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Cylindrical Imploding Strong Shock Wave in Uniform Real Dusty Gas

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Abstract: Chester-Chisnell-Whitham method has been used to study the behavior of dusty real gas on adiabatic propagation of cylindrical imploding strong shock waves. It is assumed that the dusty gas is the mixture of a real gas and a large number of small spherical solid particles of uniform size. Neglecting the effect of overtaking disturbances, the analytical expressions for shock strength and shock velocity immediately behind the shock have been derived for uniform initial density distribution. The expressions for the pressure and particle velocity have also been obtained. The variation of the flow variables have been calculated numerically and discussed through graphs. Finally the results accomplished here are compared with earlier results as well as those for ideal dusty gas.

Keywords: CCW, dusty, imploding, real gas, shock.

I. INTRODUCTION

The study of the shock wave propagation in a dusty gas has received considerable attention due to its application to space science, bomb blast, lunar ash flow, coal mine explosions, nozzle flow, missiles and other engineering problems. Many scientists (Sedov[1], Pai et al[2], Miura and Glass[3], Gretler and Regenfelder [4], Hirschler and Steiner [5], Igra et al[6]. Vishwakarma[7] generalized Ray and Bhowmik[8] solution in gas to the mixture of gas and small solid particles with exponentially varying density, using a non-similarity method. He found that the presence of small dust particles in the gaseous medium has significant effects on the variation of flow variables. The problem of propagation of shock wave in dusty ideal gas have been tackled by Vishwakarma[9] using similarity method. Vishwakarma and Pandey[10] have studied one-dimensional unsteady self-similar adiabatic flow of a dusty ideal gas behind a spherical shock wave with time- dependent energy input. Motion of shock wave in a mixture of ideal gas and small dust particles with radiation heat flux and exponentially varying density has been studied by Vishwakarma et. al.[11]. Yadav et al.[12] studied the propagation of weak cylindrical shock in the mixture of ideal gas and dust particle in presence of constant axial magnetic field. Singh and Gogoi [13] have found similarity solution for the propagation of spherical shock wave in the mixture of non-ideal gas and small solid dust particles. They use the equation of state for non-ideal gases simplified by Anisimov and Spiner[14], which describes the behavior of non-idealness of gas on the problem of strong shock explosion.

The aim of the present study is to investigate the motion of cylindrical imploding shock in a real dusty gas having uniform initial density distribution by using CCW (Chester[15]-Chisnell[16]-Whitham[17]) method, without considering the effect of overtaking disturbances. It is assumed that the real (non-ideal) dusty gas is the mixture of real gas and a large number of small spherical solid particles of uniform size. The particles are inert and uniformly distributed in the gas. Initial volume fraction of the solid particles is also assumed constant in this particular study. The particles do not interact with each other therefore their thermal motion is negligible. Initial density of the medium is taken to be constant and medium ahead of the shock front is at rest with small counter pressure. We also assumed that the particles behave like a pseudo-fluid. Maintaining the equilibrium flow condition in the flow field, the analytical expressions for the shock velocity, shock strength, pressure, and flow velocity have been derived. The variation of flow variables with propagation distance (r), mass concentration of solid particles in the mixture (k_p) and the ratio of the density of solid particles to the initial density of gas (G) are obtained and discussed through figures. The results accomplished are compared with those for dusty ideal gas[13].

II. BASIC EQUATIONS

The basic equations for one dimensional, unsteady, adiabatic and cylindrical symmetrical flow of a mixture of real (non-ideal) gas and small spherical solid particles can be written as[18]-[20]

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \tag{1}$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \tag{2}$$

$$\frac{\partial \varepsilon}{\partial t} + u \frac{\partial \varepsilon}{\partial r} + \frac{p}{\rho^2} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0 \tag{3}$$

where, u , p , ρ and ρ_{sp} denote respectively, the flow velocity, the pressure and density at a distance ' r ' from the origin at time ' t '. It is assumed that the medium (dusty real gas) to be a mixture of a real (non-ideal) gas and small spherical solid particles. The equation of state of the pseudo-fluid of solid particles in the mixture is simply [14], [21]

$$\rho_{sp} = \text{constant} \tag{4}$$

where, ρ_{sp} is the species density of solid particles.

The equation of state of the non-ideal gas in the mixture is taken to be [14] and [21]

$$p_g = R^* \rho_g (1 + b \rho_g) T_g \tag{5}$$

$$p_g = R^* (1 - Z) \rho_g T_g = (1 - Z) P \tag{6}$$

where R^* denotes the gas constant, b is the internal volume of the molecule of gas, T_g is the temperature of the gas and of the solid particles as equilibrium flow condition is mentioned and, p_g and ρ_g are the partial pressure and the partial density of the gas in mixture. In this equation the deviation of a real gas from the ideal state are taken into account. The gas is assumed to be so rarefied that its molecules interact only through binary collisions, while triple, quadruple etc collisions are negligible.

Therefore, the equation of state of the mixture of perfect gas and small solid particles is given by [13] and [22]

$$p = \frac{(1 - k_p)}{1 - Z} [1 + b \bar{\rho} (1 - k_p)] \rho R^* T \tag{7}$$

where R is the gas constant, T the temperature, Z is the volume fraction of solid particles in the mixture, K_p is the mass concentration of solid particles in the mixture.

The relation between k_p and Z is as follows [18]

$$k_p = \frac{Z \rho_{sp}}{\rho} \tag{8}$$

where ρ_{sp} is the species density of solid particles. In equilibrium flow, K_p is a constant in the whole flow-field. Therefore from the equations (4) and (10)

$$k_p = \frac{Z}{\rho} = \text{constant} \tag{9}$$

we have the relation for Z is [18,23]

$$Z = \frac{K_p}{\sigma(1 - k_p) + k_p} \tag{10}$$

where $\sigma = \rho_{sp} / \rho$ the ratio of density to solid particle to the specific density of a non-ideal gas (specific density or density ratio), Hence the fundamental parameters of the Pai model are K_p and σ which describe the effects of the dust-loading. For the dust loading parameter σ , we have range of $\sigma=1$ to $\sigma \rightarrow \infty$.

The internal energy of the mixture can be written as follows [18]

$$\varepsilon = [K_p C_{sp} + (1 - K_p) C_v] T = C_{vm} T \tag{11}$$

where C_{sp} is the specific heat of solid particles, C_v the specific heat of the gas at constant volume process and C_{vm} the specific heat of the mixture at constant volume process.

The specific heat of the mixture at constant pressure process is

$$C_{pm} = K_p C_{sp} + (1 - K_p) C_p \tag{12}$$

where C_p is the specific heat of the gas at constant pressure process.

The ratio of specific heat of the mixture is given by [18,24]

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma + \delta \Phi}{1 + \delta \Phi} \tag{13}$$

where $\gamma = \frac{C_p}{C_v}$, $\delta = \frac{K_p}{1 - K_p}$ and $\Phi = \frac{C_{sp}}{C_v}$,

The internal energy is therefore, given by

$$\varepsilon = \frac{p(1 - Z)}{\rho(\Gamma - 1)[1 + b\rho(1 - k_p)]} \tag{14}$$

Let u_o , p_o , ρ_o , F_o and Z_o denote the undisturbed values of particle velocity, pressure, density, heat flux and volume fraction of solid particles in the mixture just ahead of the shock and u , p , ρ , F and Z be the values of respective quantities at any point immediately after the passage of the shock (just behind the shock). It is assumed that a strong cylindrical shock is propagating into the medium (mixture of real gas and small solid particles) having constant initial density ρ_o at rest ($u_o = 0$) with negligible small counter pressure $p \approx 0$. The boundary conditions at the shock are given by

$$\rho(U - u) = \rho_o U \tag{15}$$

$$p = \rho u (U - u) \tag{16}$$

$$\varepsilon + \frac{p}{\rho} + \frac{1}{2}(U - u)^2 - \frac{F}{\rho_o U} = \frac{1}{2}U^2 - \frac{F_o}{\rho_o U} \tag{17}$$

$$\frac{Z}{\rho} = \frac{Z_o}{\rho_o} \tag{18}$$

The initial volume fraction of the solid particles Z_o is, in general not a constant, but the volume occupied by the solid particles is very small because the density of the solid particles is much larger than that of the gas [13], hence Z_o may be assumed a small constant and is given by [18]

$$Z_o = \frac{K_p}{G(1 - k_p) + k_p} \tag{19}$$

where $U = dR/dt$ denotes the shock velocity, R is the shock radius, $U/a_o = M$ is the Mach number, the suffix “o” refers to the values in front of the shock.

The speed of sound ‘a’ in the equilibrium two phase flow for an isentropic change of state of the mixture of the real gas and small spherical solid particles may be calculated as [25]

$$a^2 = \left(\frac{dp}{dp}\right)_s = \frac{[\Gamma + (2\Gamma - Z)b\rho(1 - k_p)]p}{(1 - Z)\{1 + b\rho(1 - k_p)\}\rho} \tag{20}$$

neglecting $b^2\rho^2$ and higher order terms[14], where the subscript ‘s’ is the process of constant entropy.

On simplifying equation (22), we get

$$a^2 = \frac{P}{\rho(1-Z)} [\Gamma + (\Gamma - Z)b\rho(1-k_p)] \tag{21}$$

The speed of sound in unperturbed medium a_o ’ is given by the relation

$$a_o = \sqrt{\frac{p_o \{ \Gamma + (\Gamma - Z_o)\bar{b}(1-k_p) \}}{\rho_o(1-Z_o)}} \tag{22}$$

To represent the quantities u, p, ρ and Z in terms of their undisturbed values, the jump conditions across the strong shock are given by[18], [13]and [26]

$$u = (1 - \beta)U \tag{23}$$

$$\rho = \frac{\rho_o}{\beta} \tag{24}$$

$$p = (1 - \beta)\rho_o^2 U^2 \tag{25}$$

$$Z = \frac{Z_o}{\beta} \tag{26}$$

The quantity $\beta(0 < \beta < 1)$ is obtained by the relation

$$\frac{2(\beta - Z)\beta}{\rho(\Gamma - 1)[\beta + \bar{b}(1 - k_p)]} + \beta - \left[1 + \frac{2(F_2 - F_1)}{pU} \right] = 0 \tag{27}$$

where $\bar{b} = b\rho_o$ (say)

As the shock is strong, we assume $F_1 - F_2$ to be negligible in the comparison with the product of p and U [27], therefore above equation may be written as

$$\beta^2(\Gamma + 1) + \beta \left[\bar{b}(1 - k_p) - 1 \right] (\Gamma - 1) - 2Z_o - (\Gamma - 1)\bar{b}(1 - k_p) = 0 \tag{28}$$

where the quantity β is the shock density ratio which is an unknown parameter to be determined.

Using boundary conditions, the speed of sound is given by

$$a = \left[\frac{(1 - \beta) \{ \Gamma\beta^2 + (\Gamma\beta - Z_o)\bar{b}(1 - k_p) \}}{(\beta - Z_o)} \right]^{1/2} U \tag{29}$$

For cylindrical imploding shock, the characteristic form of the system of equations (1)-(3), i.e. the form in which each equation contain derivatives in only one direction in (r, t) plane, is

$$dp - \rho a du + \frac{\rho a^2 u}{(u-a)} \frac{dr}{r} = 0 \tag{30}$$

Substituting the values from equations (23)-(26) and (30) in above equation, we have

$$(1-\beta) \left[\rho_o dU^2 + U^2 d\rho_o \right] - \frac{\rho_o}{\beta} \left[\frac{(1-\beta) \{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}}{(\beta - Z_o)} \right]^{\frac{1}{2}} U(1-\beta) dU$$

$$+ \frac{\rho_o}{\beta} \left[\frac{(1-\beta) \{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}}{(\beta - Z_o)} \right] U^2(1-\beta) U \frac{dr}{r} = 0$$

$$(1-\beta) U - \left[\frac{(1-\beta) \{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}}{(\beta - Z_o)} \right]^{\frac{1}{2}} U$$

At constant initial density distribution ($\rho_o = \text{constant}$), we have

$$\frac{dU}{U} + \left[\frac{1}{\frac{2\beta(\beta - Z_o)}{\{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}} - \sqrt{\frac{(1+\beta)^2(\beta - Z_o)}{(1-\beta) \{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}} + 1}} + 1 \right] \frac{dr}{r} = 0 \tag{31}$$

On solving, we have

$$\log_e U = \log_e K + \log_e r^{\left[\frac{1}{\sqrt{\frac{(\beta - Z_o)(\beta + 1)^2}{(1-\beta) \{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}} - \frac{2\beta(\beta - Z_o)}{\{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}} - 1}} + 1 \right]}$$

where K is the constant of integration.

The expression for the shock velocity is given by

$$U = Kr^{\left[\frac{1}{\sqrt{\frac{(\beta - Z_o)(\beta + 1)^2}{(1-\beta) \{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}} - \frac{2\beta(\beta - Z_o)}{\{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}} - 1}} + 1 \right]} \tag{32}$$

Using equation (22) and (32), the expression for the propagating shock strength in the region behind the shock is given by

$$\frac{U}{a_o} = K^* \sqrt{\frac{(1 - Z_o)}{\{ \Gamma + (\Gamma - Z_o) \bar{b}(1-k_p) \}}} r^{\left[\frac{1}{\sqrt{\frac{(\beta - Z_o)(\beta + 1)^2}{(1-\beta) \{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}} - \frac{2\beta(\beta - Z_o)}{\{ \Gamma\beta^2 + (\Gamma\beta - Z_o) \bar{b}(1-k_p) \}} - 1}} + 1 \right]} \tag{33}$$

III. RESULTS AND DISCUSSIONS

Expressions for non-dimensional pressure and particle velocity in the region behind the strong shock can be obtained by using (22), (23), (25) and (33), respectively

$$\frac{p}{p_o} = (1-\beta) K^* r^2 \left[\frac{\sqrt{\frac{(\beta-Z_o)(\beta+1)^2}{(1-\beta)\{\Gamma\beta^2+(\Gamma\beta-Z_o)\bar{b}(1-k_p)}\}} - \frac{2\beta(\beta-Z_o)}{\{\Gamma\beta^2+(\Gamma\beta-Z_o)\bar{b}(1-k_p)}\}}}{1} \right] \tag{34}$$

$$\frac{u}{a_o} = (1-\beta) K^* \sqrt{\frac{(1-Z_o)}{\{\Gamma+(\Gamma-Z_o)\bar{b}(1-k_p)\}}} r \left[\frac{1}{\sqrt{\frac{(\beta-Z_o)(\beta+1)^2}{(1-\beta)\{\Gamma\beta^2+(\Gamma\beta-Z_o)\bar{b}(1-k_p)}\}} - \frac{2\beta(\beta-Z_o)}{\{\Gamma\beta^2+(\Gamma\beta-Z_o)\bar{b}(1-k_p)}\}} - 1} \right] \tag{35}$$

where $K^* = K\sqrt{p_o/\rho_o}$

Initial volume fraction of solid particles in medium(Z_o) depends on mass concentration of solid particles in the mixture(k_p) and ratio of the density of solid particle to the density of gas. Values of initial volume fraction Z_o for some values of k_p and G are calculate from the equation(19).

Initially taking $U/a_o=15$ at $r=1.1$ for $k_p=0.1$, $G=50$, $\bar{b}=0.02$, $Z_o=0.00222$, $\beta=0.16$, the variation of shock velocity, shock strength, non-dimensional pressure and non-dimensional flow velocity with propagation distance have been calculated and displayed through figures(1-4). It is found from figure (1-4) hat all the flow variables increases as as imploding strong shock advances in dusty real gas. The dotted line(...) shows the variation of all flow variables for ideal ($\bar{b}=0$) gas[12]. The similar variation of all flow variables with propagation distance have been reported by [13] by using similarity method. It is also observed from figure (2) that strength of the strong cylindrical shock is decreases as concentration of dust particles in the mixture (k_p)is decreases. The effect of specific density of dust particles on shock velocity as well as other flow variables are also depicts in the figures and it is clear that all the parameters decreases as increase in G . Variation of shock strength as well as other flow variables at $\bar{b}=0$ and 0.02 has been shown in the figures(1-4). The similar results have also been obtained by [13] by using similarity method. It may be concluded from the general observation of these figures that the consideration of non-idealness (real) of the gas has significant role on the variation of all flow variables [cf Fig (1-4)].

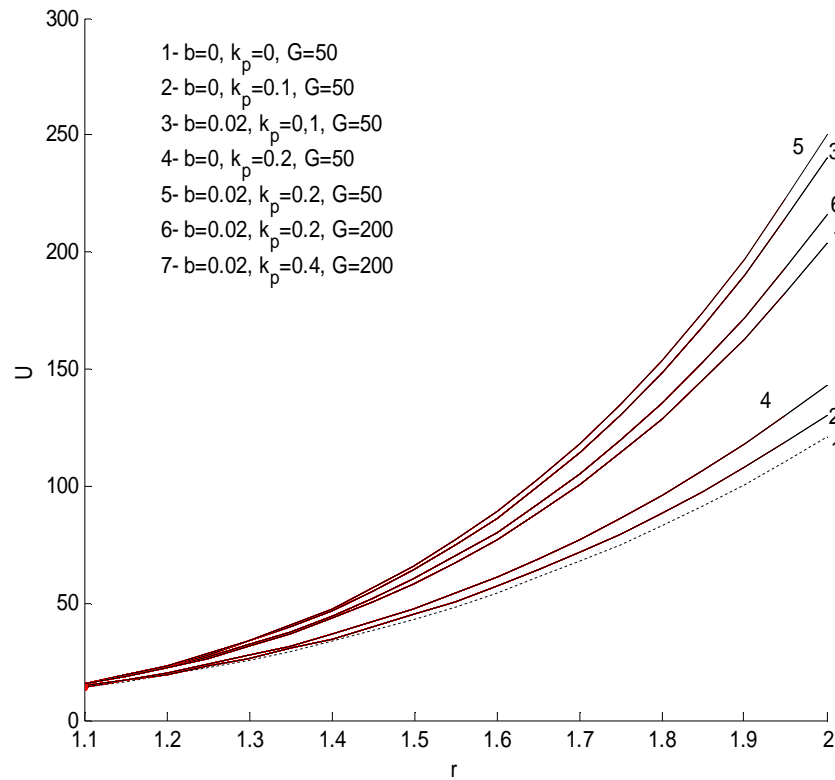


Figure 1 Variation of shock velocity with propagation distance r

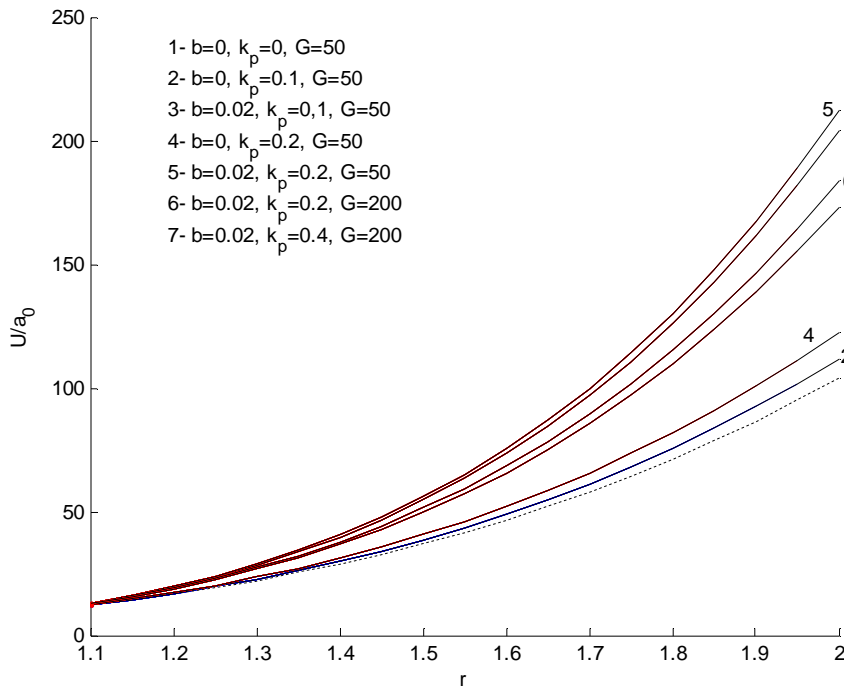


Figure 2 Variation of shock strength U/a_0 with propagation distance r.

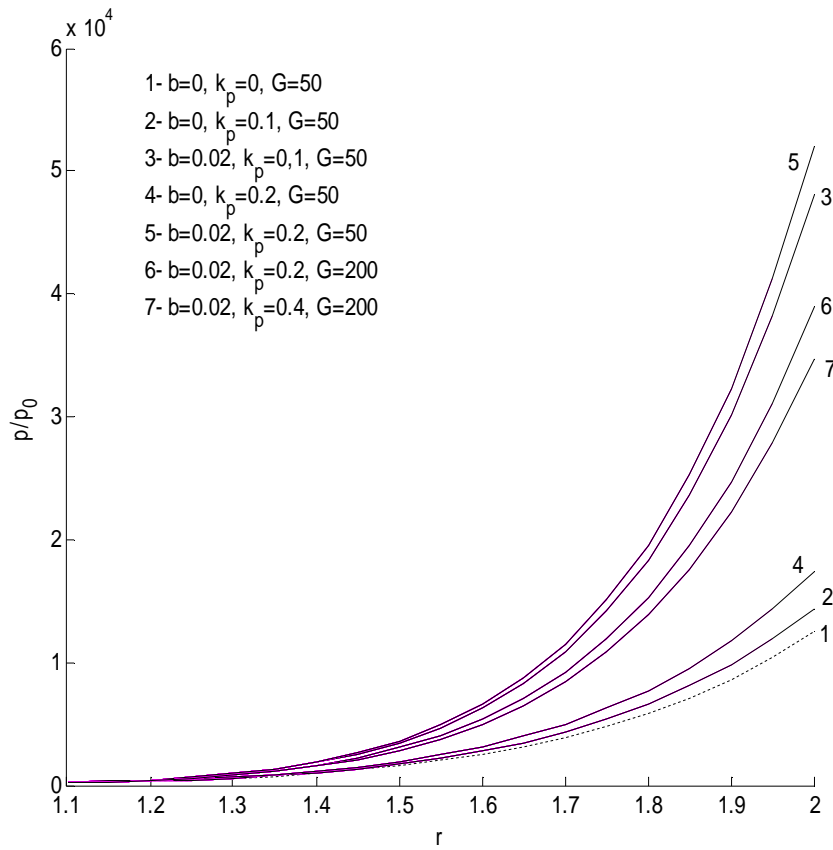


Figure 3 Variation of non-dimensional pressure p/p_0 with r

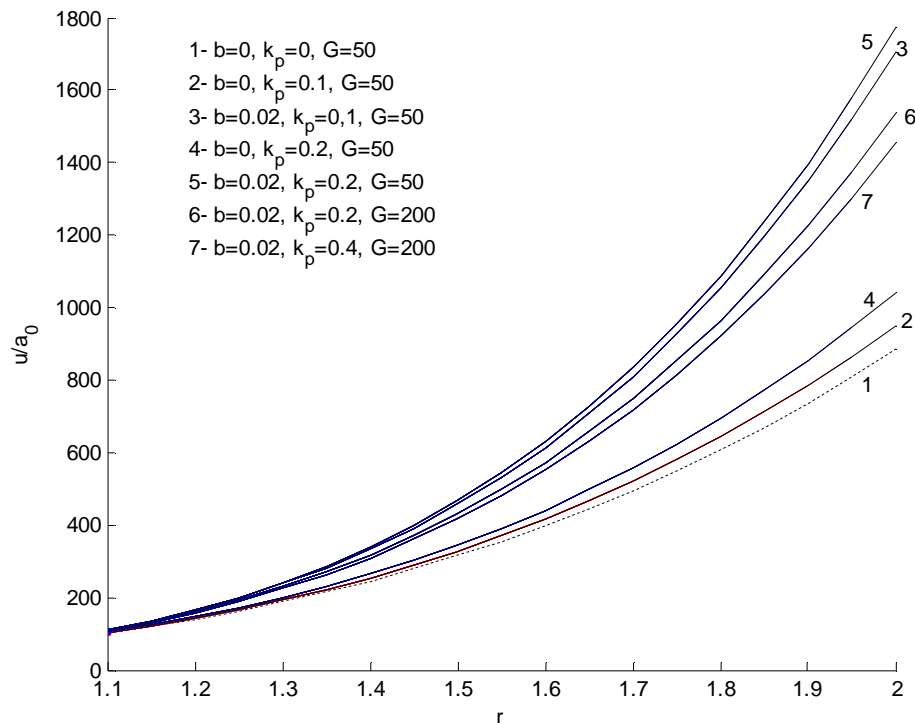


Figure 4 Variation of flow velocity u/a_0 with propagation distance r .

REFERENCES

- [1] L. I. Sedov, "Similarity and Dimensional methods in mechanics" (academic Press New York) pp. 166, 1959.
- [2] S.I. Pai, S. Menon, Z. Q. Fan, "Similarity solution of a strong shock wave propagation in a mixture of a gas and dust particles". Int J Eng Sci vol.18 pp.1365-1373, 1980.
- [3] H. Miura, I. I. Glass, "On the passage of a shock wave through a dusty gas layer". Proc. R Soc Lond., vol. A 385, pp85-105, 1983.
- [4] W. Gretler and R. Regenfelder, "Strong shock wave generated by a piston moving in a dust-laden gas under isothermal condition". Eur. J Mech. B, Fluids vol.24, pp205-218, 2005.
- [5] T. Hirschler and H. Steiner, "A self similar solution for the implosion problem in a dusty gas", Fluid Dyn. Res., vol. 32, pp 61-67, 2003.
- [6] O. Igra, G. Hu, J. Falcovitz, and B. Wang, "Shock wave reflection from a wedge in a dusty gas" Int., J., Multiphase Flow, vol. 30, pp 1139-1169, 2004.
- [7] J. P. Vishwakarma, "Propagation of shock waves in a dusty gas with exponential varying density", The Euro. J. B, vol.16, pp369-372, 2000.
- [8] G. D. Ray and J.B. Bhowmik, "Propagation of cylindrical and spherical explosion wave in an exponential medium", Def. Sci. J. vol.9, pp9-14, 1974.
- [9] J. P. Vishwakarma, G. Haidar, "Propagation of shock waves in a dusty gas with density obeying the Power law", Mod. Meas. Cont. B, vol.71(8), pp37, 2002.
- [10] J.P. Vishwakarma and S.N. Pandey, "Propagation of strong spherical shock waves in a dusty gas". Phys Scr. vol.68, pp259-263, 2003.
- [11] J.P. Vishwakarma, G.Nath and K.K. Singh, "Propagation of shock waves in a dusty gas with heat conduction, radiation heat flux and exponentially varying density, Phys. Scr. Vol.78, pp11,2008..
- [12] R.P. Yadav, A. Sharma, R. P. Vats and P.S. Rawat, "Effect of axial magnetic field on weak cylindrical shock in a dusty gas." Ultra sciences, vol. 19(2), pp331, 2007.
- [13] K. K. Singh and S. Gogoi, "Self-similar flow of a dusty gas behind an exponential shock with heat conduction and radiation heat flux". J. Raj. Acad. Phy. Sci.vol.12(4), pp419-440, 2013.
- [14] S. I. Anisimov, O.M. Spiner, "Motion of an almost ideal gas in the presence of a strong point explosion." J Appl. Math. Mech. vol.36, pp 883-887, 1972.
- [15] W. Chester, "The diffraction and reflection of shock wave", Quar. J. Mech. Appl. Math., vol. 7, pp57-82, 1954.
- [16] R. F. Chisnell, "The Normal Motion of a Shock Wave through a Non-Uniform One-Dimensional Medium", Proc Roy Soc. London, vol. A 232, pp350-370, 1955.
- [17] G.B. Whitham, "On the propagation of shock waves through regions of non-uniform area or flow", J. Fluid Mech., vol. 4, pp337- 360. 1958.
- [18] S. I. Pai, "Two phase flows, Vieweg Tracts in Pure and Appl. Phys, Chapter-V, vol 3. Vieweg Verlag Braunschweig, 1977.
- [19] J. P. Vishwakarma and G. Nath, "Similarity solutions for unsteady flow behind an exponential shock in a dusty gas", Phys. Scripta, vol.74, pp493-372, 2006.
- [20] H. Steiner and T. Hirschler, "A self-similar solution of shock propagation in a dusty gas", Eur J Mech B, Fluids, vol. 21, pp371-380, 2002.
- [21] M. P. Ranga Rao and N.K. Purohit, "Self-similar piston problem in non-ideal gas", Int. J Eng. Sci., vol.14, pp91-97, 1976.
- [22] J. P. Vishwakarma and G. Nath, "A self-similar solution of shock propagation in a mixture of a non-ideal gas and small solid particles", Meccanica vol. 44, pp239-254, 2009.



- [23] G. N. Naidu, K. Venkatanandam and M.P. Ranga Rao, "Approximate analytical solutions for self-similar flows of a dusty gas with variable energy" Int. J. Eng. Sci., vol. 23, pp39-49, 1985.
- [24] F. E. Marble, "Dynamics of dusty gases". Annu. Rev. Fluid Mech. vol.2, pp397-446, 1970.
- [25] H. Miura and I. I. Glass, "Development of the flow induced by a piston moving impulsively in a dusty gas", Proc. Roy. Soc., London vol. A, pp397, pp 295-309, 1985.
- [26] K. K. Singh and S. Gogoi, "Propagation of strong spherical shock waves in a mixture of a non-ideal gas and small solid particles", Int. J. App. Mech., vol. 9(14), pp 54-74, 2013.
- [27] D. D. Laumbach and R.F. Probstein, "Self-similar strong shocks with radiations in a decreasing exponential atmosphere", Phys Fluids, vol.13, pp1178-1183, 1970.



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