in V'$ *and* $u^1v^1 \in B_{1i}$

$$\mu_{B_i}(u^1w, v^1w) = \mu_{A_2}(w) \wedge \mu_{B_{1i}}(u^1v^1) = \mu_{A_2}(w) \wedge [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1)] \quad (\because \tilde{G}_1 \text{ be a } B_{1i} - \text{strong IFGS})$$

$$= [\mu_{A_2}(w) \wedge \mu_{A_1}(u^1)] \wedge [\mu_{A_2}(w) \wedge \mu_{A_1}(v^1)] = \mu_{A_1\circ A_2}(u^1w)] \wedge [\mu_{A_1\circ A_2}(v^1w)$$

$$= \mu_A(u^1w)] \wedge [\mu_A(v^1w)$$

$$\nu_{B_i}(u^1w, v^1w) = \nu_{A_2}(w) \vee \nu_{B_{1i}}(u^1v^1) = \nu_{A_2}(w) \vee [\nu_{A_1}(u^1) \vee \nu_{A_1}(v^1)] \quad (\because \tilde{G}_1 \text{ be a } B_{1i} - \text{strong IFGS})$$

$$= [\nu_{A_2}(w) \vee \nu_{A_1}(u^1)] \vee [\nu_{A_2}(w) \vee \nu_{A_1}(v^1)] = [\nu_{A_1\circ A_2}(u^1w)] \vee [\nu_{A_1\circ A_2}(v^1w)]$$

$$= \nu_A(u^1w)] \wedge [\nu_A(v^1w)$$

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Case(ii) : $\forall u \in V_1$ and $u^2v^2 \in B_{2i}$,

$$(\mu_{B_{1i}\circ B_{2i}})(uu^2, uv^2) = \mu_{A_1}(u)\wedge\mu_{B_{2i}}(u^2v^2) = \mu_{A_1}(u)\wedge[\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)] \quad (\because G_2 \text{ be a } B_{2i} - \text{strong IFGS})$$

$$=[\mu_{A_1}(u)\wedge\mu_{A_2}(u^2)]\wedge[\mu_{A_2}(v^2)\wedge\mu_{A_1}(u)] = [(\mu_{A_1\circ A_2})(uu^2)]\wedge[(\mu_{A_1\circ A_2})(uv^2)]$$

$$=\mu_A(uu^2)\wedge\mu_A(uv^2)]$$

$$(\nu_{B_{1i}\circ B_{2i}})(uu^2, uv^2) = \nu_{A_1}(u)\vee\nu_{B_{2i}}(u^2v^2) = \nu_{A_1}(u)\vee[\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)] \quad (\because G_2 \text{ be a } B_{2i} - \text{strong IFGS})$$

$$=[\nu_{A_1}(u)\vee\nu_{A_2}(u^2)]\vee[\nu_{A_2}(v^2)\vee\nu_{A_1}(u)] = \nu_{A_1\circ A_2}(uu^2)\vee\nu_{A_1\circ A_2}(uv^2)$$

$$=\nu_A(uu^2)\wedge\nu_A(uv^2)]$$

Case(iii) : $u^1v^1 \in B_{1i}, u^2 \neq v^2$

$$\mu_{B_i}(u^1u^2, v^1v^2) = \mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)\wedge\mu_{B_{1i}}(u^1v^1) = \mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)\wedge[\mu_{A_1}(u^1)\wedge\mu_{A_1}(v^1)]$$

$$=[\mu_{A_1}(u^1)\wedge\mu_{A_2}(u^2)]\wedge[\mu_{A_1}(v^1)\wedge\mu_{A_1}(v^2)] = \mu_{A_1\circ A_2}(u^1u^2)]\wedge[\mu_{A_1\circ A_2}(v^1v^2)$$

$$\nu_{B_i}(u^1u^2, v^1v^2) = \nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)\vee\nu_{B_{1i}}(u^1v^1) = \nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)\vee[\nu_{A_1}(u^1)\vee\nu_{A_1}(v^1)]$$

$$=[\nu_{A_1}(u^1)\vee\nu_{A_2}(u^2)]\vee[\nu_{A_1}(v^1)\wedge\nu_{A_1}(v^2)] = [\nu_{A_1\circ A_2}(u^1u^2)]\vee[\nu_{A_1\circ A_2}(v^1v^2)]$$

$\therefore G_1 \circ G_2$ is B_i – strong.

The above result holds for i = 1, 2, 3,….,k.

Result (4.8): Let ϕ_1 and ϕ_2 be permutations on the sets $\{B_{11}, B_{12}, ..., B_{1k}\}$ and $\{B_{21}, B_{22}, ..., B_{2k}\}$ respectively. Define ϕ on $\{B_{11}\circ B_{21}, B_{12}\circ B_{22}, ..., B_{1k}\circ B_{2k}\}$ as follows

$\phi(B_{1i}\circ B_{2i}) = \phi_1(B_{1i})\circ\phi_2(B_{2i})$ then ϕ is a permutation on the set $\{B_{11}\circ B_{21}, B_{12}\circ B_{22}, ..., B_{1k}\circ B_{2k}\}$.

Theorem (4.9): Let $G_1 = (A_1, B_{11}, B_{12}, ..., B_{1k})$ be a B_{1i}- strong IFGS and $G_2 = (A_2, B_{21}, B_{22}, ..., B_{2k})$ is a B_{2i}- strong IFGS. Let $\mu_A = \mu_{A_1\circ A_2}$ *and* $\nu_A = \nu_{A_1\circ A_2}$ and $B_i = B_{1i}\circ B_{2i}$ for i = 1, 2, 3,….,k . Then $B_i^{\phi}(uv) = (B_{1i}^{\phi_1}\circ B_{2i}^{\phi_2})(uv) \quad \forall\ uv \in V\times V'$ where ϕ is a permutation on the set $\{B_{11}\circ B_{21}, B_{12}\circ B_{22}, ..., B_{1k}\circ B_{2k}\}$ defined as above.

Proof : *Given* $A = A_1\circ A_2$ and $B_i = B_{1i}\circ B_{2i}$.

$\therefore \mu_A(u) = \mu_{A_1\circ A_2}(u)$ and $\nu_A(u) = \nu_{A_1\circ A_2}(u)$, $\mu_{B_i}(uv) = \mu_{B_{1i}\circ B_{2i}}(uv)$ and $\nu_{B_i}(uv) = \nu_{B_{1i}\circ B_{2i}}(uv)$.

Case(i) : $\forall\ u \in V$ and $u^2v^2 \in B_{2i}$,

$$\mu_{B_i}{}^{\phi}(uu^2, uv^2) = \mu_A(uu^2)\wedge\mu_A(uv^2) - \sum_{j\neq i}(\phi\mu_{B_j})(uu^2, uv^2) = \mu_A(uu^2)\wedge\mu_A(uv^2) - [\mu_A(uu^2)\wedge\mu_A(uv^2)]$$

$$(\because \phi\mu_{B_j} = \mu_{B_i} \text{ for some } j\neq i \text{ using case ii of theorem (4.6)})$$

$$\therefore \mu_{B_i}{}^{\phi}(uu^2, uv^2) = 0$$

$$(\mu_{B_{1i}}{}^{\phi_1}\circ\mu_{B_{2i}}{}^{\phi_2})(uu^2, uv^2) = \mu_{A_1}(u)\wedge\mu_{B_{2i}}{}^{\phi_2}(u^2v^2) = \mu_{A_1}(u)\wedge[\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2) - \sum_{j\neq i}(\phi_2\mu_{B_{2j}})(u^2v^2)]$$

$$=[\mu_{A_1}(u)\wedge\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)] - [\mu_{A_1}(u)\wedge\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)]$$

$$(\because u^2v^2 \in B_{2i}, \phi_2\mu_{B_{2j}} = \mu_{B_{2i}} \text{ for some } i \text{ and } G_2 \text{ be a } B_{2i} - \text{strong})$$

$$\therefore (\mu_{B_{1i}}{}^{\phi_1}\circ\mu_{B_{2i}}{}^{\phi_2})(uu^2, uv^2) = 0$$

$$\Rightarrow \mu_{B_i}{}^{\phi}(uu^2, uv^2) = (\mu_{B_{1i}}{}^{\phi_1}\circ\mu_{B_{2i}}{}^{\phi_2})(uu^2, uv^2)$$

$$\nu_{B_i}^{\phi}\left(uu^2,uv^2\right)=\nu_A(uu^2)\vee\nu_A(uv^2)-\sum_{j\neq i}(\phi\nu_{B_j})\left(uu^2,uv^2\right)=\nu_A\left(uu^2\right)\vee\nu_A\left(uv^2\right)-[\nu_A\left(uu^2\right)\vee\nu_A\left(uv^2\right)]$$

$$(\because \phi\nu_{B_j}=\nu_{B_i}\text{ for some } j\neq i \text{ using case ii of theorem (4.6)})$$

$$\therefore \nu_{B_i}^{\phi}\left(uu^2,uv^2\right)=0$$

$$\Rightarrow \mu_{B_i}^{\phi}\left(uu^2,uv^2\right)=(\mu_{B_{1i}}^{\phi_1}\circ\mu_{B_{2i}}^{\phi_2})\left(uu^2,uv^2\right)$$

$$\nu_{B_i}^{\phi}\left(uu^2,uv^2\right)=\nu_A(uu^2)\vee\nu_A(uv^2)-\sum_{j\neq i}(\phi\nu_{B_j})\left(uu^2,uv^2\right)=[\nu_{A_1}(u)\vee\nu_{A_2}(u^2)]\vee[\nu_{A_1}(u)\vee\nu_{A_2}(v^2)]-\sum_{j\neq i}(\phi\nu_{B_j})\left(uu^2,uv^2\right)$$

$$=[\nu_{A_1}(u)\vee\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)]-[\nu_{A_1}(u)\vee(\phi_2\nu_{B_{2p}})\left(u^2v^2\right)\text{ for some } p \text{ in } \{1,2,3\ldots,k\}$$

$$=\nu_{A_1}(u)\vee[\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)-\sum_{j\neq i}(\phi_2\nu_{B_{2j}})\left(u^2v^2\right)]=\nu_{A_1}(u)\vee\nu_{B_{2i}}^{\phi_2}(u^2v^2)=(\nu_{B_{1i}}^{\phi_1}\circ\nu_{B_{2i}}^{\phi_2})\left(uu^2,uv^2\right)$$

$$\Rightarrow \nu_{B_i}^{\phi}\left(uu^2,uv^2\right)=(\nu_{B_{1i}}^{\phi_1}\circ\nu_{B_{2i}}^{\phi_2})\left(uu^2,uv^2\right).$$

$$(\nu_{B_{1i}}^{\phi_1}\circ\nu_{B_{2i}}^{\phi_2})\left(uu^2,uv^2\right)=\nu_{A_1}(u)\vee\nu_{B_{2i}}^{\phi_2}(u^2v^2)=\nu_{A_1}(u)\vee[\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)-\sum_{j\neq i}(\phi_2\nu_{B_{2j}})\left(u^2v^2\right)]$$

$$=[\nu_{A_1}(u)\vee\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)]-[\nu_{A_1}(u)\vee\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)]$$

$$(\because u^2v^2\in B_{2i},\phi_2\nu_{B_{2j}}=\nu_{B_{2i}}\text{ for some } i \text{ and } G_2 \text{ be a } B_2-\text{strong})$$

$$\therefore (\nu_{B_{1i}}^{\phi_1}\circ\nu_{B_{2i}}^{\phi_2})\left(uu^2,uv^2\right)=0$$

$$\Rightarrow \nu_{B_i}^{\phi}\left(uu^2,uv^2\right)=(\nu_{B_{1i}}^{\phi_1}\circ\nu_{B_{2i}}^{\phi_2})\left(uu^2,uv^2\right)$$

$$=\nu_A\left(uu^2\right)\wedge\nu_A\left(uv^2\right)]$$

$$\text{Case (ii)}: \forall u\in V \text{ and } u^2v^2\notin B_2, u^2\neq v^2$$

$$\mu_{B_i}^{\phi}\left(uu^2,uv^2\right)=\mu_A(uu^2)\wedge\mu_A(uv^2)-\sum_{j\neq i}(\phi\mu_{B_j})\left(uu^2,uv^2\right)=[\mu_{A_1}(u)\wedge\mu_{A_2}(u^2)]\wedge[\mu_{A_1}(u)\wedge\mu_{A_2}(v^2)]-\sum_{j\neq i}(\phi\mu_{B_j})\left(uu^2,uv^2\right)$$

$$=[\mu_{A_1}(u)\wedge\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)]-[\mu_{A_1}(u)\wedge(\phi_2\mu_{B_{2p}})\left(u^2v^2\right)\text{ for some } p \text{ in } \{1,2,3\ldots,k\}$$

$$=\mu_{A_1}(u)\wedge[\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)-\sum_{j\neq i}(\phi_2\mu_{B_{2j}})\left(u^2v^2\right)]=\mu_{A_1}(u)\wedge\mu_{B_{2i}}^{\phi_2}(u^2v^2)=(\mu_{B_{1i}}^{\phi_1}\circ\mu_{B_{2i}}^{\phi_2})\left(uu^2,uv^2\right)$$

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Case (iii): $\forall\ w \in V'$ *and* $u^1v^1 \in B_{1i}$

$\mu_{B_i}{}^{\phi}\left(u^1w, v^1w\right) = 0$ as shown in case (i)

$$(\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1w, v^1w\right) = \mu_{A_2}(w) \wedge \mu_{B_{1i}}{}^{\phi}(u^1v^1) = \mu_{A_2}(w) \wedge [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) - \sum_{j\neq i}(\phi_1\mu_{B_{1j}})\left(u^1v^1\right)]$$

$$= [\mu_{A_2}(w) \wedge \mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1)] - [\mu_{A_2}(w) \wedge \sum_{j\neq i}(\phi_1\mu_{B_{1j}})\left(u^1v^1\right)]$$

$$= [\mu_{A_2}(w) \wedge \mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1)] - [\mu_{A_2}(w) \wedge \mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1)] = 0$$

$$\Rightarrow \mu_{B_i}{}^{\phi}\left(u^1w, v^1w\right) = (\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1w, v^1w\right)$$

$\nu_{B_i}{}^{\phi}\left(u^1w, v^1w\right) = 0$ as shown in case (i)

$$(\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1w, v^1w\right) = \nu_{A_2}(w) \vee \nu_{B_{1i}}{}^{\phi}(u^1v^1) = \nu_{A_2}(w) \vee [\nu_{A_1}(u^1) \wedge \nu_{A_1}(v^1) - \sum_{j\neq i}(\phi_1\nu_{B_{qj}})\left(u^2v^2\right)]$$

$$= [\nu_{A_2}(w) \vee \nu_{A_1}(u^1) \vee \nu_{A_1}(v^1)] - [\nu_{A_2}(w) \vee \sum_{j\neq i}(\phi_1\nu_{B_{qj}})\left(u^2v^2\right)]$$

$$= [\nu_{A_2}(w) \vee \nu_{A_1}(u^1) \vee \nu_{A_1}(v^1)] - [\nu_{A_2}(w) \vee \nu_{A_1}(u^1) \vee \nu_{A_1}(v^1)] = 0$$

$$\Rightarrow \nu_{B_i}{}^{\phi}\left(u^1w, v^1w\right) = (\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1w, v^1w\right).$$

Case (iv): $\forall\ w \in V'$ *and* $u^1v^1 \notin B_{1i}$

$$\mu_{B_i}{}^{\phi}\left(u^1w, v^1w\right) = \mu_A(u^1w) \wedge \mu_A(v^1w) - \sum_{j\neq i}(\phi\mu_{B_j})\left(u^1w, v^1w\right)]$$

$$= [\mu_{A_1}(u^1) \wedge \mu_{A_2}(w)] \wedge [\mu_{A_1}(v^1) \wedge \mu_{A_2}(w)] - \sum_{j\neq i}(\phi\mu_{B_j})\left(u^1w, v^1w\right)]$$

$$= [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) \wedge \mu_{A_2}(w)] - [\mu_{A_2}(w)] \wedge \phi_1\mu_{B_{1q}})\left(u^1v^1\right)] \quad \text{for some q } \in \{\text{i}= 1,2,3,\ldots..\text{k}\}$$

$$(\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1w, v^1w\right) = \mu_{A_2}(w) \wedge \mu_{B_{1i}}{}^{\phi_1}(u^1v^1) = \mu_{A_2}(w) \wedge [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) - \sum_{j\neq i}(\phi_1\mu_{B_{1j}})\left(u^1v^1\right)]$$

$$= [\mu_{A_2}(w) \wedge \mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1)] - [\mu_{A_2}(w) \wedge \sum_{j\neq i}(\phi_1\mu_{B_{1j}})\left(u^1v^1\right)]$$

$$= [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) \wedge \mu_{A_2}(w) - [\mu_{A_2}(w)] \wedge \phi_1\mu_{B_{1q}})\left(u^1v^1\right)] \quad \text{for some q } \in \{\text{i}= 1,2,3,\ldots..\text{k}\}$$

$$\Rightarrow \mu_{B_i}{}^{\phi}\left(u^1w, v^1w\right) = (\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1w, v^1w\right)$$

$$\nu_{B_i}{}^{\phi}\left(u^1w, v^1w\right) = \nu_A(u^1w) \vee \nu_A(v^1w) - \sum_{j\neq i}(\phi_1\nu_{B_j})\left(u^1w, v^1w\right)]$$

$$= [\nu_{A_1}(u^1) \vee \nu_{A_2}(w)] \vee [\nu_{A_1}(v^1) \vee \nu_{A_2}(w)] - \sum_{j\neq i}(\phi_1\nu_{B_j})\left(u^1w, v^1w\right)]$$

$$= [\nu_{A_1}(u^1) \vee \nu_{A_1}(v^1) \vee \nu_{A_2}(w) - [\nu_{A_2}(w)] \vee \phi_1\nu_{B_{1q}})\left(u^1v^1\right)] \quad \text{for some q } \in \{\text{i}= 1,2,3,\ldots..\text{k}\}$$

$$(\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1w, v^1w\right) = \nu_{A_2}(w) \vee \nu_{B_{1i}}{}^{\phi}(u^1v^1) = \nu_{A_2}(w) \vee [\nu_{A_1}(u^1) \wedge \nu_{A_1}(v^1) - \sum_{j\neq i}(\phi_1\nu_{B_{1j}})\left(u^1v^1\right)]$$

$$= [\nu_{A_2}(w) \vee \nu_{A_1}(u^1) \vee \nu_{A_1}(v^1)] - [\nu_{A_2}(w) \vee \sum_{j\neq i}(\phi_1\nu_{B_{1j}})\left(u^1v^1\right)]$$

$$= [\nu_{A_1}(u^1) \vee \nu_{A_1}(v^1) \vee \nu_{A_2}(w) - [\nu_{A_2}(w)] \vee \phi_1\nu_{B_{1q}})\left(u^1v^1\right)]$$

$$\Rightarrow \nu_{B_i}{}^{\phi}\left(u^1w, v^1w\right) = (\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1w, v^1w\right).$$

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Case(v) : $u^1v^1 \in B_{1i}$ and $u^2v^2 \in B_{2i}, u^2 \neq v^2$

$\mu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = 0$ as shown in case (i)

$$(\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right) = \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \mu_{B_{1i}}{}^{\phi_1}(u^1v^1) = \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) - \sum_{j \neq i}(\phi_1\mu_{B_{1j}})\left(u^1v^1\right)$$

$$= \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) - [\mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \sum_{j \neq i}(\phi_1\mu_{B_{1j}})\left(u^1v^1\right)]$$

$$= \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) - [\mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1)]$$

$(\because u^1v^1 \in B_{1i}, \phi_1\mu_{B_{1j}} = \mu_{B_{1i}}$ for some $j \neq i$ and G_1 be a B_{1i} – strong)

$\therefore (\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right) = 0$

$\Rightarrow \mu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = (\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right)$

and $\nu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = 0$ as shown in case (i)

$$(\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right) = \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \nu_{B_{1i}}{}^{\phi_1}(u^1v^1) = \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee [\nu_{A_1}(u^1) \vee \nu_{A_1}(v^1) - \sum_{j \neq i}(\phi_1\nu_{B_{1j}})\left(u^1v^1\right)$$

$$= \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \nu_{A_1}(u^1) \vee \nu_{A_1}(v^1) - [\nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \sum_{j \neq i}(\phi_1\nu_{B_{1j}})\left(u^1v^1\right)]$$

$$= \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \nu_{A_1}(u^1) \vee \nu_{A_1}(v^1) - [\nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \nu_{A_1}(u^1) \vee \nu_{A_1}(v^1)]$$

$(\because u^1v^1 \in B_{1i}, \phi_1\nu_{B_{1j}} = \nu_{B_{1i}}$ for some $j \neq i$ and G_1 be a B_{1i} – strong)

$\therefore (\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right) = 0$

$\Rightarrow \nu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = (\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right).$

Case(vi) : $u^1v^1 \notin B_{1i}$, $u^2 \neq v^2$

$$\mu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = \mu_A(u^1u^2) \wedge \mu_A(v^1v^2) - \sum_{j \neq i}(\phi\mu_{B_j})\left(u^1u^2, v^1v^2\right)]$$

$= [\mu_{A_1}(u^1) \wedge \mu_{A_2}(u^2)] \wedge [\mu_{A_1}(v^1) \wedge \mu_{A_2}(v^2)] - [\mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \phi_1\mu_{B_{1p}})\left(u^1v^1\right)]$ for some $p \in \{i = 1,2,3,.....k\}$

$$(\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right) = \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \mu_{B_{1i}}{}^{\phi_1}(u^1v^1)$$

$$= \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge [\mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) - \sum_{j \neq i}(\phi_1\mu_{B_{1j}})\left(u^1v^1\right)]$$

$$= \mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \mu_{A_1}(u^1) \wedge \mu_{A_1}(v^1) - [\mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \phi_1\mu_{B_{1p}})\left(u^1v^1\right)]$$

$\therefore \mu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = (\mu_{B_{1i}}{}^{\phi_1} \circ \mu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right)$

and $\nu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = \nu_A(u^1u^2) \vee \nu_A(v^1v^2) - \sum_{j \neq i}(\phi\nu_{B_j})\left(u^1u^2, v^1v^2\right)]$

$= [\nu_{A_1}(u^1) \vee \nu_{A_2}(u^2)] \vee [\nu_{A_1}(v^1) \vee \nu_{A_2}(v^2)] - [\nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \phi_1\nu_{B_{1p}})\left(u^1v^1\right)]$ for some $p \in \{i = 1,2,3,.....k\}$

$$(\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right) = \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \nu_{B_{1i}}{}^{\phi_1}(u^1v^1)$$

$$== \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee [\nu_{A_1}(u^1) \vee \nu_{A_1}(v^1) - \sum_{j \neq i}(\phi_1\nu_{B_{1j}})\left(u^1v^1\right)$$

$$= \nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \nu_{A_1}(u^1) \vee \nu_{A_1}(v^1) - [\nu_{A_2}(u^2) \vee \nu_{A_2}(v^2) \vee \phi_1\nu_{B_{1p}})\left(u^1v^1\right)]$$

$\Rightarrow \nu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = (\nu_{B_{1i}}{}^{\phi_1} \circ \nu_{B_{2i}}{}^{\phi_2})\left(u^1u^2, v^1v^2 \right).$

Case(vii) : $u^1v^1 \notin B_{1i}$, $u^2v^2 \notin B_{2i}$,

$$\mu_{B_i}{}^{\phi}\left(u^1u^2, v^1v^2 \right) = \mu_A(u^1u^2) \wedge \mu_A(v^1v^2) - \sum_{j \neq i}(\phi\mu_{B_j})\left(u^1u^2, v^1v^2\right)]$$

$= [\mu_{A_1}(u^1) \wedge \mu_{A_2}(u^2)] \wedge [\mu_{A_1}(v^1) \wedge \mu_{A_2}(v^2)] - [\mu_{A_2}(u^2) \wedge \mu_{A_2}(v^2) \wedge \phi_1\mu_{B_{1q}})\left(u^1v^1\right)]$ for some $q \in \{i = 1,2,3,.....k\}$

If $u^2 = v^2 = w$, as in case (iv),

$$(\mu_{B_{1i}}{}^{\phi_1}\circ\mu_{B_{2i}}{}^{\phi_2})\left(u^1u^2,v^1v^2\right)=[\mu_{A_1}(u^1)\wedge\mu_{A_1}(v^1)\wedge\mu_{A_2}(w)]-[\mu_{A_2}(w)]\wedge\phi_1\mu_{B_{1q}})\left(u^1v^1\right)]$$

If $u^2\neq v^2$, as in case (vi),

$$(\mu_{B_{1i}}{}^{\phi_1}\circ\mu_{B_{2i}}{}^{\phi_2})\left(u^1u^2,v^1v^2\right)=\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)\wedge\mu_{A_1}(u^1)\wedge\mu_{A_1}(v^1)-[\mu_{A_2}(u^2)\wedge\mu_{A_2}(v^2)\wedge\phi_1\mu_{B_{1p}})\left(u^1v^1\right)]$$

$$\therefore\mu_{B_i}{}^{\phi}\left(u^1u^2,v^1v^2\right)=(\mu_{B_{1i}}{}^{\phi_1}\circ\mu_{B_{2i}}{}^{\phi_2})\left(u^1u^2,v^1v^2\right)$$

and $\nu_{B_i}{}^{\phi}\left(u^1u^2,v^1v^2\right)=\nu_A(u^1u^2)\vee\nu_A(v^1v^2)-\sum_{j\neq i}(\phi\nu_{B_j})\left(u^1u^2,v^1v^2\right)]$

$$=[\nu_{A_1}(u^1)\vee\nu_{A_2}(u^2)]\vee[\nu_{A_1}(v^1)\vee\nu_{A_2}(v^2)]-[\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)\vee\phi_1\nu_{B_{1q}})\left(u^1v^1\right)]\quad\text{for some } q\in\{i=1,2,3,.....k\}$$

If $u^2=v^2=w$, as in case (iv),

$$(\nu_{B_{1i}}{}^{\phi_1}\circ\nu_{B_{2i}}{}^{\phi_2})\left(u^1u^2,v^1v^2\right)=[\nu_{A_1}(u^1)\vee\nu_{A_1}(v^1)\vee\nu_{A_2}(w)]-[\nu_{A_2}(w)]\vee\phi_1\nu_{B_{1q}})\left(u^1v^1\right)]$$

If $u^2\neq v^2$, as in case (vi),

$$(\nu_{B_{1i}}{}^{\phi_1}\circ\nu_{B_{2i}}{}^{\phi_2})\left(u^1u^2,v^1v^2\right)=\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)\vee\nu_{A_1}(u^1)\vee\nu_{A_1}(v^1)-[\nu_{A_2}(u^2)\vee\nu_{A_2}(v^2)\vee\phi_1\nu_{B_{1p}})\left(u^1v^1\right)]$$

$$\Rightarrow\nu_{B_i}{}^{\phi}\left(u^1u^2,v^1v^2\right)=(\nu_{B_{1i}}{}^{\phi_1}\circ\nu_{B_{2i}}{}^{\phi_2})\left(u^1u^2,v^1v^2\right).$$

Combining all the cases, we get ,

$$\mu_{B_i}{}^{\phi}\left(uv\right)=(\mu_{B_{1i}}{}^{\phi_1}\circ\mu_{B_{2i}}{}^{\phi_2})\left(uv\right)\text{ and }\nu_{B_i}{}^{\phi}\left(uv\right)=(\nu_{B_{1i}}{}^{\phi_1}\circ\nu_{B_{2i}}{}^{\phi_2})\left(uv\right),\forall uv\in V\times V'.$$

Definition (4.10): If an IFGS $\tilde{\tilde{G}}$ = (A, B_1, B_2,….,B_k) of G = (V, R_1, R_2,....,R_k) is a B_i- strong for all i = 1,2,3,….,k then $\tilde{\tilde{G}}$ is said to be a strong IFGS.

The following result is clear from theorem 4.9.

Corollary (4.11): Let $\tilde{G}_1$ = (A_1,B_{11},B_{12},...,B_{1k}) and $\tilde{G}_2$ = (A_2, B_{21},B_{22},...,B_{2k}) be two strong IFGS, then $\tilde{G}_1\circ\tilde{G}_2$ is a strong IFGS and $\left(\tilde{G}_1\circ\tilde{G}_2\right)^{\phi}=\tilde{G}_1{}^{\phi_1}\circ\tilde{G}_2{}^{\phi_2}$ where ϕ is defined as in (4.8).

V. CONCLUSION

Since intuitionistic fuzzy sets have more advantages in handling vagueness and uncertainty than fuzzy sets so the study of intuitionistic graph structures is more powerful than the study of fuzzy graph structures. Here in this paper, some elementary operations on IFGSs and their ϕ- complements are defined and discussed which will play a prominent role in the future study of the subject.

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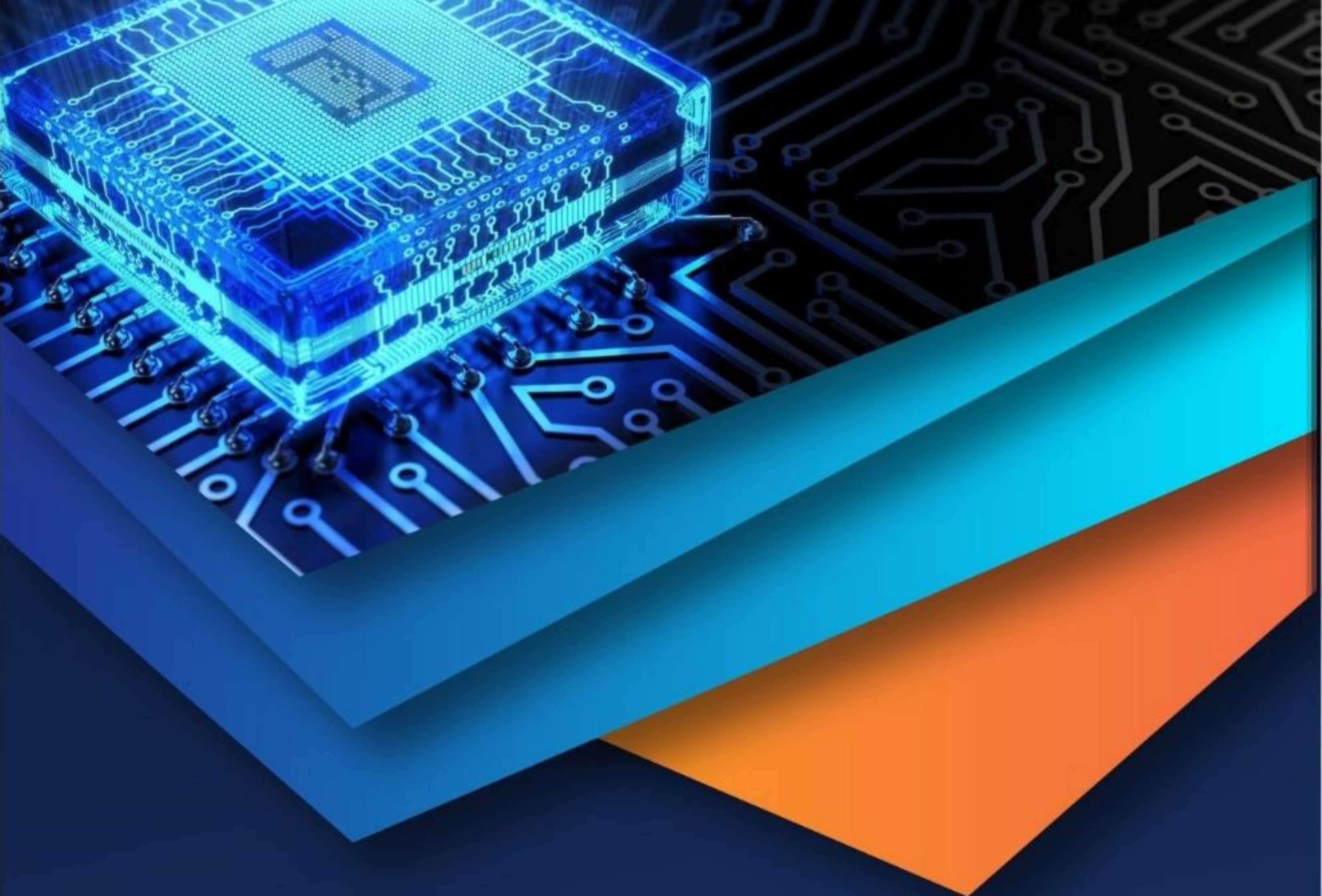

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