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New functions in BiČech sgβ-Biclosure spaces

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Abstract: The aim of this paper is to introduce the concept of BiČech sgβ –continuous functions, BiČech sgβ-Irresolute functions, Totally BiČech sgβ –continuous functions and Slightly BiČech sgβ –continuous functions in Biclosure spaces and investigate their characterizations.
Keywords: BiČech sgβ-closed sets, BiČech sgβ-open sets, BiČech sgβ –continuous, BiČech sgβ Irresolute, Totally BiČech sgβ –continuous functions and Slightly BiČech sgβ -continuous functions.

I. INTRODUCTION

Čech spaces were introduced by Eduard Čech [3] (i.e., sets endowed with a grounded , Extensive and additive closure operators) and studied by many others[6][12]. BiČech closure spaces were introduced by K.Chandrasekhara Rao, R. Gowri and V. Swaminathan [4]. N. Levine [13] introduced g-closed sets. D. Andrijevic [1] initiated the study of β-open sets and β-closed sets. In this paper, we analyze the concept of BiČech sgβ –continuous functions and BiČech sgβ –Irresolute functions, Totally BiČech sgβ –continuous functions and Slightly BiČech sgβ –continuous functions in biclosure spaces and discuss some of their basic properties.

II. PRELIMINARIES

Definition 2.1: Two maps k_1 and k_2 from power set X to itself are called BiČech closure operator on X and the pair (X, k_1,k_2) is called a BiČech closure spaces if the following axioms are satisfied

$k_1(\varphi) = \varphi$ & $k_2(\varphi) = \varphi$

$A \subseteq k_1(A)$ & $A \subseteq k_2(A)$ for every $A \subseteq X$

$k_1(A \cup B) = k_1(A) \cup k_1(B)$ and $k_2(A \cup B) = k_2(A) \cup k_2(B)$ for all $A,B \subseteq X$.

Definition 2.2 [5] A subset A in a BiČech closure space (X, k_1, k_2) is said to be

k_i-regular open if $A = int_{k_i}(k_i(A))$, i = 1, 2

k_i -regular closed if $A = k_i(int_{k_i}(A))$, i = 1, 2

k_i-semi open if $A \subseteq k_i(int_{k_i}(A))$, i = 1, 2

k_i-semi closed if $int_{k_i}(k_i(A)) \subseteq A$, i = 1, 2

k_i-pre open if $A \subseteq int_{k_i}(k_i(A))$, i = 1, 2

k_i -pre closed if $k_i(int_{k_i}(A)) \subseteq A$, i = 1, 2

k_i -α open if $A \subseteq int_{k_i}(k_i(int_{k_i}(A)))$, i = 1, 2

k_i -α closed if $k_i(int_{k_i}(k_i(A))) \subseteq A$, i = 1, 2

k_i -β open if $A \subseteq k_i(int_{k_i}(k_i(A)))$, i = 1, 2

k_i -β closed if $(int_{k_i}(k_i(intA))) \subseteq A$, i = 1, 2.

Definition 2.3: A subset A of a BiČech closure space (X, k_1,k_2) is called biclosed if $k_1A = A = k_2A$ and called biopen if its complement is biclosed.

Definition 2.4: A subset A of a BiČech closure space (X, k_1,k_2) is said to be (k_1,k_2)-g biclosed if $k_2(A) \subseteq G$ whenever $A \subseteq G$ and G is k_1 open set in X.

Definition 2.5: A subset A of a BiČech closure space (X, k_1,k_2) is said to be (k_1,k_2)-πgα biclosed if $k_{2\alpha}(A) \subseteq G$ whenever $A \subseteq G$ and G is k_1 π -open set in X.

Definition 2.6: Let (X, k_1,k_2) be a BiČech closure space. A subset A⊆ X is said to be (k_1,k_2)-w -biclosed set if $k_2(A) \subseteq G$ whenever $A \subseteq G$ and G is k_1 semi-open set in X.
Definition 2.7: Let (X, k_1,k_2) be a BiČech closure space. A subset A⊆ X is said to be (k_1,k_2)-J- Čech-biclosed set if $k_\alpha(A) \subseteq G$ whenever $A \subseteq G$ and G is k_1 semi-open set in X, where $k_\alpha(A)$ is the smallest α-closed set containing A.
Definition 2.8: Let (X, k_1,k_2) be a BiČech closure space. A subset A⊆ X is called (k_1,k_2)-sgβ closed set if $k_{2\beta}(A) \subseteq G$ whenever $A \subseteq G$ and G is semi-open subset of (X,k_1) where $k_{2\beta}(A)$ is the smallest β-closed set containing A.

III. BIČECH sgβ – CONTINUOUS FUNCTIONS

Definition 3.1: Let (X, k_1,k_2) and (Y, v_1,v_2) be a BiČech biclosure space. A map f: (X, k_1,k_2) ⟶ (Y,v_1,v_2) is called BiČech sgβ-continuous if every $f^{-1}(v)$ is BiČech sgβ - open set in (X, k_1,k_2) for every biopen set V in (Y, v_1,v_2).
Definition 3.2: A function f:(X, k_1,k_2) ⟶ (Y, v_1,v_2) is called
(a) continuous if $f^{-1}(V)$ is biclosed in X for each biclosed set V of Y.
(b) g-continuous if $f^{-1}(V)$ is g-biclosed in X for each biclosed set V of Y.
(c) w-continuous if $f^{-1}(V)$ is w-biclosed in X for each biclosed set V of Y.
(d) J-continuous if $f^{-1}(V)$ is J-biclosed in X for each biclosed set V of Y.
(e) πgα-continuous if $f^{-1}(V)$ is πgα-biclosed in X for each biclosed set V of Y.
Proposition 3.3:
(a) Every BiČech continuous is BiČech sgβ – continuous.
(b) Every BiČech g – continuous is BiČech sgβ – continuous.
(c) Every BiČech w – continuous is BiČech sgβ – continuous.
(d) Every BiČech J -continuous is BiČech sgβ – continuous.
(e) Every BiČech πgα -continuous is BiČech sgβ – continuous.
Proof: (a) Let f be a BiČech continuous. Let V be a BiČech open set in (Y, v_1,v_2). Since f is BiČech continuous, $f^{-1}(V)$ is BiČech open set of (X, k_1,k_2). Every BiČech open set is BiČech sgβ – open set.
This implies that $f^{-1}(V)$ is BiČech sgβ – open set of (X, k_1,k_2), for every BiČech open set V in (Y, v_1,v_2). (i.e.,) f is BiČech sgβ – continuous. Therefore every BiČech continuous is BiČech sgβ – continuous.
Note: The proof is obvious for others.
Remark 3.4: Converse of the above theorem need not be true which can be seen from the following example.
Example 3.5: (a) Let X = {a, b, c}, Y = {1, 2, 3}.
Define a closure operator k_1,k_2 on X by k_1{φ} ={φ}, k_1{a}= k_1{a,b} = {a,b}, k_1{c}= k_1{b,c}={b,c}, k_1{b}= {b}, k_1{a,c}= k_1{X} =X, k_2{φ} ={φ}, k_2{a} = k_2{a,c}={a,c}, k_2{b} = k_2{b,c} = {b,c}, k_2{c} = {c}, k_2{a,b} = k_2{X} =X.
BiČech closed set of X = { X, φ, {b,c}}.
(k_1,k_2) sgβ – closed set of X = { X, φ, {a}, {b}, {c}, {a,c},{b,c}}.
Define a closure operator v_1,v_2 on Y by v_1{φ} ={φ}, v_1{1}= {1}, v_1{3}= {3}, v_1{1,3}={1,3}, v_1{2}= v_1{1,2} = v_1{2,3}=v_1{Y} =Y, v_2{φ} ={φ}, v_2{3} = {3}, v_2{1,2} ={1,2}, v_2{1} = v_2{2} = v_2{1,3} = v_2{2,3}= v_2{Y} =Y.
Biclosed set of Y = { Y, φ, {3}}.
Let f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) be defined by f(a) = 2, f(b) = 3, f(c) = 1. Then f is (k_1,k_2) sgβ – continuous but not continuous. Since for the biclosed set {3} in Y, the inverse image f^{-1}{3} = {b} is not BiČech closed set in X.
Example 3.6: (b)Let X = {a, b, c}, Y = {p, q, r}. Define a closure operator k_1,k_2 on X by k_1{φ}={φ}, k_1{a}= k_1{a,c} = {a,c}, k_1{b}= k_1{b,c}={b,c}, k_1{c}= {c}, k_1{a,b}= k_1{X} =X, k_2{φ} ={φ}, k_2{b} = k_2{c}=k_2{b,c} = {b,c}, k_2{a} = {a}, k_2{a,b} = k_2{a,c} = k_2{X} =X.
g- biclosed set of X = { X, φ,{a},{c},{a, c}, {b, c}}.
(k_1,k_2) sgβ – closed set of X = { X, φ, {a}, {b}, {c},{a,b}, {a,c},{b,c}}.
Define a closure operator v_1,v_2 on Y by v_1{φ} ={φ}, v_1{p}={p}, v_1{q}={q}, v_1{p,q}= {p,q}, v_1{r}=v_1{p,r}=v_1{q,r}=v_1{Y}=Y,v_2{φ}={φ}, v_2{p} = v_2{q} = v_2{p,q} ={p,q}, v_2{r} = v_2{p, r} = v_2{q, r}=v_2{Y}=Y. Biclosed set of Y = { Y, φ, {p,q}}.Let f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) be defined by f(a) = q, f(b) = p, f(c) = r. Then f is (k_1,k_2) sgβ – continuous but not g-continuous. Since for the biclosed set {p,q} in Y, the inverse image f^{-1}{p,q} = {a,b} is not g-biclosed set in X.

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Example 3.7: (c)Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$.Define a closure operator k_1 , k_2 on X by $k_1\{\varphi\} =\{\varphi\}$, $k_1\{c\}= k_1\{a,c\} = \{a,c\}$, $k_1\{a\}= \{a\}$, $k_1\{b\}= k_1\{b,c\}= k_1\{a,b\}= k_1\{X\} =X$, $k_2\{\varphi\} =\{\varphi\}$, $k_2\{a\} = \{a\}$, $k_2\{b\}= k_2\{a,b\} = \{a,b\}$, $k_2\{c\} = k_2\{a,c\}=\{a,c\}$, $k_2\{b,c\} = k_2\{X\} =X$.

w-biclosed set of $X = \{ X, \varphi, \{a\},\{a, b\}, \{a, c\}\}$.

(k_1,k_2) sgβ – closed set of $X = \{ X, \varphi, \{a\}, \{b\}, \{c\}, \{a,c\},\{a,b\}\}$.

Define a closure operator v_1,v_2 on Y by $v_1\{\varphi\} =\{\varphi\}$, $v_1\{1\}= \{1\}$, $v_1\{2\} = v_1\{1,2\} = \{1,2\}$, $v_1\{3\}= v_1\{1,3\}=\{1,3\}$, $v_1\{2,3\}=v_1\{Y\} =Y$, $v_2\{\varphi\} =\{\varphi\}$, $v_2\{1\} = \{1\}$, $v_2\{2\} = v_2\{3\}= v_2\{2,3\} = \{2,3\}$, $v_2\{1,3\}= v_2\{1,2\} =v_2\{Y\} =Y$.

Biclosed set of $Y = \{ Y, \varphi, \{1\}\}$. Let $f: (X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ be defined by $f(a) = 3$, $f(b) = 1$, $f(c) = 2$. Then f is (k_1,k_2) sgβ – continuous but not w-continuous. Since for the biclosed set $\{1\}$ in Y, the inverse image $f^{-1}\{1\} = \{b\}$ is not w-biclosed set in X.

Example 3.8: (d) Let $X = \{1,2,3\}$, $Y = \{a,b,c\}$. Define a closure operator k_1,k_2 on X by $k_1\{\varphi\} =\{\varphi\}$, $k_1\{2\} = \{2\}$, $k_1\{3\} = k_1\{1,3\} = \{1,3\}$, $k_1\{1\} = k_1\{1,2\}= k_1\{2,3\} = k_1\{X\} =X$, $k_2\{\varphi\} =\{\varphi\}$, $k_2\{2\}= k_2\{2,3\} = \{2,3\}$, $k_2\{3\}= \{3\}$, $k_2\{1\}= k_2\{1,3\}=\{1,3\}$, $k_2\{1,2\}= k_2\{X\} =X$.

BiČech J- closed set of $X = \{ X, \varphi, \{1\},\{1,2\}, \{1,3\}, \{2,3\}\}$.

(k_1,k_2) sgβ – closed set of $X = \{ X, \varphi, \{1\},\{2\},\{3\},\{1,2\}, \{1,3\}, \{2,3\}\}$.

Define a closure operator v_1,v_2 on Y by $v_1\{\varphi\} =\{\varphi\}$, $v_1\{b\}= v_1\{c\}= v_1\{b,c\} = \{b,c\}$, $v_1\{a\}=\{a\}$,$v_1\{a,b\}= v_1\{a,c\}=v_1\{Y\} =Y$, $v_2\{\varphi\} =\{\varphi\}$, $v_2\{a\} = \{a\}$, $v_2\{b\} = v_2 \{a,b\}=\{a,b\}$, $v_2\{c\} = v_2\{a,c\} =\{a,c\}$, $v_2\{b,c\}= v_2\{Y\} =Y$.

Biclosed set of $Y = \{ Y, \varphi, \{a\}\}$.Let $f: (X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ be defined by $f(1) = c$, $f(2) = a$, $f(3) = b$.Then f is (k_1,k_2) sgβ – continuous but not J-continuous. Since for the biclosed set $\{a\}$ in Y, the inverse image $f^{-1}\{a\} = \{2\}$ is not in J -biclosed set in X.

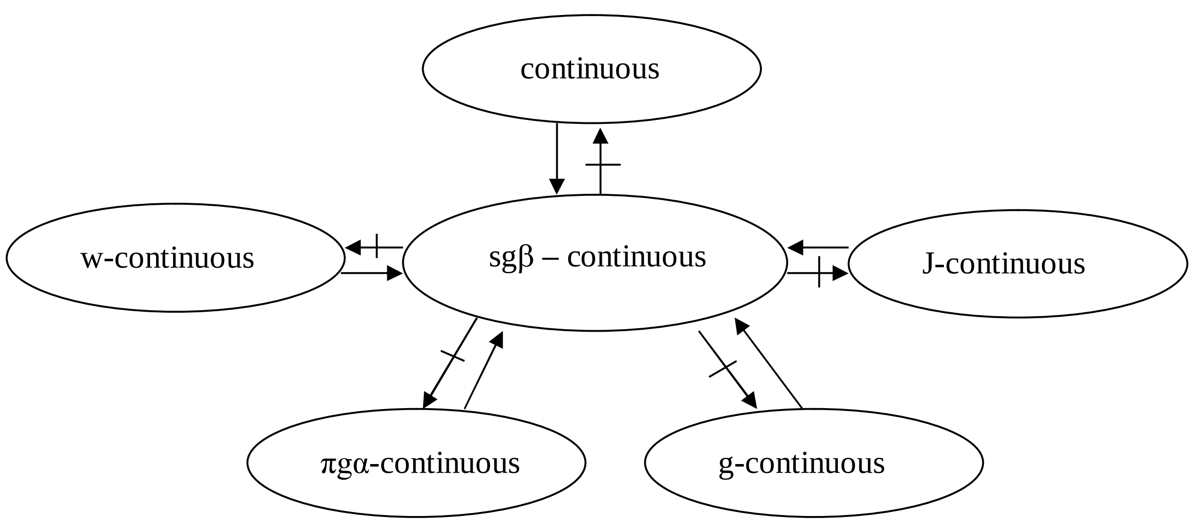
Example 3.9: (e) Let $X = \{a, b, c\}$, $Y = \{p,q,r\}$. Define a closure operator k_1,k_2 on X by $k_1\{\varphi\} =\{\varphi\}$, $k_1\{a\} = \{a\}$, $k_1\{b\} = k_1\{a,b\} = \{a,b\}$, $k_1\{c\} = k_1\{a,c\}=\{a,c\}$, $k_1\{b,c\} = k_1\{X\}=X$, $k_2\{\varphi\} =\{\varphi\}$, $k_2\{b\}= k_2\{c\}= k_2\{b,c\} = \{b,c\}$, $k_2\{a\}= \{a\}$, $k_2\{a,c\}= k_2\{a,b\}= k_2\{X\} =X$.

BiČech $\pi g\alpha$ – closed set of $X = \{ X, \varphi, \{a\},\{a, b\}, \{a, c\}, \{b, c\}\}$.

(k_1,k_2) sgβ – closed set of $X = \{ X, \varphi, \{a\}, \{b\}, \{c\}, \{a,c\},\{b,c\},\{a,b\}\}$.

Define a closure operator v_1,v_2 on Y by $v_1\{\varphi\} =\{\varphi\}$, $v_1\{p\} = \{p\}$, $v_1\{q\} = v_1\{r\}= v_1\{q,r\} = \{q,r\}$, $v_1\{p,q\}= v_1\{q,r\} = v_1\{Y\} =Y$, $v_2\{\varphi\} =\{\varphi\}$, $v_2\{p\} = \{p\}$, $v_2\{q\} = v_2\{p,q\}=\{p,q\}$, $v_2\{r\}=v_2\{p,r\} = \{p,r\}$, $v_2\{q,r\} = v_2\{Y\} =Y$.

Biclosed set of $Y = \{ Y, \varphi, \{p\}\}$.Let $f: (X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ be defined by $f(a) = r$, $f(b) = p$, $f(c) = q$. Then f is (k_1,k_2) sgβ – continuous but not $\pi g\alpha$-continuous. Since for the biclosed set $\{p\}$ in Y, the inverse image $f^{-1}\{p\} = \{b\}$ is not in $\pi g\alpha$ -biclosed set in X.



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Proposition 3.10: Let (X, k_1,k_2) and (Y, v_1,v_2) be biclosure spaces and let f: $(X, k_1,k_2) \longrightarrow (Y,v_1,v_2)$ be a map. Then f is BiČech sgβ – continuous if and only if the inverse image of every BiČech closed subset of (Y, v_1,v_2) is BiČech – sgβ closed in (X, k_1,k_2).

Proof: Let F be BiČech closed subset in (Y, v_1,v_2). Then Y – F is BiČech open in (Y, v_1,v_2). Since f is BiČech sgβ – continuous, $f^{-1}(Y - F)$ is BiČech sgβ – open. But $f^{-1}(Y - F) = X - f^{-1}(F)$ thus $f^{-1}(F)$ is BiČech sgβ – closed in (X, k_1,k_2). Conversely let G be an BiČech open subset in (Y, v_1,v_2). Then Y – G is BiČech closed in (Y, v_1,v_2). Since the inverse image of each BiČech closed subset in (Y, v_1,v_2) is BiČech sgβ – closed in (X, k_1,k_2). We have $f^{-1}(Y - G)$ is to be BiČech sgβ – closed in (X, k_1,k_2). But $f^{-1}(Y - G) = X - f^{-1}(G)$. Thus $f^{-1}(G)$ is BiČech sgβ – open. Therefore f is BiČech sgβ – continuous.

Remark 3.11: The composition of two BiČech sgβ– continuous need not be BiČech sgβ– continuous.

Definition 3.12: A Biclosure space (X, k_1,k_2) is said to be a T_d – space if every BiČech sgβ– open set in (X,k_1,k_2) is BiČech open.

Proposition 3.13: Let (X, k_1,k_2) and (Z, w_1,w_2) be biclosure spaces and (Y, v_1,v_2) be a T_d – space. If f: $(X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ and g: $(Y, v_1,v_2) \longrightarrow (Z, w_1,w_2)$ are BiČech sgβ – continuous, then g o f is BiČech sgβ – continuous.

Proof: Let H be BiČech open in (Z, w_1,w_2). Since g is BiČech sgβ– continuous, $g^{-1}(H)$ is BiČech sgβ – open in (Y, v_1,v_2). But (Y, v_1,v_2) is a T_d – space, hence $g^{-1}(H)$ is BiČech open in (Y, v_1,v_2). Thus $f^{-1}(g^{-1}(H)) = (g \text{ o } f)^{-1}(H)$ is BiČech sgβ – open in (X, k_1,k_2). Therefore, g o f is BiČech sgβ – continuous.

Proposition 3.14: Let (X, k_1,k_2), (Y, v_1,v_2) and (Z, w_1,w_2) be biclosure spaces.If f: $(X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ is BiČech sgβ – continuous and g: $(Y, v_1,v_2) \longrightarrow (Z, w_1,w_2)$ is continuous then g o f is BiČech sgβ – continuous.

Proof: Let H be an BiČech open subset of (Z, w_1,w_2). Since g is continuous, $g^{-1}(H)$ is BiČech open in (Y, v_1,v_2). Since f is BiČech sgβ – continuous, $f^{-1}(g^{-1}(H))$ is BiČech sgβ – open in (X, k_1,k_2). But $f^{-1}(g^{-1}(H)) = (g \text{ o } f)^{-1}(H)$. Therefore, g o f is BiČech sgβ – continuous.

IV. BIČECH sgβ – IRRESOLUTE FUNCTION

Definition 4.1: Let (X, k_1,k_2) and (Y, v_1,v_2) be biclosure space and a map f: $(X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ is called BiČech sgβ – irresolute, if $f^{-1}(G)$ is BiČech sgβ – open set (closed set) in (X, k_1,k_2) for every BiČech sgβ – open set (closed set) G in (Y, v_1,v_2).

Proposition 4.2: Every BiČech sgβ – irresolute map is BiČech sgβ – continuous.

Proof: Assume that f is BiČech sgβ – irresolute. Let V be a BiČech closed set in Y. Every BiČech closed set is BiČech sgβ – closed. That implies V be a BiČech sgβ – closed set in Y. Since f is BiČech sgβ – irresolute, $f^{-1}(V)$ is BiČech sgβ – closed set in X. Thus $f^{-1}(V)$ is BiČech sgβ – closed set in X, ∀ BiČech closed set V in Y. That implies f is BiČech sgβ – continuous.

Remark 4.3: The converse is not true as can be seen from the following example:

Example 4.4: Let X = {a,b,c} and Y = {1,2,3}. Define a closure operator k_1, k_2 on X by $k_1\{\varphi\} = \{\varphi\}$, $k_1\{a\} = k_1\{a,c\} = \{a,c\}$, $k_1\{b\} = k_1\{b,c\} = \{b,c\}$, $k_1\{c\} = \{c\}$, $k_1\{a,b\} = k_1\{X\} = X$, $k_2\{\varphi\} = \{\varphi\}$, $k_2\{b\} = \{b\}$, $k_2\{a\} = k_2\{a,b\} = \{a,b\}$,
$k_2\{c\} = k_2\{b,c\} = \{b,c\}$, $k_2\{a,c\} = k_2\{X\} = X$.

Biclosed set of X = { X, φ, {b,c}}.

BiČech sgβ – closed set of X = { X, φ, {a}, {b}, {c}, {a, b}, {b, c}}.

Define a closure operator v_1,v_2 on Y by $v_1\{\varphi\} = \{\varphi\}$, $v_1\{2\} = v_1\{1,2\} = \{1,2\}$, $v_1\{3\} = v_1\{1,3\} = \{1,3\}$, $v_1\{1\} = \{1\}$, $v_1\{2,3\} = v_1\{Y\} = Y$, $v_2\{\varphi\} = \{\varphi\}$, $v_2\{1\} = \{1\}$, $v_2\{2\} = v_2\{3\} = v_2\{2,3\} = \{2,3\}$,
$v_2\{1,2\} = v_2\{1,3\} = v_2\{Y\} = Y$.

Biclosed set of Y = { Y, φ, {1}}.

BiČech sgβ – closed set of Y ={ Y, ϕ, {1}, {2}, {3},{1, 2},{1,3},{2,3}}.

Define a function f: $(X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ such that f(a) = 3, f(b) = 2, f(c) = 1.Here f is BiČech sgβ – continuous.
$f^{-1}\{1,3\} = \{a,c\}$ is not BiČech sgβ – closed in (X, k_1,k_2). Therefore f is not BiČech sgβ – irresolute.

Proposition 4.5: Let (X, k_1,k_2) and (Y, v_1,v_2) be biclosure spaces and f: $(X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ be a map. Then f is BiČech sgβ – irresolute if and only if $f^{-1}(B)$ is BiČech sgβ – closed in (X, k_1,k_2) whenever B is BiČech sgβ – closed in (Y, v_1,v_2).

Proof: Suppose B be a BiČech sgβ – closed subset of (Y, v_1,v_2).Then Y – B is BiČech sgβ – open in (Y, v_1,v_2).
Since f: $(X, k_1,k_2) \longrightarrow (Y, v_1,v_2)$ is BiČech sgβ – irresolute, $f^{-1}(Y - B)$ is BiČech sgβ – open in (X, k_1,k_2).

But, $f^{-1}(Y-B) = X - f^{-1}(B)$, so that $f^{-1}(B)$ is BiČech sgβ – closed in (X, k_1,k_2). Conversely, Let A be a BiČech sgβ – open subset in (Y, v_1,v_2).Then Y – A is BiČech sgβ – closed in (Y, v_1,v_2).By the assumption, $f^{-1}(Y-A)$ is BiČech sgβ – closed in (X, k_1,k_2). But $f^{-1}(Y-A) = X - f^{-1}(A)$. Thus $f^{-1}(A)$ is BiČech sgβ – open in (X,k_1,k_2). Therefore, f is BiČech sgβ – irresolute.

Proposition 4.6: Let (X, k_1,k_2), (Y, v_1,v_2) and (Z, w_1,w_2) be biclosure spaces. If f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) is a BiČech sgβ – irresolute map and g: (Y, v_1,v_2) ⟶ (Z, w_1,w_2) is a BiČech sgβ – continuous map, then the composition g o f: (X, k_1,k_2) ⟶ (Z, w_1,w_2) is BiČech sgβ – continuous.

Proof: Let G be an open subset of (Z, w_1,w_2). Then $g^{-1}(G)$ is a BiČech sgβ-open in (Y,v_1,v_2) as g is BiČech sgβ – continuous. Hence, $f^{-1}(g^{-1}(G))$ is BiČech sgβ – open in (X, k_1,k_2) because f is BiČech sgβ – irresolute. Thus gof is BiČech sgβ – continuous.

Proposition 4.7: Let (X, k_1,k_2), (Y, v_1,v_2) and (Z, w_1,w_2) be biclosure spaces.If f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) and g: (Y,v_1,v_2) ⟶ (Z, w_1,w_2) are BiČech sgβ – irresolute, then g o f: (X, k_1,k_2) ⟶ (Z, w_1,w_2) is BiČech sgβ – irresolute.

Proof: Let F be BiČech sgβ – open set in (Z, w_1,w_2). As g is BiČech sgβ – irresolute, $g^{-1}(F)$ is BiČech sgβ – open in (Y, v_1,v_2). Since, f is BiČech sgβ – irresolute. $f^{-1}(g^{-1}(F))$ is BiČech sgβ – open in (X, k_1,k_2) implies $(g \circ f)^{-1}F = (f^{-1}g^{-1}(F))$ is BiČech sgβ – open in (X, k_1,k_2). Hence g o f is BiČech sgβ – irresolute.

Proposition 4.8: Let (X, k_1,k_2) and (Z, w_1,w_2) be biclosure spaces and (Y, v_1,v_2) be a T_d – space. If f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) be a BiČech sgβ –continuous map and g: (Y, v_1,v_2) ⟶(Z,w_1,w_2) is a BiČech sgβ – irresolute, then the composition g o f: (X, k_1,k_2) ⟶ (Z, w_1,w_2) is BiČech sgβ – irresolute.

Proof: Let V be BiČech sgβ – open in Z. Since g is BiČech sgβ – irresolute, $g^{-1}(V)$ is BiČech sgβ – open in Y. As Y is a T_d – space, $g^{-1}(V)$ is BiČech open in Y. Since f is BiČech sgβ-continuous , $f^{-1}(g^{-1}(V))$ is BiČech sgβ – open in X. Thus $(g \circ f)^{-1}(V)$ is BiČech sgβ – open in X. Hence g o f is BiČech sgβ – irresolute.

V. TOTALLY BIČECH sgβ – CONTINUOUS FUNCTION

Definition 5.1: Let (X, k_1,k_2) and (Y, v_1,v_2) be a BiČech closure space. A function f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) is called Totally BiČech continuous if the inverse image of every biopen subset of (Y, v_1,v_2) is a BiČech clopen subset in (X, k_1,k_2).

Definition 5.2: Let (X, k_1,k_2) and (Y, v_1,v_2) be a BiČech closure space. A function f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) is called Totally BiČech sgβ-continuous if the inverse image of every biopen subset of (Y, v_1,v_2) is a BiČech sgβ - clopen subset in (X, k_1,k_2).

Theorem 5.3: A function f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) is totally BiČech sgβ-continuous if and only if the inverse image of every biclosed subset of (Y, v_1,v_2) is a BiČech sgβ - clopen subset in (X, k_1,k_2).

Proof: Assume that f is totally BiČech sgβ-continuous. Let A be any biclosed subset in Y. Then A^c is a biopen subset in Y. Since f is totally BiČech sgβ-continuous. Thus $f^{-1}(A^c)$ is BiČech sgβ - clopen subset in (X, k_1,k_2). But $f^{-1}(A^c)$ = X - $f^{-1}(A)$ and so $f^{-1}(A)$ is both BiČech sgβ - closed subset and BiČech sgβ-open subset in X. Conversely, let G be a biopen subset in Y.Then G^c is biclosed subset in X. By assumption $f^{-1}(G^c)$ is BiČech sgβ - clopen subset in (X, k_1,k_2). But $f^{-1}(G^c)$ = X - $f^{-1}(G)$ and so $f^{-1}(G)$ is both BiČech sgβ – closed and BiČech sgβ-open. Hence $f^{-1}(G)$ is BiČech sgβ - clopen subset in (X, k_1,k_2). Therefore f is totally BiČech sgβ-continuous.

Theorem 5.4: Every totally BiČech sgβ-continuous f: (X, k_1,k_2) ⟶ (Y,v_1,v_2) is BiČech sgβ – continuous function.

Proof: Let A be any biopen subset in Y. Since f is totally BiČech sgβ-continuous. Thus $f^{-1}(A)$ is BiČech sgβ - clopen subset in (X, k_1,k_2).(i.e) $f^{-1}(A)$ is both BiČech sgβ - closed subset and BiČech sgβ-open subset in X. Thus $f^{-1}(A)$ is BiČech sgβ-open subset in (X, k_1,k_2). Therefore f is a sgβ-continuous.

Remark 5.5: The converse is not true as can be seen from the following example:

Example 5.6: Let X = {a,b,c}, Y = {1,2,3}. Define a closure operator k_1,k_2 on X by k_1{φ}={φ},
k_1{a}= k_1{a,c} = {a,c}, k_1{b}= k_1{b,c}={b,c}, k_1{c}= {c}, k_1{a,c}= k_1{X} =X, k_2{φ} ={φ},
k_2{b} ={b}, k_2{c}=k_2{b,c} = {b,c}, k_2{a} = k_2{a,b} ={a,b}, k_2{a,c} = k_2{X} =X.
Biclosed set of X = { X, φ, {b, c}}.
(k_1,k_2) sgβ – closed set of X = { X, φ, {a}, {b}, {c},{a,b},{b,c}}.
Define a closure operator v_1,v_2 on Y by v_1{φ} ={φ}, v_1{1}= {1}, v_1{2}=v_1{1,2} = {1,2}, v_1{3}= v_1{1,3}={1,3},
v_1{2,3}=v_1{Y} =Y, v_2{φ} ={φ}, v_2{1} = {1}, v_2{2} = v_2{3}= v_2{2,3} = {2,3},
v_2{1,3}= v_2{1,2} =v_2{Y} =Y.

Biclosed set of Y = { Y, φ, {1}}. Biopen set of Y = { Y, φ, {2,3}}.

Let f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) be defined by f(a) = 2, f(b) = 1, f(c) = 3. Then f is (k_1,k_2) sgβ – continuous but not totally BiČech sgβ-continuous. Since for the biopen set {2,3} in Y, the inverse image f^{-1}{2,3} = {a,c} is not totally BiČech sgβ-clopen set in X.

Theorem 5.7: Let f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) and g: (Y, v_1,v_2) ⟶ (Z, w_1,w_2) be function. Then g o f: (X, k_1,k_2) ⟶ (Z, w_1,w_2)

(i) If f is BiČech sgβ -irresolute and g is totally BiČech sgβ -continuous then gof is totally BiČech sgβ-continuous.

(ii) If f is totally BiČech sgβ -continuous and g is BiČech continuous then gof is totally BiČech sgβ -continuous.

Proof: (i) Let U be a BiČech open set in Z. Since g is totally BiČech sgβ -continuous, g^{-1}(U) is BiČech sgβ -clopen in Y. Since f is BiČech sgβ -irresolute, f^{-1}(g^{-1}(U)) is BiČech sgβ -open and BiČech sgβ -closed in X. Since gof^{-1}(U) =f^{-1}(g^{-1}(U)), gof is totally BiČech sgβ -continuous.

(ii) Let U be a BiČech open set in Z. Since g is BiČech continuous, g^{-1}(U) is BiČech open in Y. Also since f is totally BiČech sgβ -continuous, f^{-1}(g^{-1}(U)) is BiČech sgβ -clopen in X. Hence gof is totally BiČech sgβ -continuous.

VI. SLIGHTLY BIČECH sgβ – CONTINUOUS FUNCTION

Definition 6.1: Let (X, k_1,k_2) and (Y, v_1,v_2) be a BiČech closure space. A function f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) is called Slightly BiČech -continuous if the inverse image of every clopen subset of (Y, v_1,v_2) is a BiČech-open subset in (X, k_1,k_2).

Definition 6.2: Let (X, k_1,k_2) and (Y, v_1,v_2) be a BiČech closure space. A function f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) is called Slightly BiČech sgβ -continuous if the inverse image of every clopen subset of (Y, v_1,v_2) is a BiČech sgβ -open subset in (X, k_1,k_2).

Theorem 6.3: Every BiČech sgβ-continuous f: (X, k_1,k_2) ⟶ (Y,v_1,v_2) is Slightly BiČech sgβ – continuous.

Proof: Let f: (X, k_1,k_2) ⟶ (Y,v_1,v_2) be a BiČech sgβ-continuous function. Let U be a clopen set in Y.Then f^{-1}(U) is BiČech sgβ-open in X and BiČech sgβ-closed in X. Hence f is Slightly BiČech sgβ – continuous.

Theorem 6.4: Every Slightly BiČech continuous is Slightly BiČech sgβ – continuous.

Proof: Let f: (X, k_1,k_2) ⟶ (Y,v_1,v_2) be Slightly BiČech continuous function. Let U be a clopen set in Y.Then f^{-1}(U) BiČech open in X.Since every open set is sgβ-open, f^{-1}(U) BiČech sgβ –open. Hence f is Slightly BiČech sgβ – continuous.

Remark 6.5: The converse is not true as can be seen from the following example:

Example 6.6: Let X = {a,b,c}, Y = {1,2,3}. Define a closure operator k_1,k_2 on X by k_1{φ}={φ},
k_1{a}= k_1{a,c} = {a,c}, k_1{b}= k_1{b,c}={b,c}, k_1{c}= {c}, k_1{a,c}= k_1{X} =X, k_2{φ} ={φ},
k_2{φ} ={φ}, k_2{b} ={b}, k_2{c}=k_2{b,c} = {b,c}, k_2{a} = k_2{a,b} ={a,b}, k_2{a,c} = k_2{X} =X.

Biclosed set of X={X, φ,{b,c}}. Biopen set of X={X, φ,{a}}.

(k_1,k_2) sgβ – closed set of X = { X, φ, {a}, {b}, {c},{a,b},{b,c}}.

Define a closure operator v_1,v_2 on Y by v_1{φ} ={φ}, v_1{1}= v_1{1,3} = {1,3}, v_1{2}={2},
v_1{3}= v_1{2,3} =v_1{1,2}=v_1{Y} =Y, v_2{φ} ={φ}, v_2{2} = {2}, v_2{3} = {3}, v_2{1}= v_2{1,3} ={1,3},
v_2{2,3} ={2,3},v_2{1,2} =v_2{Y} =Y.

Biclopen set of Y = { Y, φ, {2},{1,3}}.

Let f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) be defined by f(a) = 3, f(b) = 1, f(c) = 2. Then f is slightly BiČech sgβ – continuous but not slightly BiČech continuous. Since for the clopen set {1,3} in Y, the inverse image f^{-1}{1,3} = {a,b} is not slightly BiČech open set in X.

Theorem 6.7: Let f: (X, k_1,k_2) ⟶ (Y, v_1,v_2) and g: (Y, v_1,v_2) ⟶ (Z, w_1,w_2) be function.

(i) If f is BiČech sgβ -irresolute and g is slightly BiČech sgβ -continuous then gof is slightly BiČech sgβ-continuous.

(ii) If f is BiČech sgβ-irresolute and g is BiČech sgβ -continuous then gof is slightly BiČech sgβ -continuous.

(iii) If f is BiČech sgβ-continuous and g is slightly BiČech continuous then gof is slightly BiČech sgβ -continuous.

Proof: (i) Let U be a BiČech clopen set in Z. Since g is slightly BiČech sgβ -continuous, g^{-1}(U) is BiČech sgβ -open in Y. Since f is BiČech sgβ -irresolute, f^{-1}(g^{-1}(U)) is BiČech sgβ -open in X. Since gof^{-1}(U) =f^{-1}(g^{-1}(U)), gof is slightly BiČech sgβ -continuous.

(ii) Let U be a BiČech clopen set in Z. Since g is BiČech sgβ - continuous, g^{-1}(U) is BiČech sgβ -open in Y. Also since f is BiČech sgβ-irresolute, f^{-1}(g^{-1}(U)) is BiČech sgβ-open in X. Hence gof is slightly BiČech sgβ -continuous.

(iii) Let U be a BiČech clopen set in Z. Since g is BiČech continuous, g^{-1}(U) is BiČech open in Y. Also since f is BiČech sgβ-continuous, f^{-1}(g^{-1}(U)) is BiČech sgβ-open in X. Hence gof is slightly BiČech sgβ –continuous.

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